LGIC 010 & PHIL 005
Problem Set 9
Spring Term, 2016
DUE IN CLASS MONDAY, APRIL 25

For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true. Each problem is worth 20 points.

1. $X : \{(\forall x)(\exists y)Lxy, \neg(\exists y)(\forall x)Lxy\}$
   $S : (\forall x)Lxx$

2. $X : \{\neg(\forall x)(\exists y)Lxy, \neg(\exists y)(\forall x)Lxy\}$
   $S : (\forall x)Lxx$

3. For each $n \geq 2$, let T^n be the schema

$$\bigwedge_{1 \leq i < j \leq n} x_i \neq x_j.$$

   $X : \{(\forall x)\neg Lxx, (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)),$
   $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)), (\exists x)(\forall y)\neg Lyx, (\exists x)(\forall y)\neg Lxy,$
   $(\forall x)((\exists y)Lyx \supset (\exists y)(Lyx \wedge (\forall z)\neg(Lyz \wedge Lzx))),$
   $(\forall x)((\exists y)Lxy \supset (\exists y)(Lxy \wedge (\forall z)\neg(Lxz \wedge Lzy)))\} \cup \{T^n \mid n \geq 2\}$
   $S : (\forall x)(\exists y)Lxy$

4. $X : \{(\forall x)(Fx \supset (\exists y)(\neg Fy \wedge (\forall z)(Lxz \equiv y = z))),$
   $(\forall x)(Fx \supset (\exists y)(\neg Fy \wedge (\forall z)(Pxz \equiv y = z))),$
   $(\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y), (\forall x)(\forall y)(\forall z)((Pxz \wedge Pyz) \supset x = y),$
   $(\forall x)(\neg Fx \supset (\exists y)(Fy \wedge Pyx))\}$
   $S : (\forall x)(\neg Fx \supset (\exists y)(Fy \wedge Lyx))$

5. For each $n \geq 3$, let R^n be the schema

$$\neg(\exists x_1)\ldots(\exists x_n)(\bigwedge_{1 \leq i < j \leq n} x_i \neq x_j \wedge \bigwedge_{1 \leq i < n} Lx_ix_{i+1} \wedge Lx_nx_1).$$

   $X : \{(\forall x)\neg Lxx, (\forall x)(\forall y)(Lxy \supset Lyx),$
   $(\forall x)(\exists y)(\exists z)(y \neq z \wedge (\forall w)(Lxw \equiv (w = y \vee w = z))),$
   $(\forall x)(\forall y)(\forall z)(\forall w)(x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee w = z)\} \cup \{R^n \mid n \geq 3\}$
   $S : (\forall x)Lxx$