LGIC 010 & PHIL 005 Problem Set 8 Spring Term, 2010

- 1. Let A be the structure interpreting a single dyadic predicate letter R with $U^A = \{1, 2, 3\}$ and $R^A = \{\langle 1, 2 \rangle\}$.
 - (a) (10 points) List all the automorphisms of A.
 - (b) (10 points) List all sets which are definable in A along with schemata which define them.
- 2. (10 points) Let A be the structure interpreting a single dyadic predicate letter R with $U^A = \{1, 2, 3\}$ and $R^A = \{\langle 1, 2 \rangle\}$. Write down a schema S so that for every structure B, B satisfies S if and only if B is isomorphic to A.
- 3. Let S be the conjunction of the following schemata.

 $\begin{aligned} (\forall x) \neg Rxx \\ (\forall x) (\forall y) (Rxy \supset Ryx) \end{aligned}$

- (a) (20 points) How long a list of pairwise non-isomorphic structures with universe of discourse {1, 2, 3, 4} satisfy the schema S?
- (b) (10 points) How long a list of structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy the schema S?
- (c) (20 points) Give an example of structures A and B such that
 - i. A and B both satisfy S;
 - ii. A is not isomorphic to B;
 - iii. $U^A = U^B = \{1, 2, 3, 4\};$
 - iv. exactly four subsets of $\{1, 2, 3, 4\}$ are definable in A and exactly four subsets of $\{1, 2, 3, 4\}$ are definable in B.
- (d) (20 points) Give an example of a structure A such that
 - i. A satisfies S;
 - ii. $U^A = \{1, 2, 3, 4\};$
 - iii. exactly eight subsets of $\{1, 2, 3, 4\}$ are definable in A.