LGIC 010 & PHIL 005
Problem Set 6
Spring Term, 2019
DUE IN CLASS MONDAY, APRIL 8

We write $\mathbb{Z}^+$ for the set of positive integers $\{1, 2, 3, \ldots\}$. The *spectrum* of a schema S (written $\mathsf{Spec}(S)$) is defined as follows.

$$\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \mathsf{mod}(S, n) \neq \emptyset\}.$$

1. (25 points) Write down a schema S_1 involving only the dyadic predicate letter "L," and the identity predicate such that

   - S_1 implies $(\forall x)Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz)$ and
   - $\mathsf{Spec}(S_1) = \{n \in \mathbb{Z}^+ \mid \text{ for some } i \in \mathbb{Z}^+,\ n = 3i - 1\}$.

2. (25 points) Write down a schema S_2 involving only the tetradic predicate letter "R," the monadic predicate letter "F" and the identity predicate such that

   - S_2 implies
     $(\forall x)(\exists y)(\exists z)(\exists w)Ryzwx \wedge$
     $(\forall x)(\forall y)(\forall u)((Fx \wedge Fy \wedge Fu) \supset (\exists z)(\forall w)(Rxyuw \equiv w = z))$
     and
   - $\mathsf{Spec}(S_2) = \{n \in \mathbb{Z}^+ \mid \text{ for some } i \in \mathbb{Z}^+,\ n = i^3\}$.

3. (25 points) Let S_3 be the conjunction of the following schemata.

- $(\forall g)(\forall x)(\forall y)(Egxy \supset (Px \wedge Py))$
- $(\forall x)(\forall g)((Px) \supset (\exists y)(\forall z)(Egxz \equiv y = z))$
- $(\forall y)(\forall g)(Py \supset (\exists x)Egxy)$
- $(\forall g)(\forall h)((\forall x)(\forall y)(Egxy \leftrightarrow Ehxy) \supset g = h)$
- $(\forall g)(\forall x)(\forall y)(\forall u)(\forall v)((Egxy \wedge Eguv) \supset (\exists h)(Ehxv \wedge Ehuy \wedge$ $(\forall z)((z \neq x \wedge z \neq u) \supset (\forall w)(Egzw \equiv Ehzw))))$

Specify the spectrum of S_3.

$\mathsf{Spec}(S_3) =$

4. (25 points) Let S_4 be the conjunction of the following schemata.

- $(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fx \wedge Gy))$
- $(\forall x)(\forall y)((Fx \wedge Gy) \supset (\exists z)(\forall w)(Hxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Hxyz$
- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Hvwz \wedge Hxyz) \supset (v = x \wedge w = y))$
- $(\exists x)(\exists y)(x \neq y \wedge (\forall z)(Gz \equiv (x = z \vee y = z)))$

Specify the spectrum of S_4.

$\mathsf{Spec}(S_4) =$