LGIC 010 & PHIL 005
Problem Set 6
Spring Term, 2016
DUE IN CLASS MONDAY, MARCH 21

We write $\mathbb{Z}^+$ for the set of positive integers $\{1, 2, 3, \ldots\}$. The *spectrum* of a schema S (written $\mathsf{Spec}(S)$) is defined as follows.

$$\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \mathsf{mod}(S, n) \neq \emptyset\}.$$

1. (25 points) Write down a schema S involving only the dyadic predicate letter "L," and the identity predicate such that

   - S implies $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset \neg Lyx)$ and
   - $\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid \text{ for some } i \in \mathbb{Z}^+ \cup \{0\},\ n = 3i + 1\}$.

2. (25 points) Recall that SLO is the conjunction of the following schemata.

   - $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
   - $(\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$
   - $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx))$

   Write down a schema S involving only the dyadic predicate letter "L," the monadic predicate letter "F," and the identity predicate such that

   - S implies SLO and
   - $\mathsf{Spec}(S) = \{n \in \mathbb{Z}^+ \mid n \text{ is even}\}$.

3. (25 points) Let S_1 be the conjunction of the following schemata.

   - $(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fy \wedge Gz)) \wedge (\forall x)(\forall y)(Fy \supset (\exists z)(\forall w)(Hxyw \equiv w = z))$
   - $(\exists x)(\exists y)(\neg(x = y) \wedge Gx \wedge Gy \wedge (\forall z)(Gz \supset (z = x \vee z = y))) \wedge (\exists x)Fx$
   - $(\forall x)(\forall y)((\forall z)(\forall w)(Hxzw \equiv Hyzw) \supset x = y)$
   - $(\forall x)(Fx \supset (\forall y)(\exists z)((\exists w)(Hyxw \wedge \neg Hzxw) \wedge (\forall v)(\neg(v = x) \supset (\forall u)(Hyvu \equiv Hzvu))))$

   Specify the spectrum of S_1.

   $\mathsf{Spec}(S_1) =$

4. (25 points) Let S_2 be the conjunction of the following schemata.

   - $(\forall x)(\forall y)(\forall z)(Hxyz \supset (Fx \wedge Gy))$
   - $(\forall x)(\forall y)((Fx \wedge Gy) \supset (\exists z)(\forall w)(Hxyw \equiv w = z))$
   - $(\exists x)(\exists y)(Fx \wedge Fy \wedge x \neq y)$
   - $(\exists x)(\exists y)(Gx \wedge Gy \wedge x \neq y)$
   - $(\forall z)(\exists x)(\exists y)Hxyz$
   - $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Hvwz \wedge Hxyz) \supset (v = x \wedge w = y))$

   Specify the spectrum of S_2.

   $\mathsf{Spec}(S_2) =$