LGIC 010 & PHIL 005
Problem Set 5
Spring Term, 2014

1. Let S_1 be the following schema.

   $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\exists y)(\exists z)(y \neq z \wedge (\forall w)(Lxw \equiv (w = y \vee w = z)))$

   (a) (10 points) Specify a structure A_1 of size at least 6 which satisfies S_1, that is, U^{A_1} has at least 6 members and $A_1 \models S_1$.

   $U^{A_1} =$

   $L^{A_1} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5, 6\}$ satisfy S_1?

2. Let S_2 be the following schema.

   $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lxz) \supset y = z) \wedge (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y) \wedge (\forall x)(\exists y)Lxy$

   (a) (10 points) Specify a structure A_2 of size at least 6 which satisfies S_2.

   $U^{A_2} =$

   $L^{A_2} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4, 5, 6\}$ satisfy S_2?

3. Let S_3 be the conjunction of the following two schema.

   - $(\forall x)(\forall y)(\forall z)((Lxy \wedge Lxz) \supset y = z) \wedge (\forall x)(\exists y)Lxy$
   - $(\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y) \wedge (\exists y)(\forall x)\neg Lxy$

   (a) (10 points) Specify a structure A_3 of size at least 3 which satisfies S_3.

   $U^{A_3} =$

   $L^{A_3} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_3?

4. Let S_4 be the following schema.

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y)$$

   (a) (10 points) Specify a structure A_4 of size at least 3 which satisfies S_4.

   $U^{A_4} =$

   $L^{A_4} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_4?

5. Let S_5 be the conjunction of the following four schemata.

- $(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y))$
- $(\forall x)(\forall y)(\forall z)(Rxyz \supset (Fx \wedge Fy))$
- $(\forall x)(\forall y)((Fx \wedge Fy) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))$
- $(\forall z)(\exists x)(\exists y)Rxyz$

(a) (10 points) Specify a structure A_5 of size at least 3 which satisfies S_5.

$U^{A_5} =$

$F^{A_5} =$

$R^{A_5} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_5?