LGIC 010 & PHIL 005
Problem Set 5
Spring Term, 2010

1. Let S_1 be the following schema.

   $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\exists y)(\exists z)(Lyz \wedge (\forall w)(Lxw \equiv (w = y \vee w = z)))$

   (a) (10 points) Specify a structure A_1 of size at least 4 which satisfies S_1, that is, U^{A_1} has at least 4 members and $A_1 \models S_1$.

   $U^{A_1} =$

   $L^{A_1} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_1?

2. Let S_2 be the following schema.

   $(\forall x)\neg Lxx \wedge (\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)(\exists y)(\exists z)(y \neq z \wedge (\forall w)(Lxw \equiv (w = y \vee w = z)))$

   (a) (10 points) Specify a structure A_2 of size at least 4 which satisfies S_2, that is, U^{A_2} has at least 4 members and $A_2 \models S_2$.

   $U^{A_2} =$

   $L^{A_2} =$

   (b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_2?

3. Let S_3 be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z)$$

(a) (10 points) Specify a structure A_3 of size at least 4 which satisfies S_3.

$U^{A_3} =$

$L^{A_3} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_3?

4. Let S_4 be the following schema.

$$(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z) \wedge (\forall y)(\exists x)Lxy \wedge \neg(\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$$

(a) (10 points) Specify a structure A_4 of size at least 4 which satisfies S_4.

$U^{A_4} =$

$L^{A_4} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_4?

5. Let S_5 be the following schema.

$$(\forall x)Lxx \wedge (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz)) \wedge (\forall x)(\forall y)(Lxy \supset Lyx)$$

(a) (10 points) Specify a structure A_5 of size at least 4 which satisfies S_5.

$U^{A_5} =$

$L^{A_5} =$

(b) (10 points) How many structures with universe of discourse $\{1, 2, 3, 4\}$ satisfy S_5?