1. (25 points) The *size* of a finite set X is the number of members of X. We say a set of integers X of size $n \geq 1$ is *good* if and only if it does not contain a nonempty subset Y, the sum of whose members is divisible by n. Is there a good set of integers of size at least 1? If so, give an example of a good set of minimal size. Explain your answer. (By the way, the sum of the members of a set $X = \{k\}$ of size 1 is k.)

2. (25 points) Brit hosted a gala attended by 100 people (inclusive of Brit). The day after the event, Brit learned that all 99 guests had danced with someone, but that no two of them had had the same number of different dance partners. (Remember, it's a gala, not a rave – nobody can be their own dance partner!) How many different dance partners did Brit have? Explain your answer.

3. (25 points) How many truth assignments to the sentence letters $p_1, \ldots, p_{10}, q_1, \ldots, q_{10}$ satisfy the following schema?

$$(p_1 \supset q_1) \vee (p_2 \supset q_2) \vee \ldots \vee (p_9 \supset q_9) \vee (p_{10} \supset q_{10})$$

4. (25 points) Recall that $\oplus$ represents exclusive disjunction. How many truth assignments to the sentence letters $p_1, \ldots, p_5$ satisfy the following schema?

$$(p_1 \oplus p_2) \wedge (p_2 \oplus p_3) \wedge (p_3 \oplus p_4) \wedge (p_4 \oplus p_5)$$