1. (30 points) Test the following schemata for validity.

   (a) $(p \equiv q) \vee ((q \equiv r) \vee (p \equiv r))$

   (b) $((p \supset r) \wedge (q \supset r)) \supset ((p \vee q) \supset r)$

   (c) $(p \supset q) \vee (q \supset p)$

2. (50 points) In each case, determine whether the first schema implies the second. (Recall that "$\oplus$" represents exclusive disjunction.)

   (a) $p \oplus p$ $\neg p$

   (b) $(\neg p \supset \neg q)$ $(p \supset q)$

   (c) $(\neg p \supset \neg q)$ $(q \supset p)$

   (d) $(p \vee q) \wedge r$ $p \vee (q \wedge r)$

   (e) $p \vee (q \wedge r)$ $(p \vee q) \wedge r$

3. (20 points) Let S be the following schema.

$$((((((p_1 \oplus p_2) \oplus p_3) \oplus p_4) \oplus p_5) \oplus p_6) \oplus p_7)$$

   (a) How many truth assignments to the seven sentence letters $p_1, \ldots, p_7$ satisfy the schema S?

   (b) Call a truth assignment good just in case it assigns exactly 4, 5, or 6 distinct sentence letters the value $\top$. How many good truth assignments to the seven sentence letters $p_1, \ldots, p_7$ satisfy the schema S?