PRINT NAME: ___________________________________

LGIC 010 & PHIL 005
Practice Final Examination
Spring Term, 2012

1. Let S be the schema

$$(\forall x)(\forall y)(Lxy \supset Lyx) \wedge (\forall x)\neg Lxx.$$

   (a) (10 points) How long a list of distinct structures with universe of discourse $\{1,2,3,4\}$ satsify the schema S? 64

   (b) (10 points) How long a list of pairwise nonisomorphic structures with universe of discourse $\{1,2,3,4\}$ satsify the schema S? 11

   (c) (10 points) Give an example of a structure A with the following properties:

   - $A \models S$;
   - $U^A = \{1,2,3,4\}$;
   - A has exactly two automorphisms;
   - exactly eight subsets of $\{1,2,3,4\}$ are definable in A.

   $L^A = \{\langle 1,2\rangle, \langle 2,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle\}$

2. (70 pts.) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\exists x)Px \wedge (\exists x)Qx\}$
   $S : (\exists x)(Px \wedge Qx)$

   $A : U^A = \{1, 2\}$

   $P^A = \{1\}$

   $Q^A = \{2\}$

Deduction

(b) $X : \{(\forall x)(Px \land Qx)\}$
$S : (\forall x)Px \land (\forall x)Qx$

$B : U^B =$

$P^B =$

$Q^B =$

Deduction

$\{1\}$	(1)	$(\forall x)(Px \land Qx)$	P
$\{1\}$	(2)	$Px \land Qx$	(1) UI
$\{1\}$	(3)	Px	(2) TF
$\{1\}$	(4)	Qx	(2) TF
$\{1\}$	(5)	$(\forall x)Px$	(3) UG
$\{1\}$	(6)	$(\forall x)Qx$	(4) UG
$\{1\}$	(7)	$(\forall x)Px \land (\forall x)Qx$	$(5,6)$ TF

(c) $X : \{(\forall x)((\exists y)Lxy \supset (\forall z)Lzx), (\exists x)(\exists y)Lxy\}$
$S : (\forall u)(\forall z)Lzu$

$C : U^C =$

$L^C =$

Deduction

$\{1\}$	(1) $(\exists x)(\exists y)Lxy$	P
$\{1, 2\}$	(2) $(\exists y)Lwy$	(1)w EII
$\{3\}$	(3) $(\forall x)((\exists y)Lxy \supset (\forall z)Lzx)$	P
$\{3\}$	(4) $(\exists y)Lwy \supset (\forall z)Lzw$	(3) UI
$\{1, 2, 3\}$	(5) $(\forall z)Lzw$	(2)(4) TF
$\{1, 2, 3\}$	(6) Luw	(5) UI
$\{1, \not2, 3\}$	(7) $(\exists y)Luy$	(6) EG; $\{2\}$ EIE
$\{3\}$	(8) $(\exists y)Luy \supset (\forall z)Lzu$	(3) UI
$\{1, 3\}$	(9) $(\forall z)Lzu$	(7)(8) TF
$\{1, 3\}$	(10) $(\forall u)(\forall z)Lzu$	(9) UG

(d) $X : \{(\exists x)(\forall y)Rxy\}$
$S : (\forall x)(\exists y)Rxy$

$D : U^D = \{1, 2\}$

$R^D = \{\langle 1, 1\rangle, \langle 1, 2\rangle\}$

Deduction

(e) $X: \{(\forall x)\neg Lxx, (\forall x)(\forall y)(Lxy \supset Lyx),$
$(\forall x)(\exists y)(\exists z)(Lyz \wedge (\forall w)(Lxw \equiv (w = y \vee w = z))),$
$(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y)),$
$(\forall x)(\forall y)(\forall z)(Rxyz \supset (Px \wedge Py)), (\forall z)(\exists x)(\exists y)Rxyz,$
$(\forall x)(\forall y)((Px \wedge Py) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))\}$
$S: (\exists x)(\exists y)(Px \wedge Py \wedge Lxy)$

$E: U^E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$L^E = \{\langle 1,2\rangle, \langle 2,1\rangle, \langle 1,3\rangle, \langle 3,1\rangle, \langle 2,3\rangle, \langle 3,2\rangle, \langle 4,5\rangle, \langle 5,4\rangle, \langle 4,6\rangle, \langle 6,4\rangle, \langle 5,6\rangle, \langle 6,5\rangle,$
$\langle 7,8\rangle, \langle 8,7\rangle, \langle 7,9\rangle, \langle 9,7\rangle, \langle 8,9\rangle, \langle 9,8\rangle\}$

$P^E = \{1, 4, 7\}$

$R^E = \{\langle 1,1,1\rangle, \langle 1,4,2\rangle, \langle 1,7,3\rangle, \langle 4,1,4\rangle, \langle 4,4,5\rangle, \langle 4,7,6\rangle, \langle 7,1,7\rangle, \langle 7,4,8\rangle, \langle 7,7,9\rangle\}$

Deduction

(f) $X : \{(\exists x)(\forall y)(Fy \equiv x = y)\}$
$S : (\exists x)Fx \wedge (\forall x)(\forall y)((Fx \wedge Fy) \supset x = y)$

$F : U^F =$

$F^F =$

Deduction

$\{1\}$	(1) $(\exists x)(\forall y)(Fy \equiv x = y)$	P
$\{1, 2\}$	(2) $(\forall y)(Fy \equiv u = y)$	(1)u EII
$\{1, 2\}$	(3) $(Fu \equiv u = u)$	(2) UI
$\{\}$	(4) $u = u$	I
$\{1, 2\}$	(5) Fu	(3)(4) TF
$\{1, 2\}$	(6) $(\exists x)Fx$	(5) EG
$\{1, 2\}$	(7) $(Fy \equiv u = y)$	(2) UI
$\{1, 2\}$	(8) $(Fx \equiv u = x)$	(2) UI
$\{\}$	(9) $u = x \supset (u = y \equiv x = y)$	III
$\{1, 2\}$	(10) $(Fx \wedge Fy) \supset x = y$	(7)(8)(9) TF
$\{1, 2\}$	(11) $(\forall y)((Fx \wedge Fy) \supset x = y)$	(10) UG
$\{1, 2\}$	(12) $(\forall x)(\forall y)((Fx \wedge Fy)$ $\supset x = y)$	(11) UG
$\{1, \not 2\}$	(13) $(\exists x)Fx \wedge$ $(\forall x)(\forall y)((Fx \wedge Fy) \supset x = y)$	(6)(12) TF; $\{2\}$ EIE

(g) $X : \{(\forall x)(\exists z)(\forall w)(Rxw \equiv w = z), (\forall z)(\exists x)Rxz\}$
$S : (\forall x)(\forall y)(\forall z)((Rxz \wedge Ryz) \supset y = x)$

$G : U^G = \{0, 1, 2, 3, \ldots\}$

$R^G = \{\langle 0, 0\rangle\} \cup \{\langle n, n-1\rangle \mid 0 < n\}$

Deduction