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LGIC 010 & PHIL 005
Practice Final Examination
Spring Term, 2011

1. Let S be the schema $(\forall x)\neg Lxx$.

   (a) (10 points) How long a list of distinct structures with universe of discourse $\{1, 2, 3\}$ satsify the schema S?

   (b) (10 points) How long a list of pairwise nonisomorphic structures with universe of discourse $\{1, 2, 3\}$ satsify the schema S?

   (c) (10 points) Give an example of a structure A with the following properties:

   - $A \models S$;
   - $U^A = \{1, 2, 3\}$;
   - A has exactly three automorphisms;
   - exactly two subsets of $\{1, 2, 3\}$ are definable in A.

   $L^A =$

2. (70 pts.) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\exists x)(Px \vee Qx)\}$
   $S : (\exists x)Px \vee (\exists x)Qx$

   $A : U^A =$

   $P^A =$

   $Q^A =$

Deduction

(b) $X : \{(\forall x)(Px \vee Qx)\}$
$S : (\forall x)Px \vee (\forall x)Qx$

$B : U^B =$

$P^B =$

$Q^B =$

Deduction

(c) $X : \{(\forall x)Rxx, \neg(\forall x)(\forall y)Rxy\}$
$S : \neg(\exists x)(\forall y)x = y$

$C : U^C =$

$R^C =$

Deduction

(d) $X : \{(\forall x)(Fx \supset (\exists y)(\neg Fy \wedge (\forall z)(Rxz \equiv z = y))), (\forall x)(\neg Fx \supset (\exists y)(Fy \wedge (\forall z)(Rzx \equiv z = y))), (\forall x)(\forall y)(\forall z)((Pxy \wedge Pxz) \supset y = z), (\forall x)(\exists y)(Fy \wedge Pyx)\}$
$S : p \wedge \neg p$

$D : U^D =$

$F^D =$

$R^D =$

$P^D =$

Deduction

(e) $X : \{(\forall x)(\forall y)(\forall z)((Rxy \wedge Ryz) \supset Rxz), (\forall x)\neg Rxx, (\forall x)(\forall y)(Rxy \vee Ryx \vee x = y), (\forall x)(\exists y)(\forall z)(Pxz \equiv z = y), (\forall x)(\exists y)Pyx, (\forall x)(\forall y)(Pxy \supset Rxy)\}$
$S : p \wedge \neg p$

$E : U^E =$

$P^E =$

$R^E =$

Deduction

(f) $X : \{(\exists x)(\forall y)((\forall z)Rzy \equiv y = x)\}$
$S : (\forall x)(\exists y)(\forall z)(Rxz \equiv z = y)$

$F : U^F =$

$R^F =$

Deduction

(g) $X : \{(\forall x)(\exists z)(\forall w)(Rxw \equiv w = z),$
$(\forall x)(\forall y)(\forall z)((Rxz \wedge Ryz) \supset y = x)\}$
$S : (\forall z)(\exists x)Rxz$

$G : U^G =$

$R^G =$

Deduction