PRINT NAME: ____________________________________

LGIC 010 & PHIL 005
Practice Final Examination
Spring Term, 2010

1. Let S be the schema

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y).$$

   (a) (10 points) How long a list of distinct structures with universe of discourse $\{1, 2, 3, 4\}$ satsify the schema S? 64

   (b) (10 points) How long a list of pairwise nonisomorphic structures with universe of discourse $\{1, 2, 3, 4\}$ satsify the schema S? 4

   (c) (10 points) Give an example of a structure A with the following properties:

   - $A \models S$;
   - $U^A = \{1, 2, 3, 4\}$;
   - A has exactly three automorphisms;
   - exactly four subsets of $\{1, 2, 3, 4\}$ are definable in A.

   $L^A = \{\langle 1, 2\rangle, \langle 1, 3\rangle, \langle 1, 4\rangle, \langle 2, 3\rangle, \langle 3, 4\rangle, \langle 4, 2\rangle\}$

2. (70 pts.) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\exists x)Px \wedge (\exists x)Qx\}$
   $S : (\exists x)(Px \wedge Qx)$

   $A : U^A = \{1, 2\}$

   $P^A = \{1\}$

   $Q^A = \{2\}$

Deduction

(b) $X : \{(\forall x)(Px \wedge Qx)\}$
$S : (\forall x)Px \wedge (\forall x)Qx$

$B : U^B =$

$P^B =$

$Q^B =$

Deduction

$\{1\}$	(1)	$(\forall x)(Px \wedge Qx)$	P
$\{1\}$	(2)	$Px \wedge Qx$	(1) UI
$\{1\}$	(3)	Px	(2) TF
$\{1\}$	(4)	Qx	(2) TF
$\{1\}$	(5)	$(\forall x)Px$	(3) UG
$\{1\}$	(6)	$(\forall x)Qx$	(4) UG
$\{1\}$	(7)	$(\forall x)Px \wedge (\forall x)Qx$	$(5, 6)$ TF

(c) $X : \{(\forall x)((\exists y)Lxy \supset (\forall z)Lzx), (\exists x)(\exists y)Lxy\}$
$S : (\forall v)(\forall z)Lvz$

$C : U^C =$

$R^C =$

Deduction

$\{1\}$	(1) $(\exists x)(\exists y)Lxy$	P
$\{1,2\}$	(2) $(\exists y)Lwy$	(1)w EII
$\{3\}$	(3) $(\forall x)((\exists y)Lxy \supset (\forall z)Lzx)$	P
$\{3\}$	(4) $(\exists y)Lwy \supset (\forall z)Lzw$	(3) UI
$\{1,2,3\}$	(5) $(\forall z)Lzw$	(2)(4) TF
$\{1,2,3\}$	(6) Luw	(5) UI
$\{1, \not{2}, 3\}$	(7) $(\exists y)Luy$	(6) EG; $\{2\}$ EIE
$\{3\}$	(8) $(\exists y)Luy \supset (\forall z)Lzu$	(3) UI
$\{1,3\}$	(9) $(\forall z)Lzu$	(7)(8) TF
$\{1,3\}$	(10) Lvu	(9) UI
$\{1,3\}$	(11) $(\forall z)Lvz$	(10) UG
$\{1,3\}$	(12) $(\forall v)(\forall z)Lvz$	(11) UG

(d) $X : \{(\exists x)(\forall y)Rxy\}$
$S : (\forall x)(\exists y)Rxy$

$D : U^D = \{1, 2\}$

$R^D = \{\langle 1, 1\rangle, \langle 1, 2\rangle\}$

Deduction

(e) $X : \{(\forall x)\neg Lxx, (\forall x)(\forall y)(Lxy \supset Lyx),$
$(\forall x)(\exists y)(\exists z)(Lyz \wedge (\forall w)(Lxw \equiv (w = y \vee w = z))),$
$(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y)),$
$(\forall x)(\forall y)(\forall z)(Rxyz \supset (Px \wedge Py)), (\forall z)(\exists x)(\exists y)Rxyz,$
$(\forall x)(\forall y)((Px \wedge Py) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))\}$
$S : (\exists x)(\exists y)(Px \wedge Py \wedge Lxy)$

$E : U^E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$L^E = \{\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 1, 3\rangle, \langle 3, 1\rangle, \langle 2, 3\rangle, \langle 3, 2\rangle, \langle 4, 5\rangle, \langle 5, 4\rangle, \langle 4, 6\rangle, \langle 6, 4\rangle, \langle 5, 6\rangle, \langle 6, 5\rangle,$
$\langle 7, 8\rangle, \langle 8, 7\rangle, \langle 7, 9\rangle, \langle 9, 7\rangle, \langle 8, 9\rangle, \langle 9, 8\rangle\}$

$P^E = \{1, 4, 7\}$

$R^E = \{\langle 1, 1, 1\rangle, \langle 1, 4, 2\rangle, \langle 1, 7, 3\rangle, \langle 4, 1, 4\rangle, \langle 4, 4, 5\rangle, \langle 4, 7, 6\rangle, \langle 7, 1, 7\rangle, \langle 7, 4, 8\rangle, \langle 7, 7, 9\rangle\}$

Deduction

(f) $X : \{(\forall x)(\exists y)Rxy\}$
$S : (\exists y)(\forall x)Rxy$

$F : U^F = \{1, 2\}$

$R^F = \{\langle 1, 1\rangle, \langle 2, 2\rangle\}$

Deduction

(g) $X : \{(\forall x)(\exists z)(\forall w)(Rxw \equiv w = z), (\forall z)(\exists x)Rxz\}$
$S : (\forall x)(\forall y)(\forall z)((Rxz \wedge Ryz) \supset y = x)$

$G : U^G = \{0, 1, 2, 3, \ldots\}$

$R^G = \{\langle 0, 0\rangle\} \cup \{\langle n, n-1\rangle \mid 0 < n\}$

Deduction