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LGIC 010 & PHIL 005
Practice Final Examination
Spring Term, 2010

1. Let S be the schema

$$(\forall x)(\forall y)(Lxy \supset \neg Lyx) \wedge (\forall x)(\forall y)(Lxy \vee Lyx \vee x = y).$$

   (a) (10 points) How long a list of distinct structures with universe of discourse $\{1, 2, 3, 4\}$ satsify the schema S?

   (b) (10 points) How long a list of pairwise nonisomorphic structures with universe of discourse $\{1, 2, 3, 4\}$ satsify the schema S?

   (c) (10 points) Give an example of a structure A with the following properties:

   - $A \models S$;
   - $U^A = \{1, 2, 3, 4\}$;
   - A has exactly three automorphisms;
   - exactly four subsets of $\{1, 2, 3, 4\}$ are definable in A.

   $L^A =$

2. (70 pts.) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

   (a) $X : \{(\exists x)Px \wedge (\exists x)Qx\}$
   $S : (\exists x)(Px \wedge Qx)$

   $A : U^A =$

   $P^A =$

   $Q^A =$

Deduction

(b) $X : \{(\forall x)(Px \wedge Qx)\}$
$S : (\forall x)Px \wedge (\forall x)Qx$

$B : U^B =$

$P^B =$

$Q^B =$

Deduction

(c) $X : \{(\forall x)((\exists y)Lxy \supset (\forall z)Lzx), (\exists x)(\exists y)Lxy\}$
$S : (\forall v)(\forall z)Lvz$

$C : U^C =$

$R^C =$

Deduction

(d) $X : \{(\exists x)(\forall y)Rxy\}$
$S : (\forall x)(\exists y)Rxy$

$D : U^D =$

$R^D =$

Deduction

(e) $X : \{(\forall x)\neg Lxx, (\forall x)(\forall y)(Lxy \supset Lyx),$
$(\forall x)(\exists y)(\exists z)(Lyz \wedge (\forall w)(Lxw \equiv (w = y \vee w = z))),$
$(\forall v)(\forall w)(\forall x)(\forall y)(\forall z)((Rvwz \wedge Rxyz) \supset (v = x \wedge w = y)),$
$(\forall x)(\forall y)(\forall z)(Rxyz \supset (Px \wedge Py)), (\forall z)(\exists x)(\exists y)Rxyz,$
$(\forall x)(\forall y)((Px \wedge Py) \supset (\exists z)(\forall w)(Rxyw \equiv w = z))\}$
$S : (\exists x)(\exists y)(Px \wedge Py \wedge Lxy)$

$E : U^E =$

$L^E =$

$P^E =$

$R^E =$

Deduction

(f) $X : \{(\forall x)(\exists y)Rxy\}$
$S : (\exists y)(\forall x)Rxy$

$F : U^F =$

$R^F =$

Deduction

(g) $X : \{(\forall x)(\exists z)(\forall w)(Rxw \equiv w = z), (\forall z)(\exists x)Rxz\}$
$S : (\forall x)(\forall y)(\forall z)((Rxz \wedge Ryz) \supset y = x)$

$G : U^G =$

$R^G =$

Deduction