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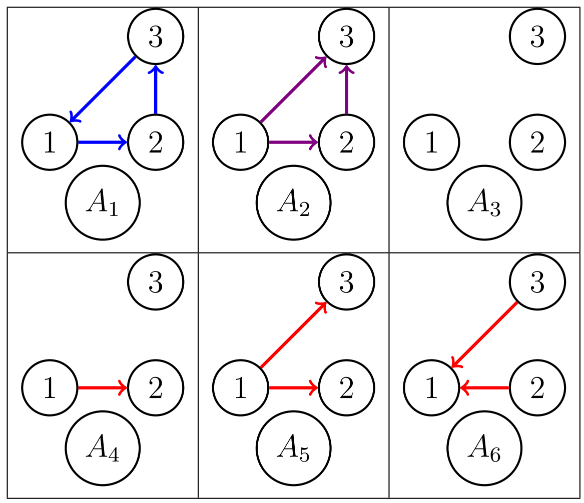
LGIC 010 & PHIL 005 Practice Final Examination Spring Term, 2016

1. Let S be the conjunction of the following schemata.

- $(\forall x)(\forall y)(Lxy \supset \neg Lyx)$
- $(\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)) \vee (\forall x)(\forall y)(\forall z)(Lxy \supset (Lyz \supset Lxz))$

(a) (15 points) Construct a maximal length succinct list l of structures such that each structure listed on l is a member of $\mathsf{mod}(S, 3)$.



(b) (15 points) For each structure A on your list l and each $O \in \mathsf{Orbs}(A)$ write down a schema $S(x)$ such that $S[A] = O$.

A	$O \in \mathsf{Orbs}(A)$	$S[A] = O$
A_1	$\{1, 2, 3\}$	$x = x$
A_2	$\{1\}$	$(\forall y)\neg Lyx$
	$\{2\}$	$(\exists y)Lyx \wedge (\exists y)Lxy$
	$\{3\}$	$(\forall y)\neg Lxy$
A_3	$\{1, 2, 3\}$	$x = x$
A_4	$\{1\}$	$(\exists y)Lxy$
	$\{2\}$	$(\exists y)Lyx$
	$\{3\}$	$\neg(\exists y)Lxy \wedge \neg(\exists y)Lyx$
A_5	$\{1\}$	$(\exists y)Lxy$
	$\{2, 3\}$	$\neg(\exists y)Lxy$
A_6	$\{1\}$	$(\exists y)Lyx$
	$\{2, 3\}$	$\neg(\exists y)Lyx$

2. Let A be a structure interpreting a single triadic predicate letter P and a single dyadic predicate letter L with $U^A = \mathbb{Z}$, $P^A = \{\langle i, j, k\rangle \mid i + j = k\}$, the sum relation on $\mathbb{Z}$, and $L^A = \{\langle i, j\rangle \mid j = |i|\}$, the absolute-value relation on $\mathbb{Z}$.

   (a) (15 points) Let $X_1 = \{i \in \mathbb{Z} \mid i < 0\} = \{-1, -2, -3, \ldots\}$. Is X_1 definable in A? If it is, write down a schema $S_1(x)$ such that $S_1[A] = X_1$; if not, specify an $h_1 \in \mathsf{Aut}(A)$ such that $h_1[X_1] \neq X_1$.

$$S_1(x) : (\forall y)\neg Lyx$$

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   (b) (15 points) Let $X_2 = \{i \in \mathbb{Z} \mid i > 0 \text{ and there is a } j \in \mathbb{Z} \text{ such that } i = 5j + 3\} = \{3, 8, 13, \ldots\}$. Is X_2 definable in A? If it is, write down a schema $S_2(x)$ such that $S_2[A] = X_2$; if not, specify an $h_2 \in \mathsf{Aut}(A)$ such that $h_2[X_2] \neq X_2$.

$$\mathbf{0}^{<}(y) : (\exists w)(\exists v)(w \neq v \wedge Lvy \wedge Lwy)$$

$$\mathbf{1}(y) : \mathbf{0}^{<}(y) \wedge (\forall r)(\forall s)((\mathbf{0}^{<}(r) \wedge \mathbf{0}^{<}(s)) \supset \neg Prsy)$$

$$\mathbf{3}(d) : (\exists a)(\exists b)(\exists c)(\mathbf{1}(a) \wedge \mathbf{1}(b) \wedge Pabc \wedge Pacd)$$

$$S_2(x) : (\exists d)(\exists e)(\exists f)(\exists g)(\exists h)(\mathbf{3}(d) \wedge (\exists y)Lye \wedge Peef \wedge Pffg \wedge Pgeh \wedge Phdx)$$

3. (40 points) For each of the following pairs consisting of a set of schemata X and a schema S determine whether X implies S. If so, provide a deduction to establish the implication. If not, specify a structure which makes S false and all the schemata in X true.

(a) $X : \{(\exists y)(\forall x)(Fx \equiv x = y), (\exists y)(\forall x)(\neg Fx \equiv x = y)\}$
$S : (\exists x)(\exists y)(x \neq y \wedge (\forall z)(z = x \vee z = y))$

$A : U^A =$

$F^A =$

Deduction

$\{1\}$	(1) $(\exists y)(\forall x)(Fx \equiv x = y)$	P
$\{1,2\}$	(2) $(\forall x)(Fx \equiv x = r)$	(1)r EII
$\{1,2\}$	(3) $(Fr \equiv r = r)$	(2) UI
$\{\}$	(4) $(\forall x)x = x$	(I)
$\{\}$	(5) $r = r$	(4) UI
$\{1,2\}$	(6) Fr	(3)(5) TF
$\{7\}$	(7) $(\exists y)(\forall x)(\neg Fx \equiv x = y)$	P
$\{7,8\}$	(8) $(\forall x)(\neg Fx \equiv x = s)$	(7)s EII
$\{7,8\}$	(9) $(\neg Fs \equiv s = s)$	(8) UI
$\{\}$	(10) $s = s$	(4) UI
$\{7,8\}$	(11) $\neg Fs$	(9)(10) TF
$\{\}$	(12) $r = s \supset (Fr \equiv Fs)$	(III)
$\{1,2,7,8\}$	(13) $r \neq s$	(6)(11)(12) TF
$\{1,2\}$	(14) $Fz \equiv z = r$	(2) UI
$\{7,8\}$	(15) $\neg Fz \equiv z = s$	(7) UI
$\{1,2,7,8\}$	(16)$z = r \vee z = s$	(14)(15) TF
$\{1,2,7,8\}$	(17)$(\forall z)(z = r \vee z = s)$	(16) UG
$\{1,2,7,8\}$	(18)$r \neq s \wedge (\forall z)(z = r \vee z = s)$	(13)(17) TF
$\{1,2,7,\not{8}\}$	(19)$(\exists y)(r \neq y \wedge (\forall z)(z = r \vee z = y))$	(18) EG; $\{8\}$EIE
$\{1,\not{2},7\}$	(20)$(\exists x)(\exists y)(x \neq y \wedge (\forall z)(z = x \vee z = y))$	(19) EG; $\{2\}$EIE

(b) $X : \{(\exists x)(Fx \wedge Gx)\}$
$S : (\exists x)Fx \wedge (\exists x)Gx$

$B : U^B =$

$F^B =$

$G^B =$

Deduction

$\{1\}$	(1) $(\exists x)(Fx \wedge Gx)$	P
$\{1, 2\}$	(2) $(Fx \wedge Gx)$	(1)x EII
$\{1, 2\}$	(3) Fx	(2) TF
$\{1, 2\}$	(4) $(\exists x)Fx$	(3) EG
$\{1, 2\}$	(5) Gx	(2) TF
$\{1, 2\}$	(6) $(\exists x)Gx$	(5) EG
$\{1, \not{2}\}$	(7) $(\exists x)Fx \wedge (\exists x)Gx$	(4)(6)TF; $\{2\}$EIE

(c) $X : \{(\forall x)(\exists y)(\forall z)(Lxz \equiv y = z), (\forall x)(\exists y)Lyx\}$
$S : (\forall x)(\forall y)(\forall z)((Lxz \wedge Lyz) \supset x = y)$

$C : U^C = \mathbb{Z}^+$

$L^C = \{\langle 2i - 1, i\rangle, \langle 2i, i\rangle \mid i \in \mathbb{Z}^+\}$

Deduction

(d) For each $n \geq 2$, let R^n be the schema

$$(\exists x_1)\dots(\exists x_n)\bigwedge_{1\leq i<j\leq n}(x_i \neq x_j \wedge (\exists y)Lyx_i \wedge (\forall z)(Lzx_i \supset (\exists w)(Lzw \wedge Lwx_i)).$$

$X : \{(\forall x)(\forall y)(\forall z)((Lxy \wedge Lyz) \supset Lxz), (\forall x)(\forall y)(x \neq y \supset (Lxy \vee Lyx)), (\forall x)\neg Lxx,$
$(\forall x)(\exists y)(Lxy \wedge (\forall z)\neg(Lxz \wedge Lzy))\} \cup \{R^n \mid n \geq 2\}$
$S : (\forall x)(\exists y)Lyx$

$D : U^D = \{\langle i, j\rangle \mid i, j \in \mathbb{Z}^+\}$

$L^D = \{\langle\langle i, j\rangle, \langle k, l\rangle\rangle \mid i, j, k, l \in \mathbb{Z}^+$ and $(i < k$ or $(i = k$ and $j < l))\}$

Deduction