21 Lecture 04.13

On 04.13, we continued our study of deduction, and began by giving a classic "argument by cases."

$$\{(\forall x)Fx \vee (\forall x)Gx\} \text{ implies } (\forall x)(Fx \vee Gx).$$

$\{1\}$	(1) $(\forall x)Fx \vee (\forall x)Gx$	P
$\{2\}$	(2) $(\forall x)Fx$	P
$\{2\}$	(3) Fx	(2) UI
$\{2\}$	(4) $Fx \vee Gx$	(3) TF
$\{2\}$	(5) $(\forall x)(Fx \vee Gx)$	(4) UG
$\{\}$	(6) $(\forall x)Fx \supset (\forall x)(Fx \vee Gx)$	$\{2\}$(5) D
$\{7\}$	(7) $(\forall x)Gx$	P
$\{7\}$	(8) Gx	(7) UI
$\{7\}$	(9) $Fx \vee Gx$	(8) TF
$\{7\}$	(10) $(\forall x)(Fx \vee Gx)$	(9) UG
$\{\}$	(11) $(\forall x)Gx \supset (\forall x)(Fx \vee Gx)$	$\{7\}$(10) D
$\{1\}$	(12) $(\forall x)(Fx \vee Gx)$	$(1, 6, 11)$ TF

We followed with an example of argument by *reductio ad absurdum*, that, in addition, illustrated use of the "conversion of quantifiers" rule, also known as "driving a negation across a quantifier."

$$(\exists y)(Py \supset (\forall x)Px) \text{ is valid}$$

$\{1\}$	(1) $\neg(\exists y)(Py \supset (\forall x)Px)$	P
$\{1\}$	(2) $(\forall y)\neg(Py \supset (\forall x)Px)$	(1) CQ
$\{1\}$	(3) $\neg(Py \supset (\forall x)Px)$	(2) UI
$\{1\}$	(4) Py	(3) TF
$\{1\}$	(5) $(\forall x)Px$	(4) UG
$\{1\}$	(6) $\neg(\forall x)Px \wedge (\forall x)Px$	(3)(5) TF
$\{\}$	(7) $\neg(\exists y)(Py \supset (\forall x)Px) \supset$ $(\neg(\forall x)Px \wedge (\forall x)Px)$	$\{1\}$(6) D
$\{\}$	(8) $(\exists y)(Py \supset (\forall x)Px)$	(7) TF

We next gave the first of a pair of deductions that legitimate extending the "conversion of quantifiers rule" to allow passing directly from $\neg(\forall x)S$ to $(\exists x)\neg S$ and *vice versa.* I include the second here, though we did not cover it in class.

$$\{\neg(\forall x)Fx\} \text{ implies } (\exists x)\neg Fx.$$

$\{1\}$	(1) $\neg(\forall x)Fx$	P
$\{2\}$	(2) $\neg(\exists x)\neg Fx$	(1) P
$\{2\}$	(3) $(\forall x)\neg\neg Fx$	(2) CQ
$\{2\}$	(4) $\neg\neg Fx$	(3) UI
$\{2\}$	(5) Fx	(4) TF
$\{2\}$	(6) $(\forall x)Fx$	(5) UG
$\{1, 2\}$	(7) $(\forall x)Fx \wedge \neg(\forall x)Fx$	(6) TF
$\{1\}$	(8) $\neg(\exists x)\neg Fx \supset ((\forall x)Fx \wedge \neg(\forall x)Fx)$	$\{2\}$(7) D
$\{1\}$	(9) $(\exists x)\neg Fx$	(8) TF

$$\{(\exists x)\neg Fx\} \text{ implies } \neg(\forall x)Fx.$$

$\{1\}$	(1) $(\forall x)Fx$	P
$\{2\}$	(2) $(\exists x)\neg Fx$	(1) P
$\{1\}$	(3) Fx	(1) UI
$\{1\}$	(4) $\neg\neg Fx$	(3) TF
$\{1\}$	(5) $(\forall x)\neg\neg Fx$	(4) UG
$\{1\}$	(6) $\neg(\exists x)\neg Fx$	(5) CQ
$\{1, 2\}$	(7) $\neg(\exists x)\neg Fx \wedge (\exists x)\neg Fx$	(6) TF
$\{2\}$	(8) $(\forall x)Fx \supset (\neg(\exists x)\neg Fx \wedge (\exists x)\neg Fx)$	$\{1\}$(7) D
$\{1\}$	(9) $\neg(\forall x)Fx$	(8) TF