Towards a formal semantics for GHC Core

Stephanie Weirich

Joachim Breitner, Antal Spector-Zabusky, Yao Li,

Christine Rizkallah, John Wiegley

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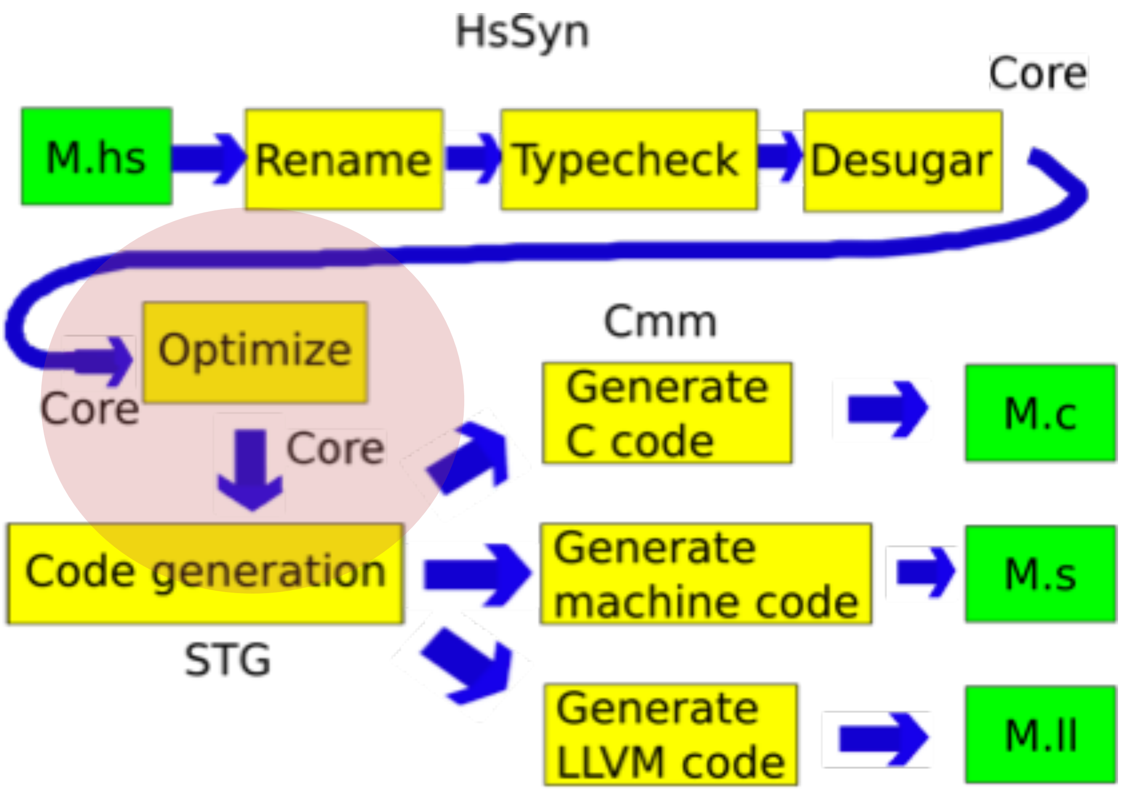
A Software Expedition

- DeepSpec is about specifying and verifying system software, such as compilers
- Functional programs are "easy" to specify and reason about
- Let's prove the Glasgow Haskell Compiler correct!
- What would it take?

What would it take?

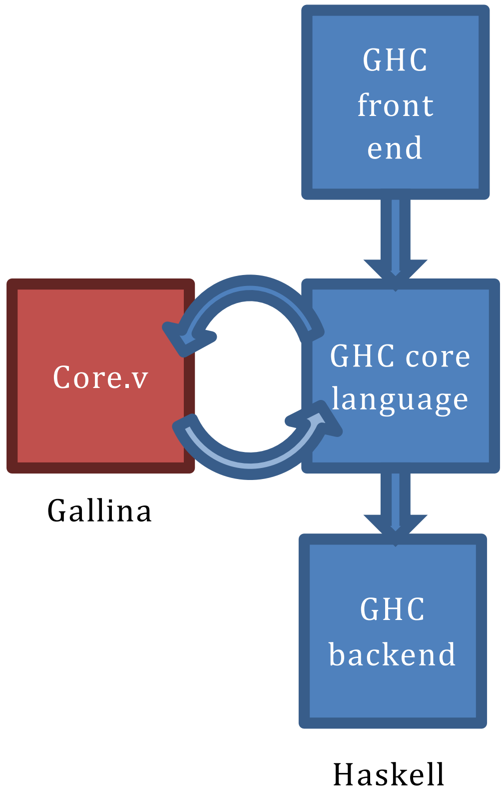
- A formal specification of Haskell, to define what correct means for the whole compiler
  - That's really big and we don't have one. Maybe we can start with something smaller? GHC Core
- A formal specification of Haskell, to prove that the *Haskell program GHC* is correct
  - That's really big and we don't have one. Maybe we can use something else? Gallina
- A lot of work

The GHC Core language



Gallina is Haskell if you squint

- Want to use Coq to reason about GHC
  - Need a semantics for Haskell in Coq
  - But that is what we are trying to build!
- "Easy" approach: shallow embedding
  - Use Gallina as a stand-in for Haskell
  - Translate Haskell functions to Gallina functions, use that as semantics



hs-to-coq

A tool for translating Haskell code to equivalent Gallina definitions via shallow embedding [CPP' 18]

```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr k z = go
          where
            go []     = z
            go (y:ys) = y `k` go ys
```

```
Definition foldr {a} {b} :
               (a -> b -> b) -> b -> list a -> b :=
  fun k z =>
    let fix go arg_0__
              := match arg_0__ with
                 | nil => z
                 | cons y ys => k y (go ys)
                 end in
    go.
```

Questions about hs-to-coq approach

1. Is there enough Haskell code out there that we can translate to make this approach worthwhile?

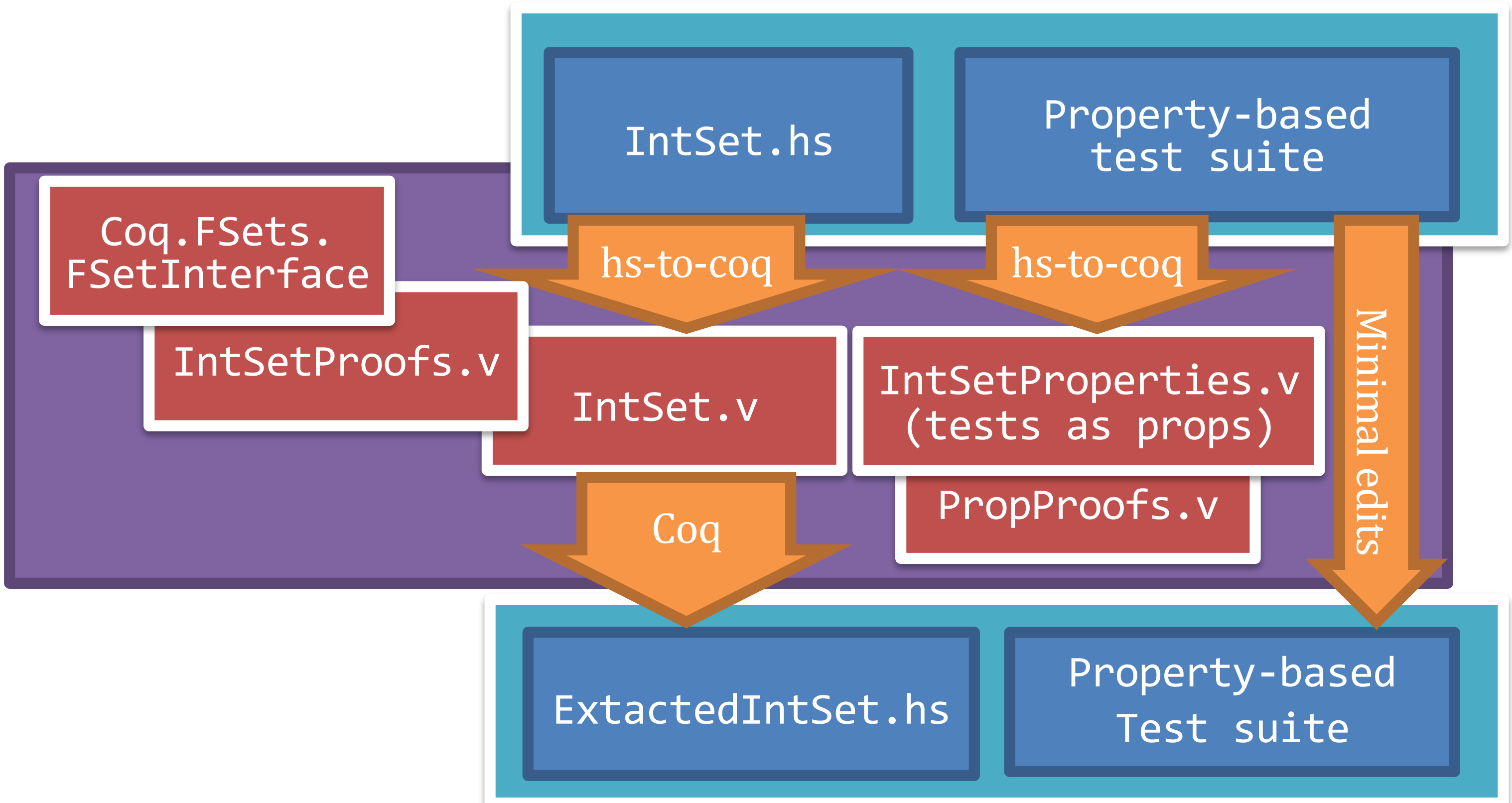
2. Even if we can find code to translate, is the result suitable for verification?

3. Even if we can do the proofs, do they mean anything about the Haskell source?

Case study: containers

- Popular Haskell libraries: Data.Set (weight-balanced trees) and Data.IntSet (big endian patricia tries)
- Used by GHC Core language implementation
- What did we prove?
  - Invariants in the source file comments (ensures the balance properties and other invariants)
  - Mathematical specification of finite sets (both our own and from Coq library)
  - Quickcheck properties interpreted as theorems
  - GHC Rewrite rules

Containers case study



What did we learn?

1. We can translate these libraries*
2. We can prove what we want to prove**
3. Gallina version is semantically equivalent to Haskell (as far as we can tell by testing)
4. Haskell code is correct ☺

*Need to address partiality

**We "edit" the code during translation in support of verification

Partiality: Unsound

```
head :: [a] -> a
head (x:_) = x
head [] = error "head: empty list"
```

```
Axiom error : forall {a} , String -> a.

Definition head {a}
    (xs : list a) : a :=
  match xs with
  | (x::_) => x
  | _      => error "head: empty list"
  end.
```

Partiality: Annoying

```
head :: [a] -> a
head (x:_) = x
head [] = error "head: empty list"
```

```
Inductive Partial (a:Type) :=
  | return : a -> Partial a
  | error  : String -> Partial a
  | …

Definition head {a} (xs : list a) : Partial a :=
  match xs with
  | (x::_) => return x
  | _      => error "head: empty list"
  end.
```

Partiality: Pragmatic approach

```
head :: [a] -> a
head (x:_) = x
head [] = error "head: empty list"
```

```
Definition error : forall {a} `{Default a},
                      String -> a := default.
Definition head {a} `{Default a} (xs : list a) : a :=
  match xs with
  | (x::_) => x
  | _      => error "head: empty list"
  end.
```

☞ "default" is an opaque definition so proofs must work for any value of the appropriate type.

Partiality: Pragmatic approach

- Can use this approach for difficult termination arguments (with classical logic/axiom of choice)

```
Definition deferredFix:
   forall {a r} `{Default r},
       ((a -> r) -> (a -> r)) -> (a -> r).

Definition deferredFix_eq_on:
   forall {a r} `{Default b}
      (f : (a -> r) -> (a -> r))
      (P : a -> Prop) (R : a -> a -> Prop),
   well_founded R -> recurses_on P R f ->
   forall x, P x ->
   deferredFix f x = f (deferredFix f) x.
```

A PRAGMATIC FORMALIZATION GAP

A Formalization Gap is a *good* thing

- Machine integers are fixed width. Do we want to reason about overflow?
- No!
  - In Data.Set, Ints track size of tree for balance
  - GHC uses Data.IntSet to generate unique names
  - Both cases will run out of memory before overflow
- Control translation with hs-to-coq rewrites
  - type GHC.Num.Int = Coq.ZArith.BinNum.Z
  - Formalization gap is explicit & recorded

A Formalization Gap is a *good* thing

- Machine integers store positive and negative numbers. Do we want that?
- No!
  - In Data.Set, Ints track size of tree for balance
  - GHC uses Data.IntSet to generate unique names
  - Both cases never need to store negative numbers
- Control translation with hs-to-coq rewrites
  - type GHC.Num.Int = Coq.NArith.BinNat.N
  - (But, need *partial* implementation of subtraction)
  - Formalization gap is explicit & recorded

What about GHC?

Core Spec

Questions about GHC

1. Is there enough code *in GHC* that we can translate to make this approach worthwhile?

2. Even if we can find code to translate, is the result suitable for verification?

3. Even if we can do the proofs, do they mean anything about the GHC implementation?
   (Note: Core plug-in option available)

GHC: Current status

- Base libraries (9k loc)
  - 45 separate modules
  - Some written by-hand: GHC.Prim, GHC.Num, GHC.Tuple
  - Most translated: GHC.Base, Data.List, Data.Foldable, Control.Monad, etc.
- Containers (6k loc)
  - Translated & (mostly) verified: 4 modules
  - (Data.Set, Data.Map, Data.IntSet, Data.IntMap)
- GHC, version 8.4.1 (19k loc)
  - 55 modules so far (327 modules total in GHC, but we won't need them all)
  - hs-to-coq edits (2k LOC)
- *First verification goal*: Exitify compiler pass

Core AST

```
data Expr b
  = Var  Id
   | Lit  Literal
   | App  (Expr b) (Arg b)
   | Lam  b (Expr b)
   | Let  (Bind b) (Expr b)
   | Case (Expr b) b Type [Alt b]
   | Cast (Expr b) Coercion
   | Tick (Tickish Id) (Expr b)
   | Type Type
   | Coercion Coercion
    deriving Data

data Bind b =
   NonRec b (Expr b)
 | Rec [(b, (Expr b))]
  deriving Data
```

```
Inductive Expr b : Type
  := Mk_Var : Id -> Expr b
   | Lit : Literal -> Expr b
   | App : Expr b -> Arg b -> Expr b
   | Lam : b -> Expr b -> Expr b
   | Let : Bind b -> Expr b -> Expr b
   | Case : Expr b -> b -> unit
       -> list (Alt b) -> Expr b
   | Cast : Expr b -> unit -> Expr b
   | Tick :
      Tickish Id -> Expr b -> Expr b
   | Type_ : unit -> Expr b
   | Coercion : unit -> Expr b
with Bind b : Type
  := NonRec : b -> Expr b -> Bind b
   |  Rec : list (b * (Expr b))
                -> Bind b
```

Core Optimization : Exitify

```
-- | Given a recursive group of a joinrec,
identifies
--   “exit paths” and binds them as
--   join-points outside the joinrec.

exitify :: InScopeSet -> [(Var,CoreExpr)] ->
           (CoreExpr -> CoreExpr)
exitify in_scope pairs =
    \body -> mkExitLets exits (mkLetRec pairs' body)
  where
     pairs' = …   // updated recursive group
     exits  = …   // exit paths

-- 215 LOC, incl comments
```

- Requires moving code from one binding scope to another
- First proof: show that well-scoped terms stay well-scoped

Bug found!

- Exitify does not always produced well-scoped code
  - Missed by GHC test suite
  - (Perhaps not exploitable at source level)
- Fixed in GHC HEAD
  - Proofs updated to new version
  - (What is the general workflow?)
- Next step: work with a model of the operational semantics
  - Use GHC API to develop a Coq interpreter, also translate via hs-to-coq

Conclusion & More questions

Let's take advantage of the semantic similarity of Haskell and Gallina to develop verified compilers

- Haskell's purity means existing code is verifiable
- "Formalization gap" makes this pragmatic
- Can we make verification incremental?
- Can we get good performance of extracted code? (And plug back into GHC?)