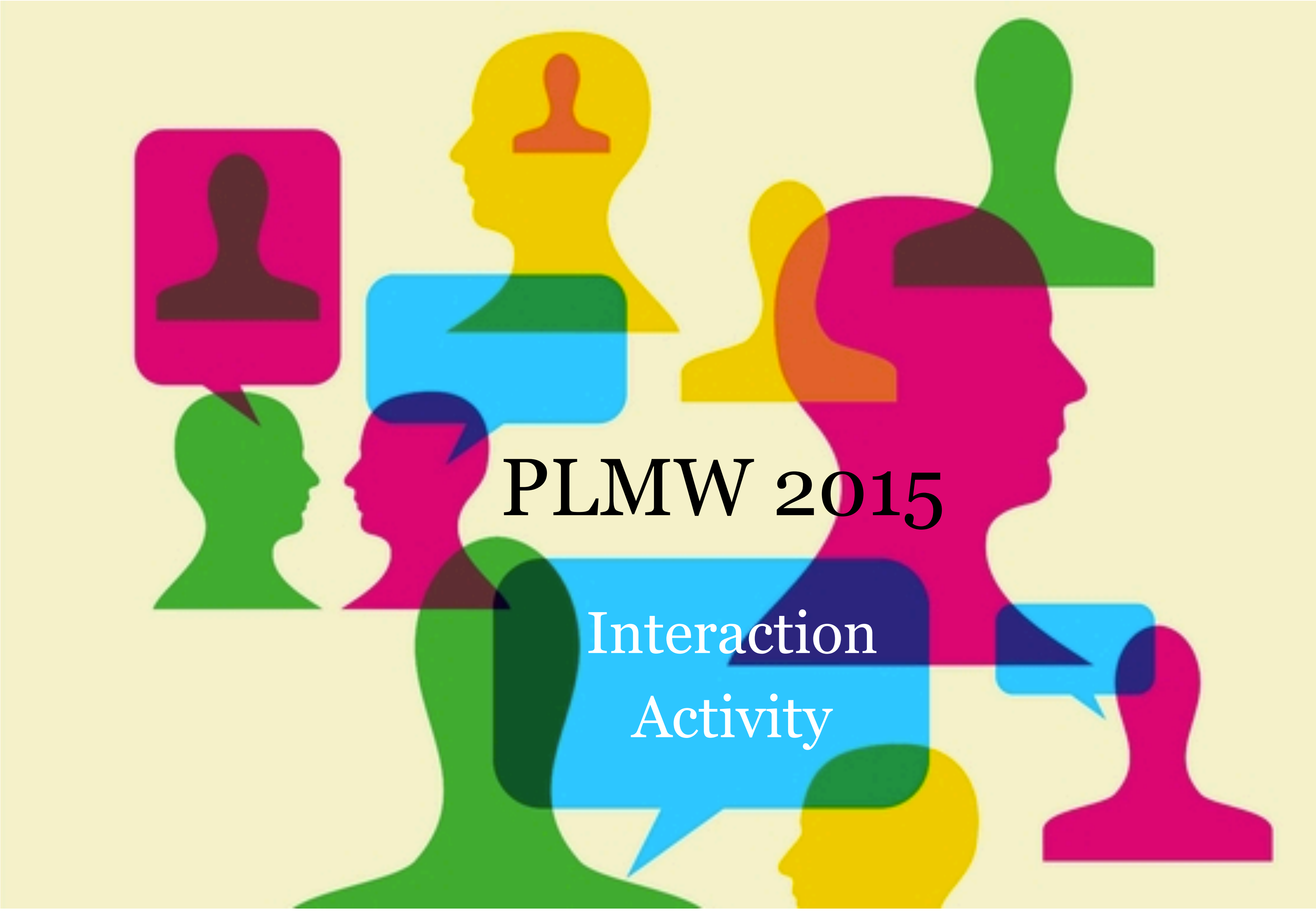
PLMW 2015
Interaction
Activity

CNF Satisfiability

CNF Boolean Formula

- Variables, x, y, z
- Literals, positive and negative variables
- Clauses, disjunctions of literals
- CNF formula, a conjunction of clauses

A formula is *satisfiable* if there is some truth assignment such that the formula evaluates to true.

$$(x \overset{\text{or}}{\lor} y) \overset{\text{and}}{\land} (\neg y \lor z)$$

literals

clauses

CNF formula

Satisfiability examples

- $(x \lor \neg x)$
- $(x) \land (\neg x)$
- $(x \lor \neg x) \land (y \lor z)$
- $(x \lor y) \land (\neg y \lor z)$
- $(x \lor y) \land (\neg y \lor z) \land (\neg y \lor \neg z)$

  $\land (y \lor \neg z) \land (\neg y \lor x)$

Resolution based satisfiability

$(x \vee y) \wedge (\neg x \vee z)$ is satisfiable
when $(y \vee z)$ is satisfiable

Use this idea to create resolution based theorem prover:

- Start with a set of clauses S.
- Find two clauses to resolve. $(x \vee y_1 ..)$ & $(\neg y \vee z_1 ...)$
- If the result $(y_1 .. \vee z_1 ...)$ is empty, then the formula is unsatisfiable.
- If the result is a tautology, ignore it.
- Otherwise add result to S, and continue until no new information can be derived.

The Formula

1. $D \lor \neg AE \lor \neg U$
2. $\neg R \lor \neg D$
3. $\neg B \lor \neg O \lor G$
4. $\neg C \lor \neg AD \lor \neg A$
5. $\neg T \lor AE \lor E$
6. $C \lor Q \lor \neg E$
7. $T \lor \neg B \lor \neg M$
8. $AD \lor \neg N \lor U$
9. $\neg G \lor A \lor S$
10. $D \lor \neg G \lor M$
11. $AE \lor \neg P \lor X$
12. $\neg N \lor \neg D \lor O$
13. $Y \lor C \lor \neg X$
14. $\neg N \lor Y \lor R$
15. $S \lor G \lor N$
16. $\neg T \lor \neg Y$
17. $\neg AE \lor P$
18. $X \lor \neg Y$

Activity

1. Find your clause (other people with same #)
2. With your clause, find another clause with complementary literal
3. RESOLVE!
4. Resolved literals out, sit and chat
5. If you find a contradiction, let me know!
6. If resolved clause is a tautology, let me know your clause number(s). Everyone out.
7. Otherwise, got back to step 2 until no more resolutions.

- Random formula has residual of
- $AE \lor \neg P \lor C \lor W \lor \neg T$
- $\neg G \lor \neg O \lor \neg T$ (never resolves)