

Par means Parallel - Multiplicative Linear Logic
Proofs as Concurrent Functional Programs
POPL 2020 - Aschieri + Genco

$$R \triangleq \nu z (x(y). z(w). P \mid z[a]. Q)$$

In CP (Wadler's Classical Processes)

$$R \rightarrow x(y). \nu z (z(w). P \mid z[a]. Q)$$

$$\lambda \& \quad A ::= \alpha \mid \underline{A \multimap B} \mid A \wp B \mid \perp$$

$$\Rightarrow A \otimes B \triangleq ((A \multimap \perp) \wp (\perp \multimap B)) \multimap \perp$$

$$(A \wedge B) = \neg(\neg A \vee \neg B)$$

$$A \multimap B \equiv \boxed{A^\perp} \wp B$$

$$A \rightarrow B \equiv \neg A \vee B$$

$x, y, z =$ channels

$t, u, v ::= o$ terminated

$\bar{x}.u$

output on channel x
continue as u
without continuation

$\bar{x}\bar{y}.u$

$u | t$

$\text{close}(t)$

$\text{close}(o) \rightarrow$

x

$u \ v$

syntactic sugar $\lambda x. u \triangleq \bar{x}.u \quad (x \in \text{fv}(u))$

$\bar{x}v.u \triangleq (\bar{x}.u) \ v$

$D[_], C[_]$::= ... | $[_]$ ← "hole" that appears once

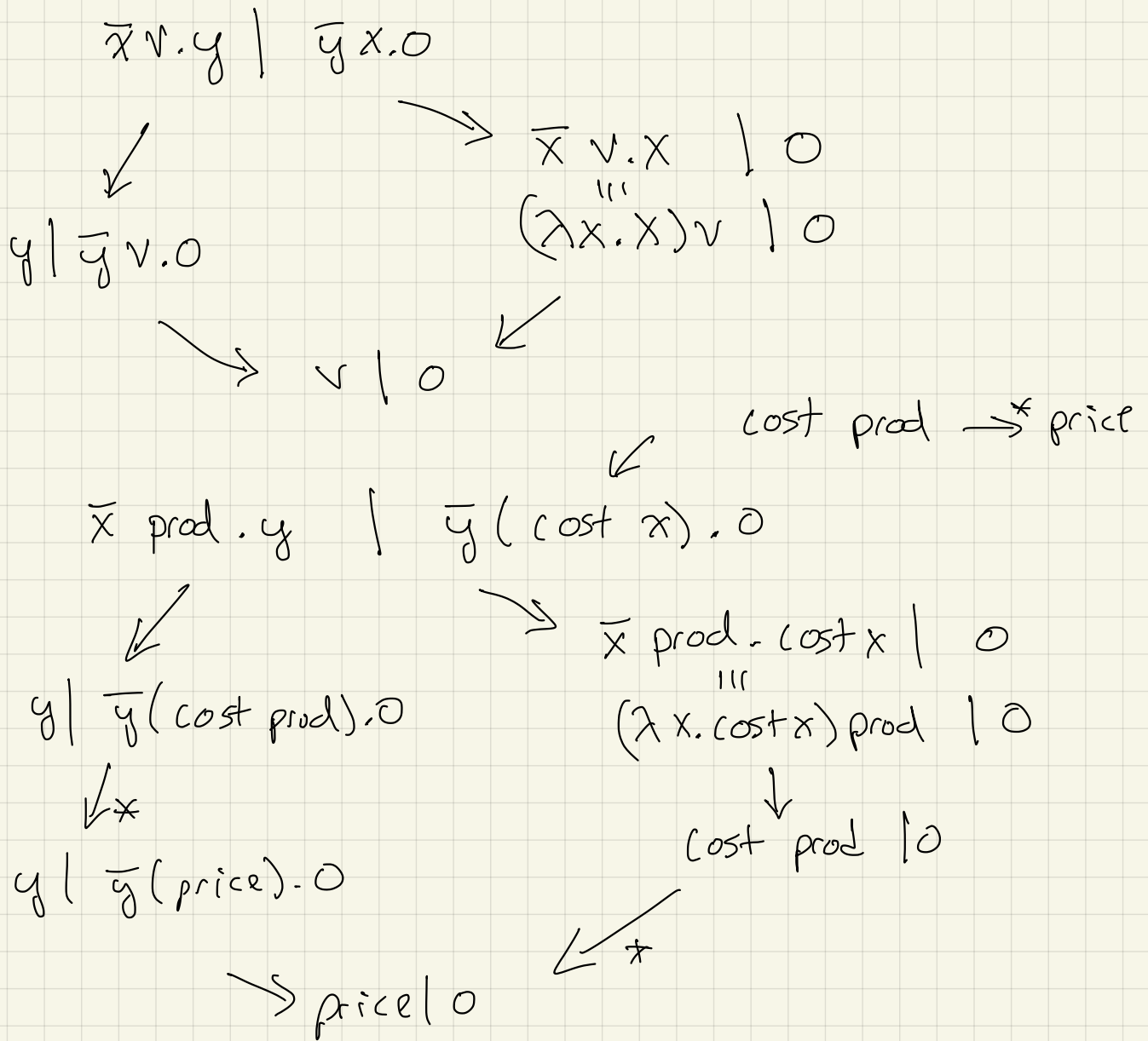
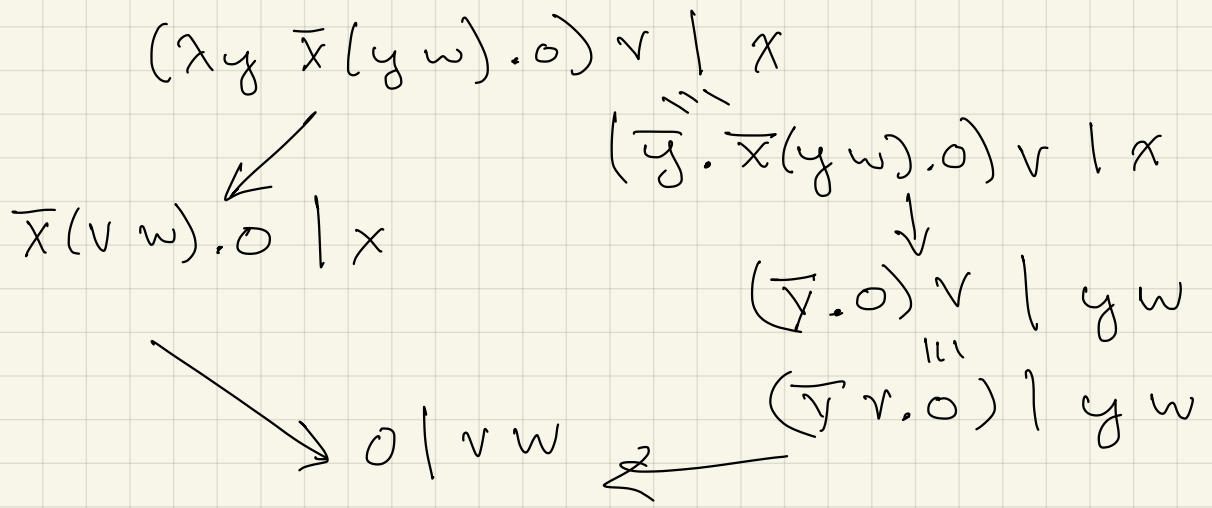
$(\lambda x. u) \ t \rightarrow u[t/x]$ (CBN) (β)

(and sym) $C[\bar{x}u.s] | D[x] \rightarrow C[s] | D[u]$ (comm)

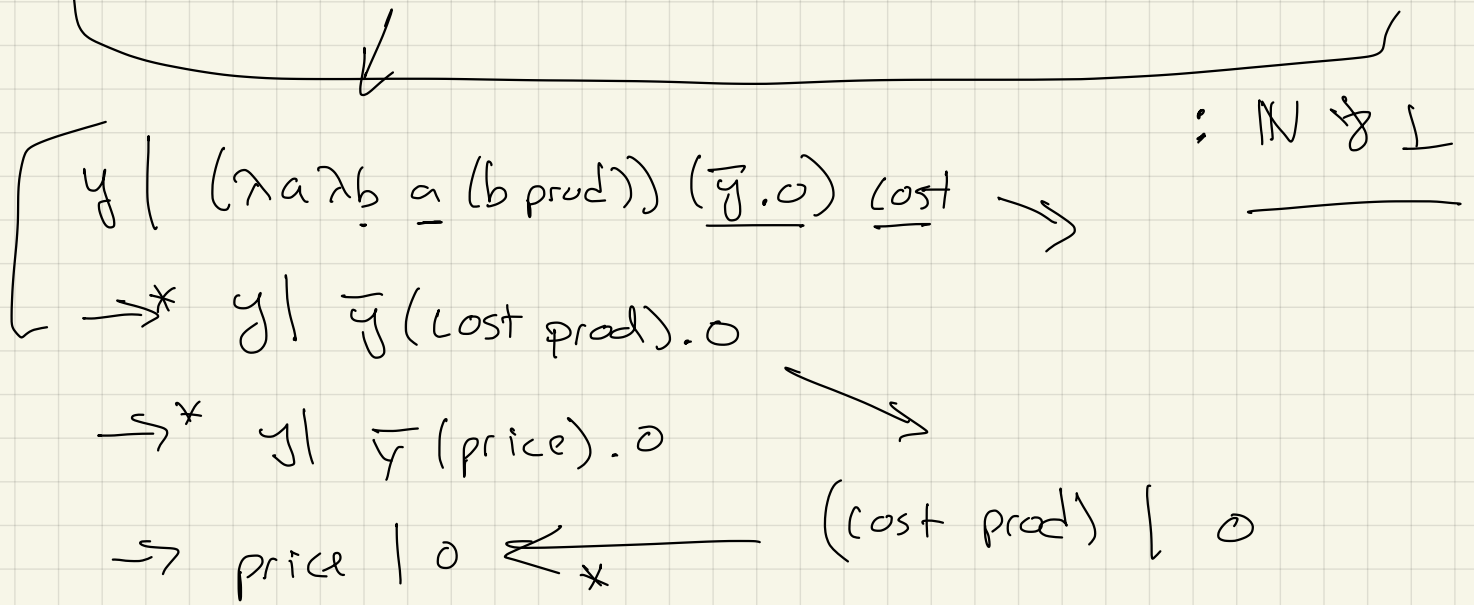
(and sym) $C[\bar{x}\bar{y}(s|t)] | D[x][y] \rightarrow C[o] | D[s][t]$

(and sym) $D[x] | C[\bar{x}\bar{y}(s|t)] | E[y] \rightarrow D[s] | C[o] | E[t]$

$\frac{t \rightarrow u}{C[t] \rightarrow C[u]}$



$\bar{x} (\lambda a \lambda b a (b \text{ prod})).y \mid x(\bar{y}.0) \text{ cost}$



Types

$A ::= \alpha \mid A \rightarrow B \mid A \times B \mid \perp$

$\Gamma ::= \cdot \mid \Gamma, x:A$

$\Delta ::= \cdot \mid \Delta, t:A \mid \Delta, u$

$\Gamma \Rightarrow \Delta$

$$x:A \Rightarrow x:A$$

$$\frac{\Gamma \Rightarrow s:A, t:B, \Delta}{\Gamma \Rightarrow (s|t): A \wp B, \Delta} \quad \wp I$$

$$\frac{\Gamma \Rightarrow s:A \wp B, \Delta \quad \Sigma_1, x:A \Rightarrow \Theta_1 \quad \Sigma_2, y:B \Rightarrow \Theta_2}{\Gamma, \Sigma_1, \Sigma_2 \Rightarrow \bar{x} \bar{y} s : \perp, \Delta, \Theta_1, \Theta_2}$$

$$\left. \begin{array}{l} \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, 0:\perp} \\ \frac{\Gamma \Rightarrow t:\perp, \Delta}{\Gamma \Rightarrow \text{close}(t), \Delta} \end{array} \right\}$$

$$= \frac{\Gamma, x:A \Rightarrow t:B, \Delta}{\Gamma \Rightarrow \lambda x t : A \multimap B, \Delta} \quad (\text{if } x \text{ occurs in } t)$$

$$\bar{x}.t \stackrel{\Delta}{=} \lambda x.t$$

$$\frac{\Gamma, x:A \Rightarrow t:B, \Delta}{\Gamma \Rightarrow \bar{x}.t : A \multimap B, \Delta} \quad (x \text{ occurs in } \Delta) \quad x \in \text{fv}(t)$$

$$\frac{\Gamma \Rightarrow s:A \multimap B, \Delta \quad \Sigma \Rightarrow t:A, \Theta}{\Gamma, \Sigma \Rightarrow st : B, \Delta, \Theta}$$

Substitution $\Sigma \Rightarrow t : A, \Theta$

$\Gamma, x : A \Rightarrow \Delta$

then $\Sigma, \Gamma \Rightarrow \underbrace{\Delta[t/x]}, \Theta$

Subject Reduction $\Gamma \Rightarrow \Delta$

and $\Delta \rightarrow \Delta'$

$\Delta = t_1 : A_1 \mid \dots \mid t_n : A_n \mid \text{close}(t_{n+1}) \dots$
 Δ'

$\Gamma \Rightarrow \Delta'$

$\Gamma \vdash A \rightarrow B$

$\Gamma \vdash A$

$\Gamma \vdash B$

$\lambda x(y). P$

$\lambda P \equiv P / \lambda P$