"Par means Parallel : Multiplicative Linear-logic proofs as concurrent functional programs"
Aschieri and Genco, POPL 2020

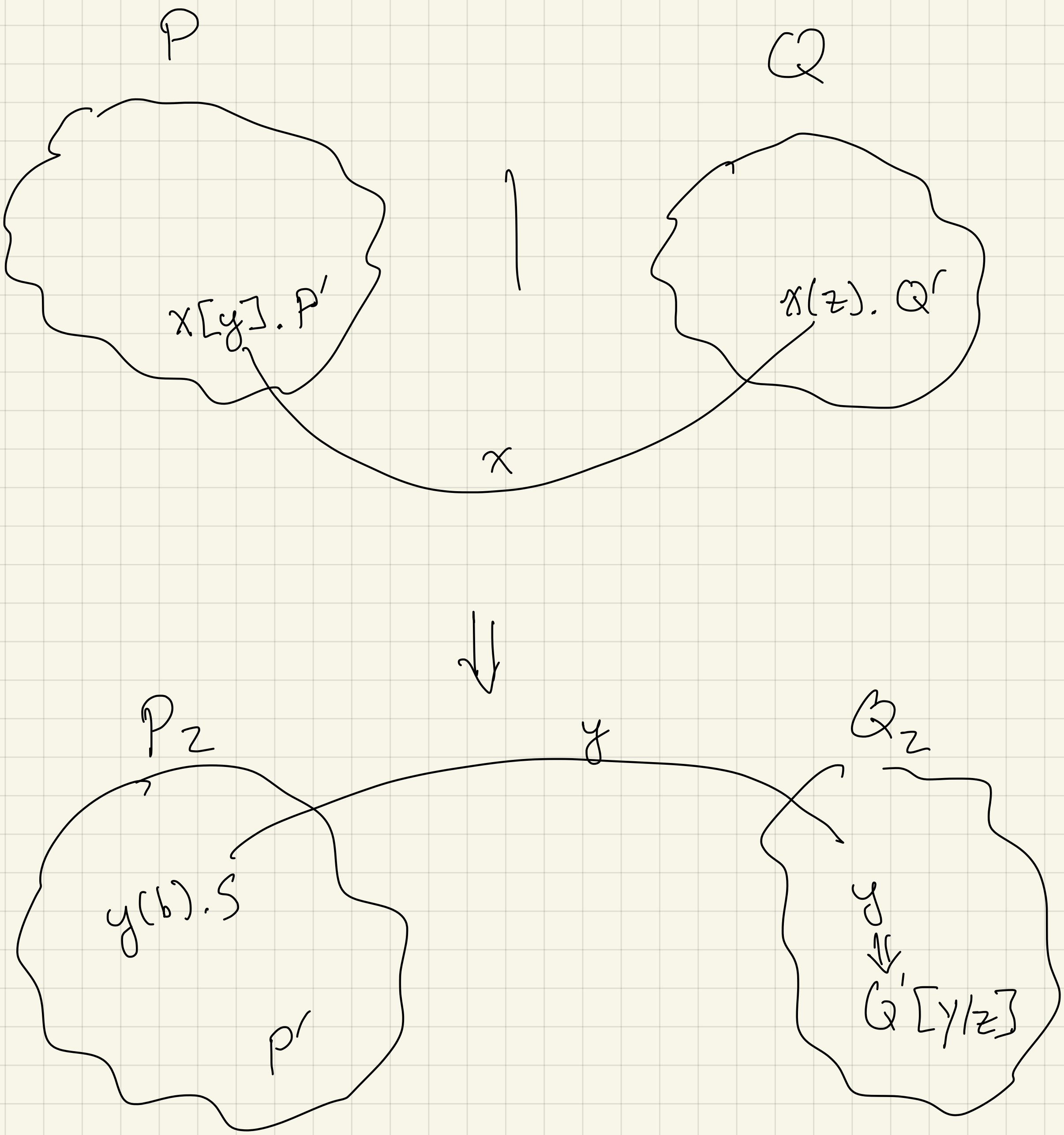
Background π-calculus

Milner, Parrow, David Walker 1992
- π-calculus

References

- Milner 1999 Comminicating and Mobile Systems the π-calculus
- Sangiorgi & Walker 2001
  π-calculus A theory of mobile processes

- Original formulation:
  finite automata ⇒ concurrent automata ⇒
  CCS ⇒ π-calculus (names)
- turing complete
- Many variants
  - synchronous, asynchronous, typed, "located"
  - SPi-calculus Abadi/Gordon
  - PICT - Pierce & Sangiori
  - many papers Honda and Yoshida
  - Session types - Pfenning et. al



P
Q
$x[y].P'$
$|$
$x(z).Q'$
x
$\Downarrow$
P_2
y
Q_2
$y(b).S$
P'
y
$\Downarrow$
$Q'[y/z]$

Syntax

$x, y, z \in$ Channel Names

$P, Q ::= x(y).P$ - receive a name on channel x (y is bound in P)

$\mid x[y].P$ - send the name y on x

$\mid P \mid Q$ - parallel composition

$\mid (\nu x)\, P$ - create a fresh channel x in scope in P

$\mid\ !P$ - repeatedly spawn P

$\mid 0$ - halt or "done"

Structural Congruence $P \equiv Q$

$\equiv$ is the least congruence satisfying

$P \equiv Q$ if $P =_\alpha Q$

$!P \equiv P \mid !P$

$P \mid Q \equiv Q \mid P$

$P \mid (Q \mid R) \equiv (P \mid Q) \mid R$

$P \mid 0 \equiv P$

$(\nu x)\, 0 \equiv 0$

$(\nu x)(\nu y)\, P \equiv (\nu y)(\nu x)\, P$

$(\nu x)(P \mid Q) \equiv ((\nu x)\, P) \mid Q$ if $x \notin fn(Q)$

Reduction Semantics

$$(\text{comm}) \quad x[z].P \mid x(y).Q \;\rightarrow\; P \mid Q[z/y]$$

$$\frac{P \rightarrow P'}{P \mid Q \rightarrow P' \mid Q} \qquad \frac{P \rightarrow P'}{(\nu x)P \rightarrow (\nu x)P'}$$

$$\frac{P \equiv P' \quad P' \rightarrow Q' \quad Q' \equiv Q}{P \rightarrow Q}$$

- Nondeterminism (no confluence)

$$\begin{tikzcd}[column sep=small]
 & x[z].0 \mid x(y).P \mid x(y).Q \arrow[dl] \arrow[dr] & \\
P[z/y] \mid x(y).Q & & x(y).P \mid Q[z/y]
\end{tikzcd}$$

$$\begin{tikzcd}[column sep=small]
 & x[z] \mid x[y] \mid x(w).P \arrow[dl] \arrow[dr] & \\
x[y] \mid P[z/w] & & x[z] \mid P[y/w]
\end{tikzcd}$$

<u>Abbrev</u> $x[y].0 \triangleq x[y]$

Polyadic π- calculus (sending tuples)

$$P ::= \dots \mid x[z_1,\dots,z_n].P \mid x(y_1,\dots,y_n).Q$$

w fresh

$$x[z_1,\dots,z_n].P \triangleq (\nu w)\, x[w].w[z_1].\dots.w[z_n].P$$

$$x(y_1,\dots y_n).Q \triangleq x(w).w(y_1)\dots w(y_n).Q$$

w fresh

x has arity n

$$x[z_1,\dots z_n].P \mid x(y_1,\dots y_n).Q \xrightarrow{*} P \mid Q[\overline{z_i/y_i}]$$

(νx)

$$(\nu z)\, x[z].\ P \quad \mid \quad x(y).\ Q$$

$$\equiv (\nu z)(x[z].P \mid x(y).Q) \qquad (z \notin fn(Q))$$

$$\to \nu z(\ P \mid Q\{z/y\}\ )$$

$$x[x] \mid x(y).y[y] \qquad (\nu y x)\ y(x).x[z] \mid x(z).y[z] \ \sim 0$$

$$(\nu x)\quad z[x] \;\mid\; z(y).\, y[y]$$

$$\to \quad (\nu x)\; x[x] \quad \sim \quad 0$$

CBN λ-calculus

$$[\![x]\!]u \;\triangleq\; x[u]$$

$$[\![\lambda x. M]\!]u \;\triangleq\; u(x,v).\, [\![M]\!]v$$

$$[\![M\, N]\!] \;\triangleq\; (\nu v)\big(\, [\![M]\!]v \;\mid\; (\nu x)\; v[x,u].\, [\![x := N]\!] \,\big)$$

where $x \notin fn(N)$ $\quad v$ is fresh

where $\quad [\![x := N]\!] \;\triangleq\; \underbrace{!\, x(w).\, [\![N]\!]w}$

$$[\![(\lambda x. x)\, N]\!]u$$

$$\to \underbrace{\sim [\![N]\!]u}$$

$$P \sim P'$$