### Problem Statement

**Problem:** How do we distribute a resource across a variety of assets in order to maximize return and minimize risk?

**Novel Aspects:**
1. Borrow additional resources to use as leverage to increase returns.
2. Utilize different allocation positions through structured hedging to reduce risk.

**Solution:** Design an online learning algorithm which learns asset correlations and uses leverage to hedge its allocation.

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### Leverage and Structured Hedging

**Leverage** is a way to increase returns such as borrow additional resources to increase allocation power. **Hedging** is a method to offset risk by taking opposing positions.

For structured hedging, we use a correlation graph over the n assets with 2 nodes per asset, one for each position.

Let $W$ be a correlation matrix and $q(i)$ and $q(j)$ be long/short positions for assets $i$ and $j$, then a structured hedging penalty function is

$$
\Omega_h = \sum_{i,j \in \mathcal{E}} W_{ij}(q(i) + q(j))^2 = p^T L p
$$

### Online Portfolio Selection

Consider a stock market with $n$ stocks over a span of $T$ days. Each day $t$ we must choose a portfolio $p_t$ over the stocks to maximize the log gain in wealth $\ln \{ p_t^T x_t / p_1^T x_1 \}$, where $x_i$ is the vector of price relatives (closing price / opening price).

### Datasets

- **DJIA:** 30 stocks with 507 trading days between 2001-2003.
- **NYSE:** 36 stocks with 5651 trading days between 1962-1984.
- **SP500:** 263 stocks with 505 trading days between 2007-2009.
- **TSX:** 89 stocks with 595 trading days between 1994-1996.

Percentage of stocks that lost value over the time period:
- **DJIA:** 83%.
- **NYSE:** 0%, all stocks increased in value.
- **SP500:** 96%.
- **TSX:** 28%.

### Experiment 1: EG with Leverage vs SHERAL

**Description:**
- **Problem Statement:**
  - Long position: Investing in an asset using cash on hand. One profits if the value of the asset increases.
  - Short position: Investing borrowed cash/shares in an asset. One profits if the value of the asset decreases.

**Portfolios:**
- $\mathcal{P} \subset \mathbb{R}^{2n}$ and $\ell = \{1, \ldots, n\}$
- $D_{\mathcal{P}}(i) = 1$ for $i \in \ell$ and $D_{\mathcal{P}}(i) = 1$ for $i \notin \ell$

**Long-only:**
- $q^{\ell} = D_{\mathcal{P}} p \geq 0$

**Long and short portfolio multiplicative gain in wealth:**
- $\text{Market change in wealth} = \text{Long} \times \text{Market change in wealth} - \text{Short} \times \text{Market change in wealth} - \text{Interest used on borrowed shares}$

**Leverage and Structured Hedging**
- Long and short portfolio multiplicative gain in wealth:
  - $\text{Market change in wealth} = \text{Market change in wealth} + \text{Market change in interest used on borrowed shares}$

**Risk:**
- The Measure of risk $\alpha_{\text{risk}}$ is the covariance of risk, $\alpha$ is a UCIP, $R_k$ is the portfolio return, $R_b$ is a benchmark portfolio return, and $D_R$ is the standard deviation of negative returns (losses).

**Key References**
- P. Das, N. Johnson, and A. Banerjee. Online resource allocation problem with additional resources to increase allocation power. SDM, 2012.
- N. Johnson and A. Banerjee. Online projected gradient descent based solution: $\min_{q \in \mathbb{R}^n} q^T L q + \frac{1}{2} \| p - p_0 \|_2^2$