### Section 1: What is the problem?

**Problem:** How to invest in a few top performing sectors while accounting for transaction costs in online portfolio selection.

- Investors group stocks as sectors by the type of business
- Not all sectors can yield profit.
- Sectors react differently during economic conditions.

**Goal:** Learn the underlying structure within stocks to identify top performing sectors.

### Section 2: Online Portfolio Selection

- Choose portfolio based on past stock performance: \( p_t = (p_t(1), \ldots, p_t(n)) \)
- Price relatives \( x(t) \) are the multiplicative factor by which a stock price changes
  - \( x(t) < 1 \) implies a loss
  - \( x(t) > 1 \) implies a gain
  - \( x(t) = 1 \) implies the price remained unchanged
- Maximize log gain in wealth: \( LS_T(p_1, \ldots, p_{T-1}, x_1, \ldots, x_T) = \sum_{t=1}^{T} \log(p_t^T x_t) \)

### Section 3: What is the solution?

**Solution:** Group sparsity inducing regularizer in an online framework.

- At time \( t \) select \( p_t \) such that the regret is sublinear in \( T \): \( R_T = \sum_{t=1}^{T} \psi(p_t) - \min_{p \in P} \sum_{t=1}^{T} \psi(p^*) \leq o(T) \)
  - \( \psi(p) = f_t(p) + \lambda_1 \Omega(p) + \lambda_2 ||p - p_{t-1}||_1 \)
    - \( f_t(\cdot) \) -- convex loss function at time \( t \)
    - \( \Omega(\cdot) \) -- groupwise \( L2 \) norm for group sparsity
    - \( ||\cdot||_1 \) -- \( L1 \) norm inducing lazy updates
  - Composite objective consisting of smooth and non-smooth terms.
  - Can pose online portfolio selection as special case with \( f_t(p) = -\log(p^T x_t) \)

### Section 4: Online Lazy Updates with Group Sparsity (OLU-GS)

**OLU-GS objective function:**

\[
p_{t+1} = \arg\min_{p \in P} (\nabla f_t(p) + \lambda_1 \Omega(p) + \lambda_2 ||p - p_t||_1 + \frac{1}{2\beta} ||p - p_t||_2^2)
\]

**ADMM Updates**

\[
\begin{align*}
&\hat{p}_{t+1} = \arg\min_{p \in P} (\nabla f_t(p) + \frac{1}{2} ||p - p_t||_2^2 + \frac{\beta}{2} ||p - y(t)||_2^2 + \frac{1}{2} ||p - y(t)||_2^2) \\
&y(t+1) = \arg\min_{y \in P} \lambda_1 \Omega(y) + \frac{1}{2} ||y - \hat{p}_{t+1}||_2^2 \quad \text{(Closed form)} \\
&w_{t+1} = w(t) + (p_{t+1} - y(t+1)) \quad \text{(Soft thresholding on groups of variables)}
\end{align*}
\]

**Analysis**

- \( f_t \) is general convex: \( R_T \leq O(\sqrt{T}) \)
- \( f_t \) is strongly convex: \( R_T \leq O(\log(T)) \)

### Section 5: Experiments and Results

**Datasets:**
- NYSE (36 stocks, 1962-1984): 8 sectors

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**Figure 1:** As \( \lambda \) increases, the number of days with high group lasso value and active groups decrease.

**Figure 2:** Cyclic and Non-cyclic sectors during bear and bull market.

**Figure 3:** Transaction cost-adjusted wealth.

**Figure 4:** (a) Total Group Lasso (b) Number of Active Group changes.

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**Acknowledgements:** The research was supported by NSF CAREER award IIS-0933274, NSF grants IIS-0916750, and IIS-1029711, and NASA grant NNX12AQ38A.