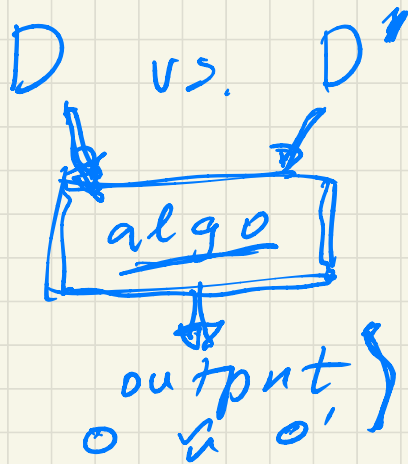



- anonymity (e.g. K-anon.)
 - aggregation
 - "no harm"
- DP: no harm to you specifically
due to your data



Randomized Response

- embarrassing questions:

"Have you ever shoplifted?"

"Do you believe in alien abductions?"

RR:

flip
coin
dice

H ~~P~~ ^{answer} truthfully

$\frac{1}{2}$

T ~~1-P~~ flip
again
(fair)

H \rightarrow "yes"

$\frac{1}{4}$

T \rightarrow "no"

$\frac{3}{4}$

Utility/accuracy analyses

- population of size n all follow $H \Rightarrow RR$
- use $\{\underline{y}, \underline{n}\}$ as underlying true answers & $\{\text{"y"}, \text{"n"}\}$ as responses to R/E.
- $t = \text{fraction of pop. with truth} = y$

$$\begin{aligned} Pr[\text{"y"} | \underline{y}] &= \underset{H}{(1/2)} \cdot 1 + \underset{T}{(1/2)} \underset{\text{"y"}}{(1/2)} \\ &= 1/2 + 1/4 = \textcircled{3/4} \end{aligned}$$

$$\begin{aligned} Pr[\text{"y"} | \underline{n}] &= \underset{H}{(1/2)} \cdot 0 + \underset{T}{(1/2)} \underset{\text{"y"}}{(1/2)} \\ &= \textcircled{1/4} \end{aligned}$$

$$\Pr["y"] = t \left(\frac{3}{4} \right) + (1-t) \left(\frac{1}{4} \right)$$

$$= \frac{1}{4} + t/2$$

frac. of
"y" responses
out

"y"

$$\Pr["g"] = q$$

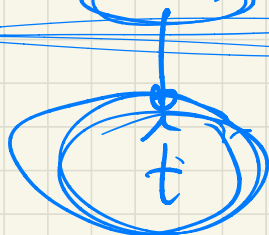
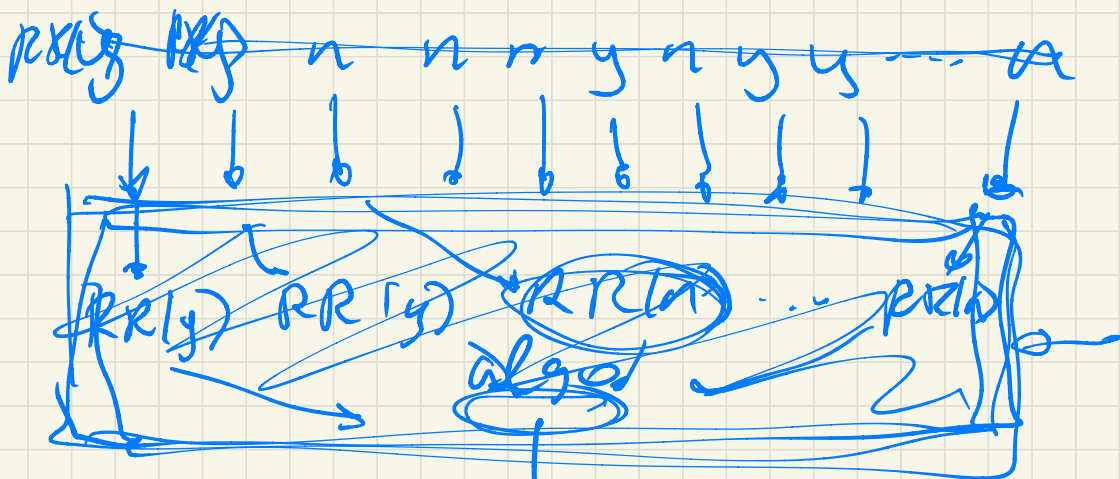
$$q = \frac{1}{4} + t/2 \quad \text{solve for } t$$

\hat{q} = estimate of q
from running exp. n times

Fact. With very high prob.

$$|\hat{q} - q| \leq 1/\sqrt{n} \rightarrow 0$$

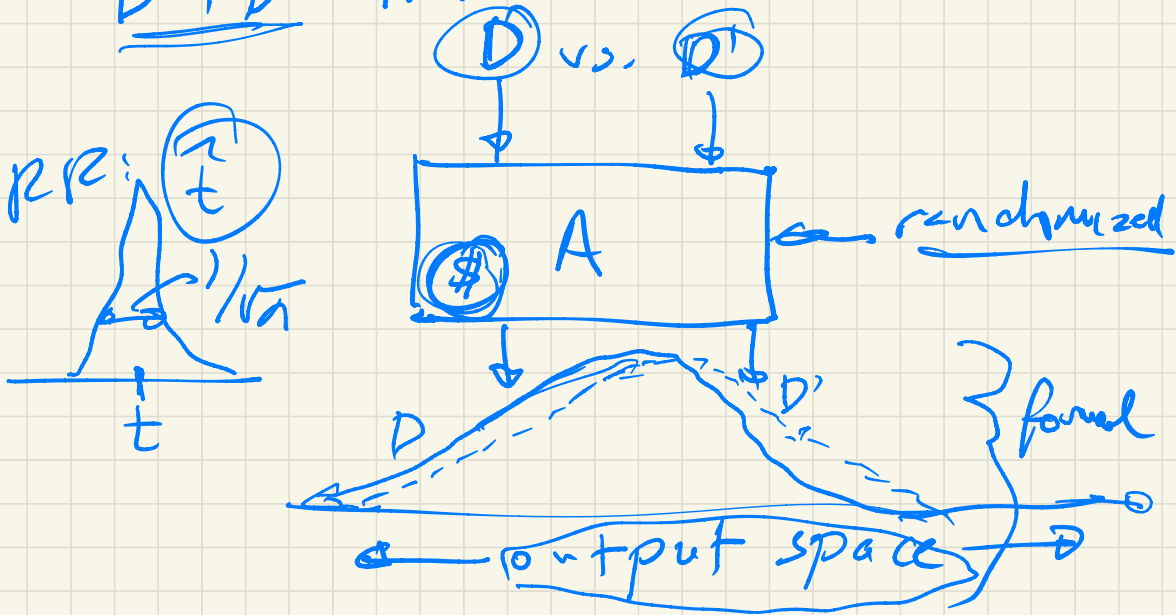
$$t \pm 1/\sqrt{n}$$



local (RR)
vs.
centralized
DP.

Definition of DP

We say that an algorithm A satisfies ϵ -DP if for any pair of neighboring inputs B/B_1 , $D \neq D'$ it:



Formally:

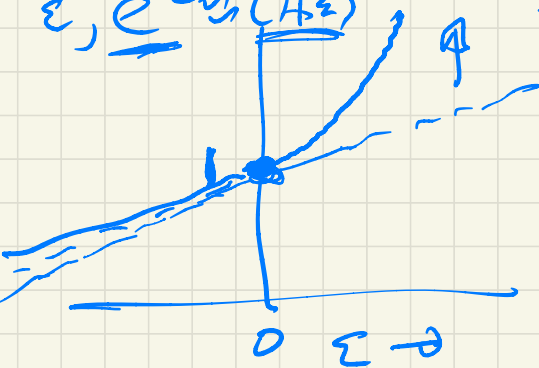
- Let $A(D), A(D')$ denote the random output of A under D, D'

We want that for any subset S of the output space of A

"bad"

$$\Pr[A(D) \in S] \leq e^\epsilon \Pr[A(D') \in S]$$

For small ϵ , $e^\epsilon \approx 1 + \epsilon$



$$\begin{aligned} e^{-\epsilon} \Pr[A(D') \in S] &\leq \Pr[A(D) \in S] \\ &\leq e^\epsilon \Pr[A(D') \in S] \end{aligned}$$