Structural Properties of Networks: Introduction

Networked Life

NETS 112

Fall 2013

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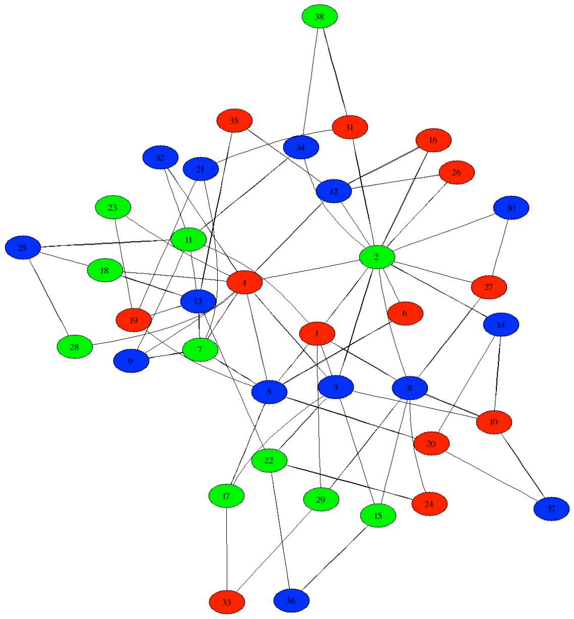
Networks: Basic Definitions

- A network (or graph) is:
  - a collection of individuals or entities, each called a vertex or node
  - a list of pairs of vertices that are neighbors, representing edges or links
- Examples:
  - vertices are mathematicians, edges represent coauthorship relationships
  - vertices are Facebook users, edges represent Facebook friendships
  - vertices are news articles, edges represent word overlap
- Networks can represent any binary relationship over individuals
- Often helpful to visualize networks with a diagram
- But to us, the network is the list of edges, not the visualization
  - same network has many different visualizations

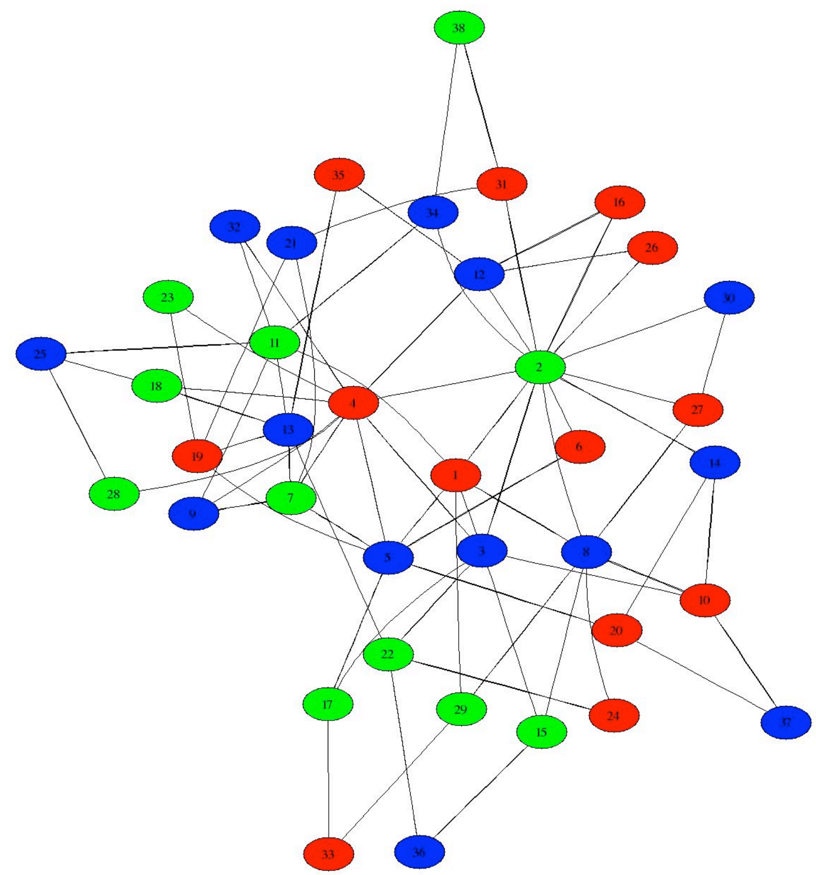


Networks: Basic Definitions

- We will use N to denote the number of vertices in a network
- Number of possible edges:

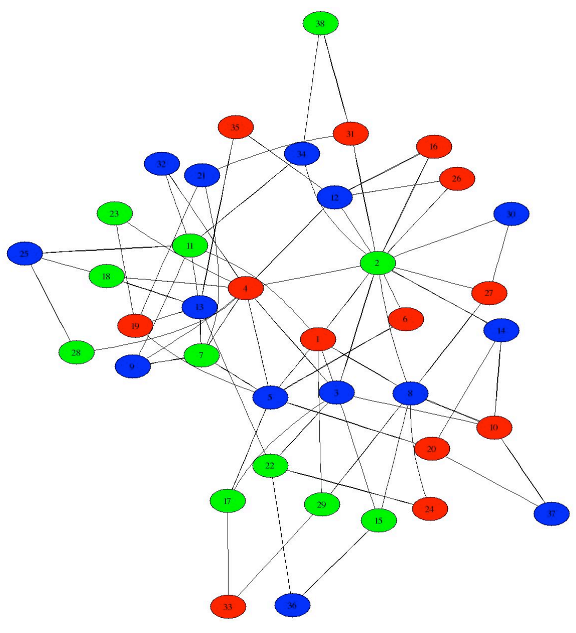
$$N(N-1)/2 \approx N^2/2$$

- The degree of a vertex is its number of neighbors



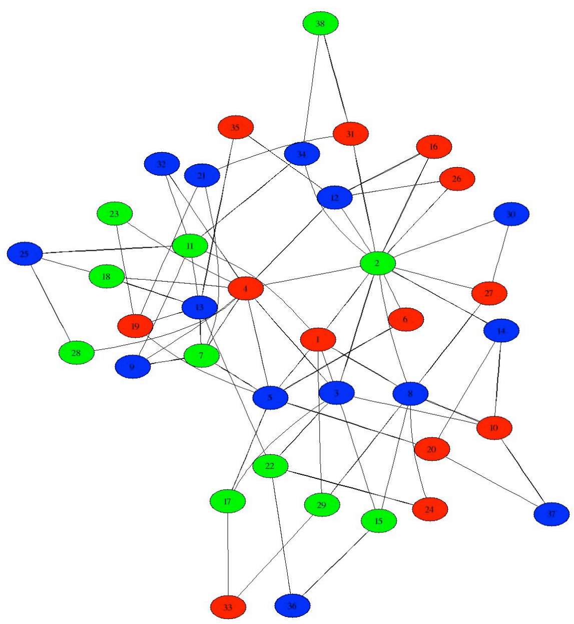
Networks: Basic Definitions

- The distance between two vertices is the length of the shortest path connecting them
- This assumes the network has only a single component or "piece"
- If two vertices are in different components, their distance is undefined or infinite
- The diameter of a network is the average distance between pairs
- It measures how near or far typical individuals are from each other



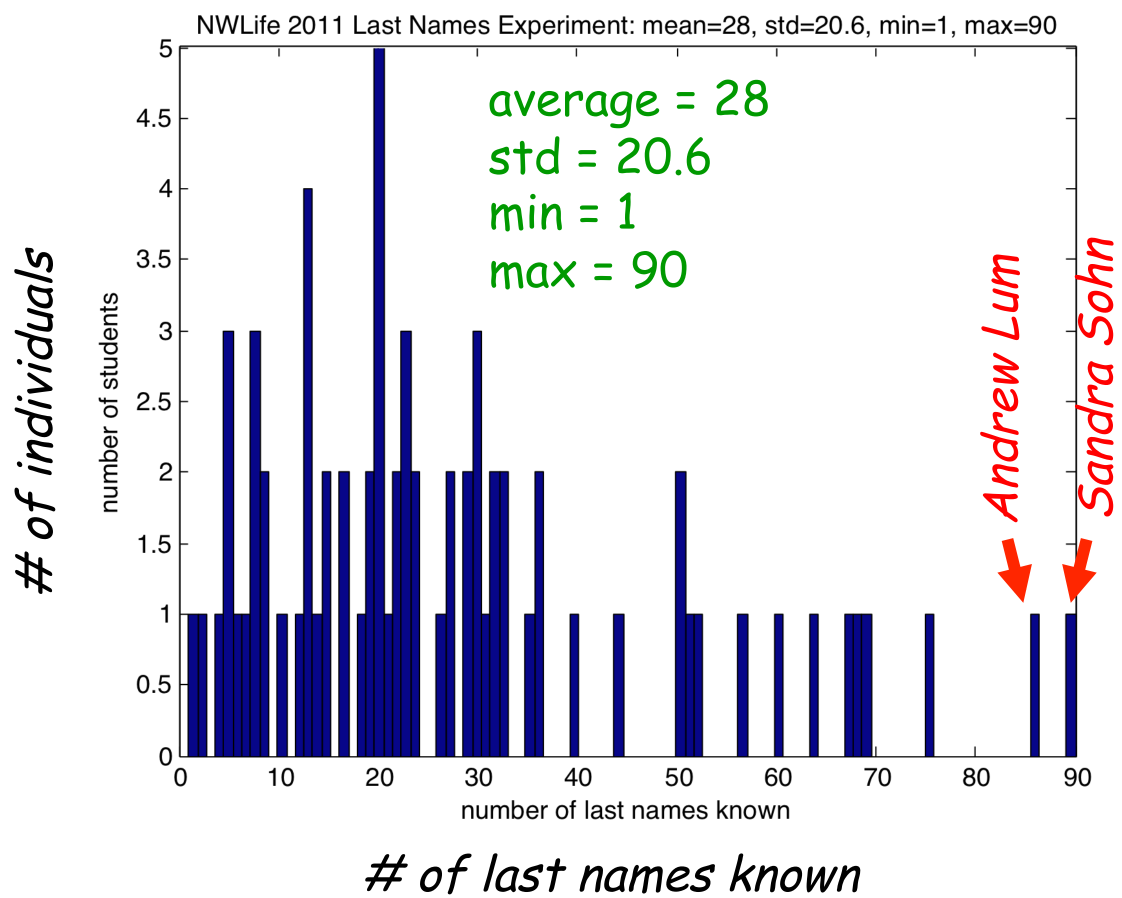
Networks: Basic Definitions

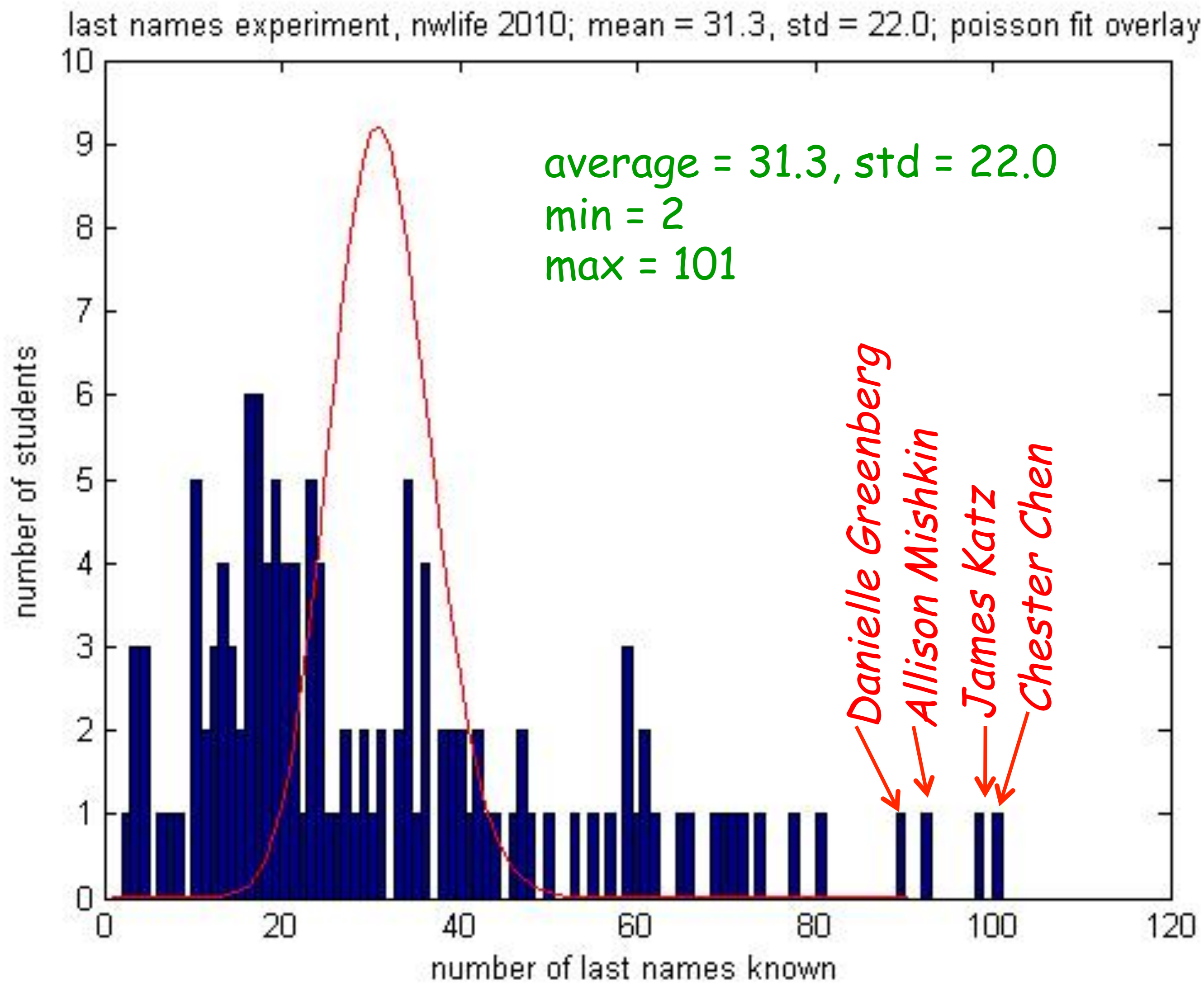
- So far, we have been discussing undirected networks
- Connection relationship is symmetric:
  - if vertex u is connected to vertex v, then v is also connected to u
  - Facebook friendship is symmetric/reciprocal
- Sometimes we'll want to discuss directed networks
  - I can follow you on Twitter without you following me
  - web page A may link to page B, but not vice-versa
- In such cases, directionality matters and edges are annotated by arrows

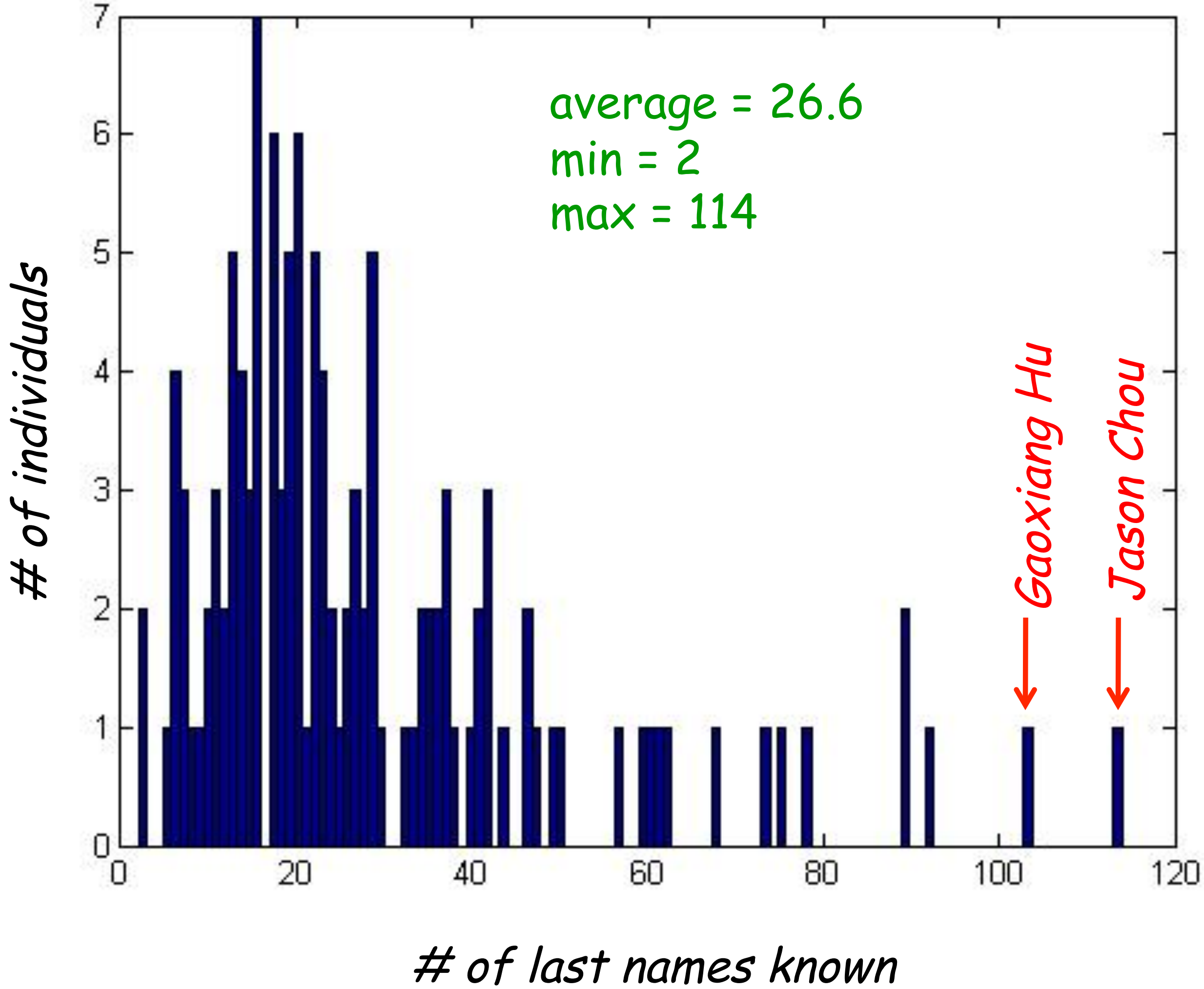


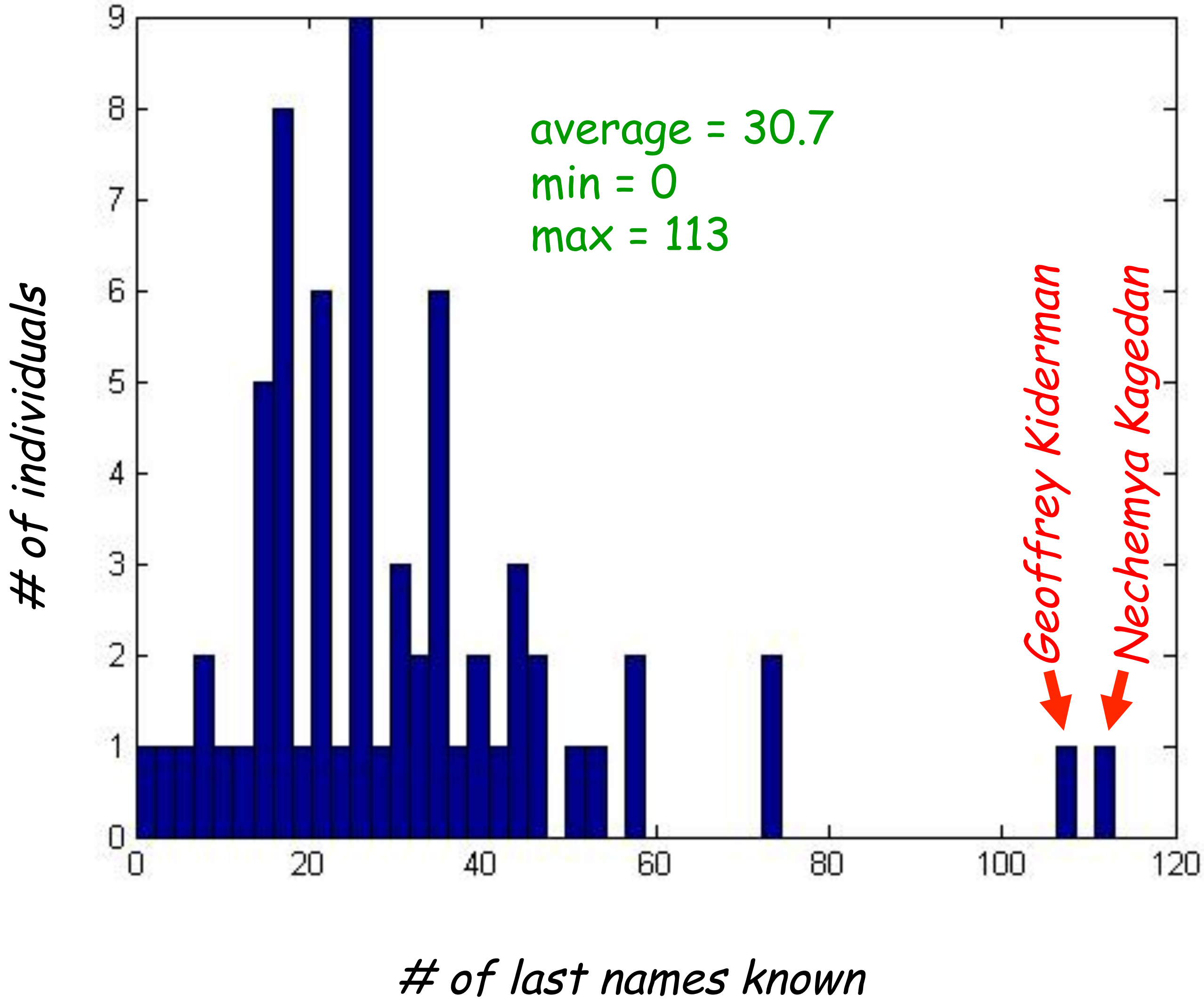
Illustrating the Concepts

- Example: scientific collaboration
  - vertices: math and computer science researchers
  - links: between coauthors on a published paper
  - *Erdos numbers* : distance to Paul Erdos
  - Erdos was definitely a *hub* or *connector;* had 507 coauthors
  - MK's Erdos number is 3, via Kearns → Mansour → Alon → Erdos
  - how do we *navigate* in such networks?
- Example: "real-world" acquaintanceship networks
  - vertices: people in the world
  - links: have met in person and know last names
  - hard to measure
  - let's examine the results of our own *last-names exercise*


NWLife 2011 Last Names Experiment: mean=28, std=20.6, min=1, max=90
average = 28
std = 20.6
min = 1
max = 90
Andrew Lum
Sandra Sohn
number of students
number of last names known
# of individuals
# of last names known


last names experiment, nwlife 2010; mean = 31.3, std = 22.0; poisson fit overlay
average = 31.3, std = 22.0
min = 2
max = 101
Danielle Greenberg
Allison Mishkin
James Katz
Chester Chen
number of students
number of last names known
0
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120


average = 26.6
min = 2
max = 114
Gaoxiang Hu
Jason Chou
# of individuals
# of last names known
0
1
2
3
4
5
6
7
0
20
40
60
80
100
120


average = 30.7
min = 0
max = 113
Geoffrey Kiderman
Nechemya Kagedan
# of individuals
# of last names known
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120

Structure, Dynamics, and Formation

Network *Structure (Statics)*

- Emphasize purely *structural* properties
  - size, diameter, connectivity, degree distribution, etc.
  - may examine statistics across many networks
  - will also use the term *topology* to refer to structure
- Structure can reveal:
  - community
  - "important" vertices, centrality, etc.
  - robustness and vulnerabilities
  - can also impose *constraints* on dynamics
- Less emphasis on what actually occurs *on* network
  - web pages are linked… but people surf the web
  - buyers and sellers exchange goods and cash
  - friends are connected… but have specific interactions

Network *Dynamics*

- Emphasis on what *happens* on networks
- Examples:
  - mapping spread of disease in a social network
  - mapping spread of a fad
  - computation in the brain
  - spread of wealth in an economic network
- Statics and dynamics often closely linked
  - rate of disease spread (dynamic) depends critically on network connectivity (static)
  - distribution of wealth depends on network topology
- Gladwell emphasizes dynamics
  - but often dynamics of *transmission*
  - what about dynamics involving deliberation, rationality, etc.?

Network *Formation*

- Why does a particular structure emerge?
- Plausible processes for network formation?
- Generally interested in processes that are
  - decentralized
  - distributed
  - limited to local communication and interaction
  - "organic" and growing
  - consistent with (some) measurement
- The Internet versus traditional telephony

Structure and Dynamics Case Study: A "Contagion" Model of Economic Exchange

- Imagine an undirected, connected network of individuals
  - no model of network formation
- Start each individual off with some amount of currency
- At each time step:
  - each vertex divides their current cash equally among their neighbors
  - (or chooses a random neighbor to give it all to)
  - each vertex thus also *receives* some cash *from* its neighbors
  - repeat
- A *transmission* model of economic exchange --- no "rationality"
- Q: How does network structure influence outcome?
- A: As time goes to infinity:
  - vertex i will have fraction deg(i)/D of the wealth; D = sum of deg(i)
  - degree distribution *entirely* determines outcome!
  - "connectors" are the wealthiest
  - not obvious: consider two degree = 2 vertices…
- How does this outcome change when we consider more "realistic" dynamics?
  - e.g. we each have goods available for trade/sale, preferred goods, etc.
- What other processes have similar dynamics?
  - looking ahead: models for web surfing behavior