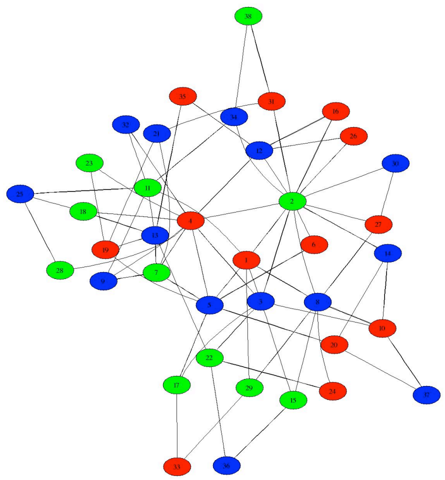
Navigation in Networks

Networked Life
NETS 112
Fall 2013
Prof. Michael Kearns

The Navigation Problem

- You are an individual (vertex) in a very large social network
- You want to find a (short) chain of friendships to another individual
- You don't have huge computers and a global/bird's-eye view
- All you (hopefully) know is who your neighbors/friends are
  - …and perhaps information about them (age, interests, religion, address, job,…)
- You can ask your friends to make introductions, which lead to more
- How would you do it?
- Also known as search in networks and the "small world problem"
- Small diameter is necessary but not sufficient!
  - …navigation is an algorithmic problem
- Related to the problem of routing data packets in the Internet



Small Worlds and the Law of the Few

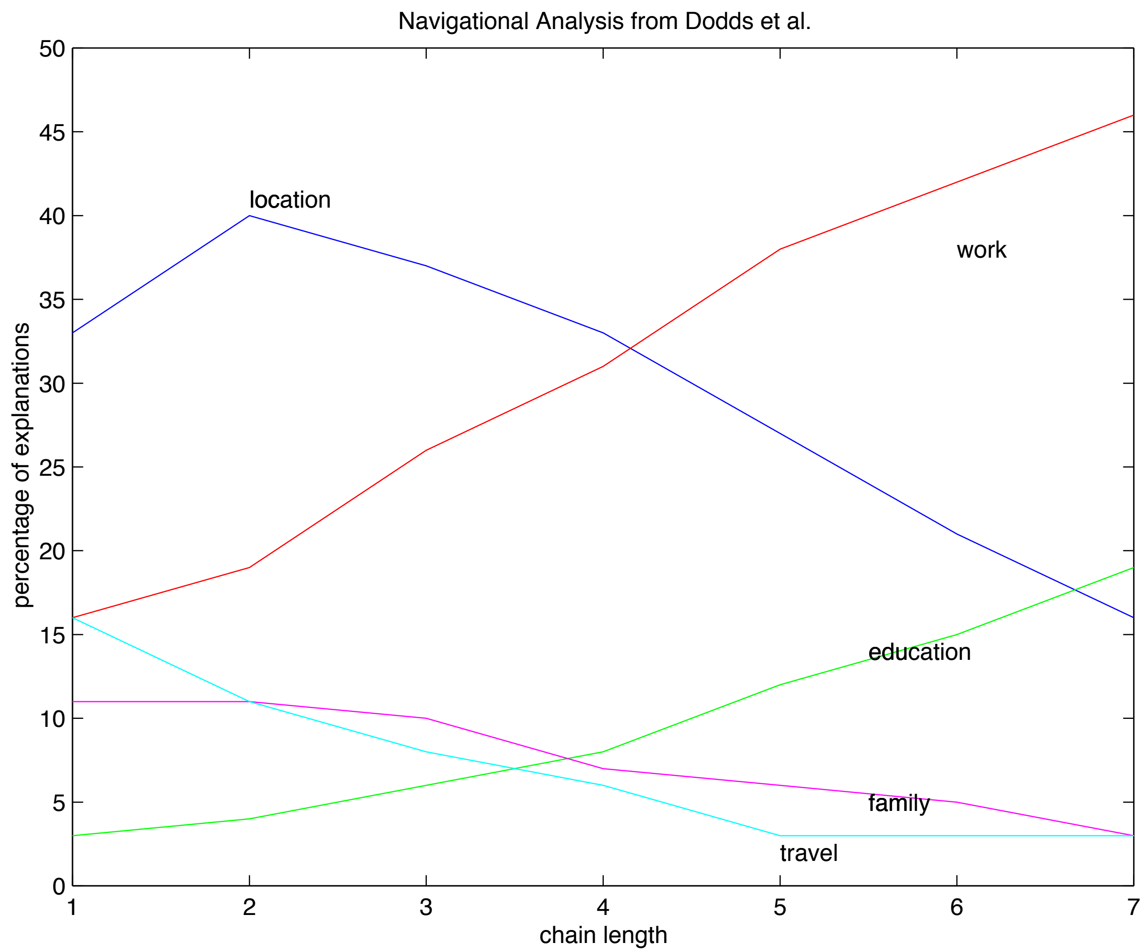
- Travers & Milgram 1969: classic early social network study
  - destination: a Boston stockbroker; lived in Sharon, MA
  - sources: Nebraska stockowners; Nebraska and Boston "randoms"
  - forward letter to a first-name acquaintance "closer" to target
  - target information provided:
    - name, address, occupation, firm, college, wife's name and hometown
    - navigational value?
- Basic findings:
  - 64 of 296 chains reached the target
  - average length of *completed* chains: 5.2
    - interaction of chain length and navigational difficulties
  - main approach routes: home (6.1) and work (4.6)
  - Boston sources (4.4) faster than Nebraska (5.5)
  - no advantage for Nebraska stockowners

The Connectors to the Target

- T & M found that many of the completed chains passed through a very small number of penultimate individuals
  - Mr. G, Sharon merchant: 16/64 chains
  - Mr. D and Mr. P: 10 and 5 chains
- Connectors are individuals with extremely high degree
  - why should connectors exist?
  - how common are they?
  - how do they get that way? (see Gladwell for anecdotes)
- Connectors can be viewed as the "hubs" of social traffic
- Note: no reason *target* must be a connector for small worlds
- Two ways of getting small worlds (low diameter):
  - truly random connection pattern → dense network
  - a small number of well-placed connectors in a sparse network

Small Worlds: A Modern Experiment

- The Columbia Small Worlds Project:
  - considerably larger subject pool, uses email
  - subject of Dodds et al. assigned paper
- Basic methodology:
  - 18 targets from 13 countries
  - on-line registration of initial participants, all tracking electronic
  - 99K registered, 24K initiated chains, 384 reached targets
- Some findings:
  - < 5% of messages through any penultimate individual
  - large "friend degree" rarely (< 10%) cited
  - Dodds et al: → no evidence of connectors!
    - (but could be that connectors are not cited for this reason…)
  - interesting analysis of reasons for forwarding
  - interesting analysis of navigation method vs. chain length


Navigational Analysis from Dodds et al.
percentage of explanations
chain length
location
work
education
family
travel
0
5
10
15
20
25
30
35
40
45
50
1
2
3
4
5
6
7

The Strength of Weak Ties

- Not all links are of equal importance
- Granovetter 1974: study of job searches
  - 56% found current job via a personal connection
  - of these, 16.7% saw their contact "often"
  - the rest saw their contact "occasionally" or "rarely"
- Your "closest" contacts might not be the most useful
  - similar backgrounds and experience
  - they may not know much more than you do
  - connectors derive power from a large fraction of weak ties
- Further evidence in Dodds et al. paper
- T&M, Granovetter, Gladwell: multiple "spaces" & "distances"
  - geographic, professional, social, recreational, political,…
  - we can reason about general principles without precise measurement

The Magic Number 150

- Social channel capacity
  - correlation between neocortex size and group size
  - Dunbar's equation: neocortex ratio → group size
- Clear implications for many kinds of social networks
- Again, a *topological* constraint on typical degree
- From primates to military units to Gore-Tex

Neocortex size and group size in primates

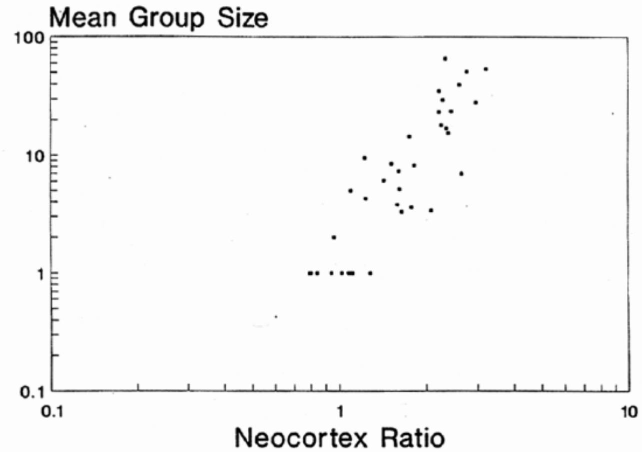


Figure 1. Group size plotted against neocortex ratio for nonhuman primates (redrawn from Dunbar 1992a).

Summary, and a Mathematical Digression

- So far:
  - large-scale social networks reliably have high-degree vertices
  - large-scale social networks have small diameter
  - furthermore, people can find or navigate the short paths from only local, distributed knowledge
  - these properties are true of other types of networks, too

- But there must be some limits to degrees
  - can't be "close friends" with too many people (150? 1000?)
- Large N, small diameter and limited degrees are in tension
  - not all combinations are possible
- Let N be population size, Δ be the maximum degree, and D be the diameter
- If $\Delta = 2$ then must have $D \sim N/4$ ($\gg 6$, $\gg \log(N)$)

Summary, and a Mathematical Digression

- The relationship between Δ, D and N has been studied mathematically
- For fixed Δ and D, largest N can be is

$$N \leq \Delta^D$$

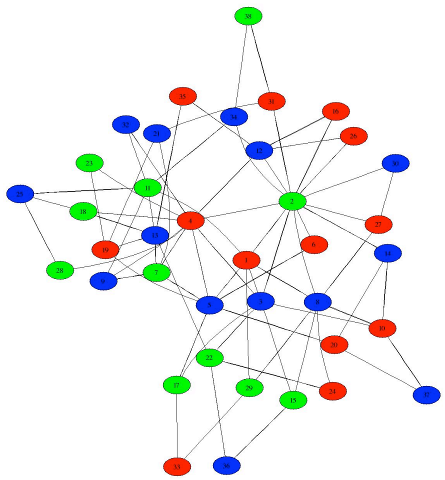
- For example: if N = 300M (U.S. population) and $\Delta = 150$, get constraint on D:

$$300{,}000{,}000 \leq (150)^D$$
$$\log(300{,}000{,}000) \leq D \log(150)$$
$$D \geq 3.9$$

- So calculation consistent with reality (whew!)
- More generally: multiple structural properties may be *competing*

Two Aspects of Navigation

- In order for people (or machines) to find short paths in networks:
  - short paths must exist (structural; small diameter)
  - people must be able to find the short paths via only local forwarding (algorithmic)
- The algorithmic constraints are strong (Travers & Milgram)
  - only know your neighbors in the network
  - limited information about the target/destination (physical location, some background)
- Look at a model incorporating structural and algorithmic constraints

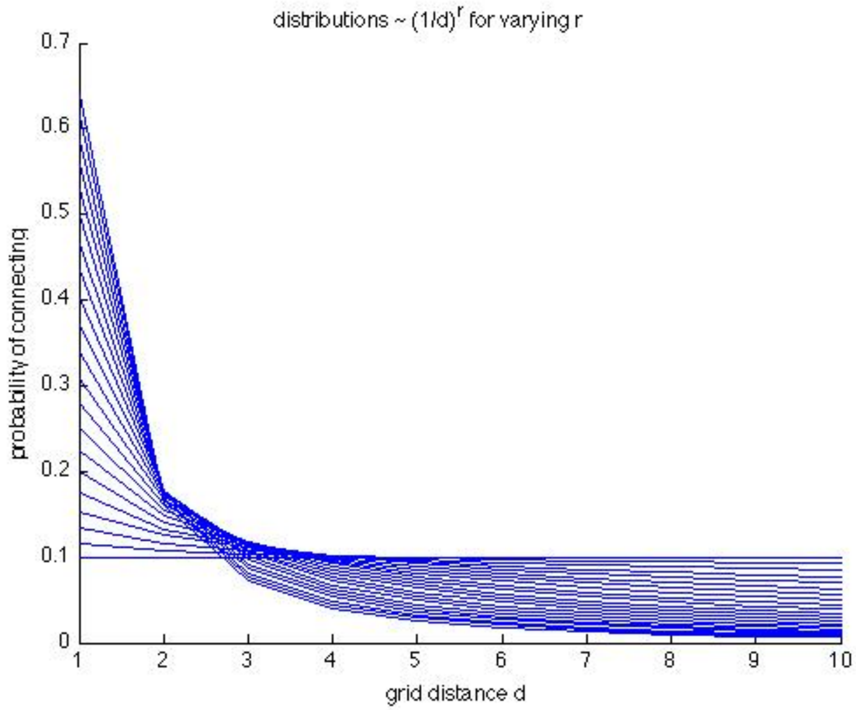


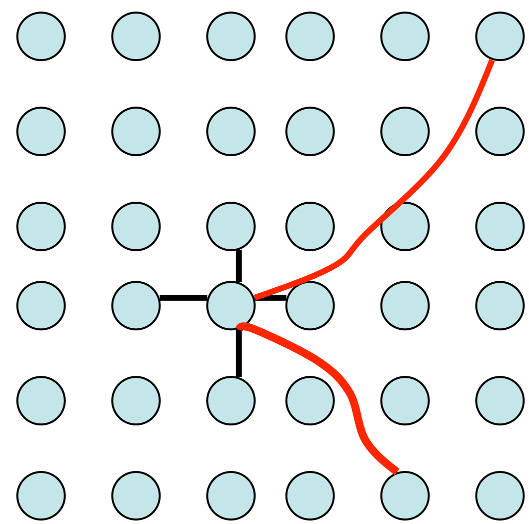
Kleinberg's Model

- Start with an k by k *grid* of vertices (so N = k^2)
  - each vertex connected to compass neighbors
  - add a few random "long-distance" connections to each vertex
  - probability p(d) of connecting to a vertex at grid distance d:

$$p(d) \propto (1/d)^r, r \geq 0$$

  - large r: heavy bias towards "more local" long-distance connections
  - small r: approach uniformly random



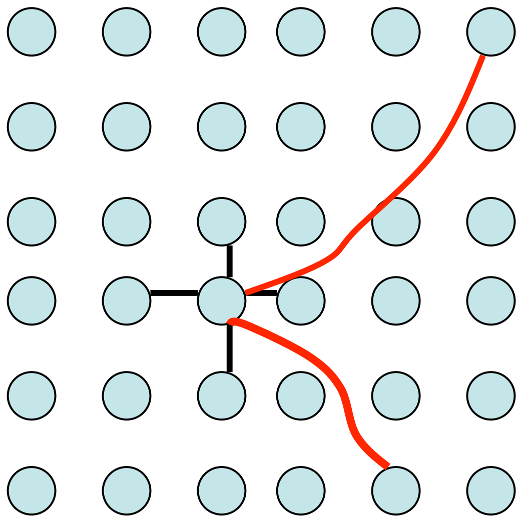


Kleinberg's Question

- Which values of r:

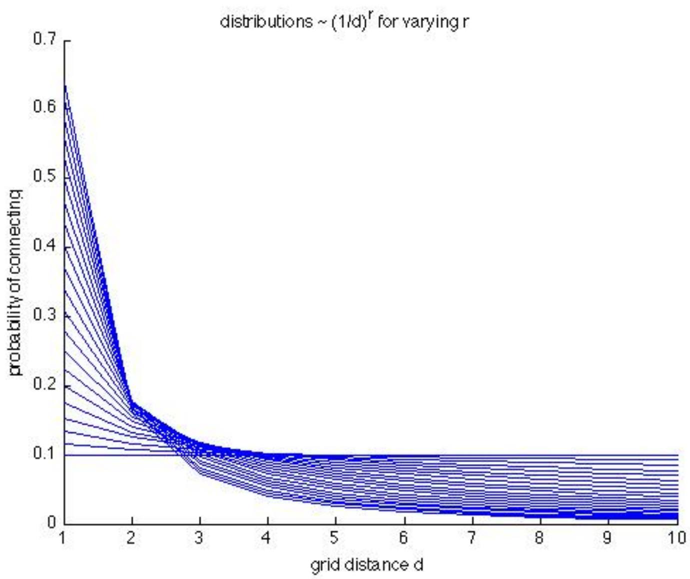
$$p(d) \propto (1/d)^r, r \geq 0$$

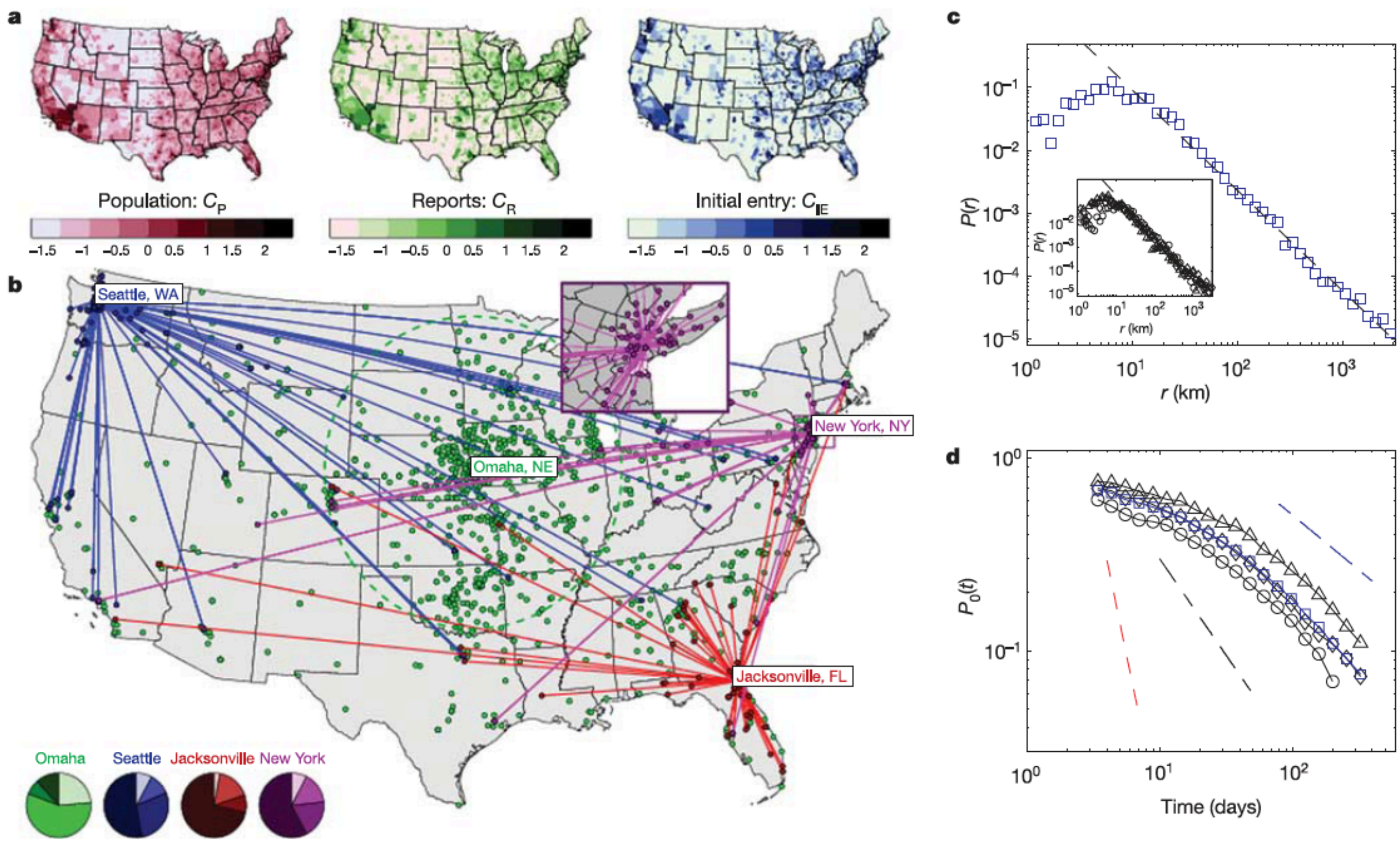
  permit efficient navigation?
- Efficient: number of hops << N, e.g. log(N)
- Algorithmic assumption:
  - vertices know the grid addresses of their neighbors
  - vertices know the grid address of the target (Sharon, MA)
  - vertices always forward the message to neighbor closest to the target in grid distance
  - no "backwards" steps, even if helpful
  - purely geographic information



Kleinberg's Result

- Intuition:
  - if r is too *large* (strong local bias), then "long-distance" connections never help much; short paths may not even *exist*
  - if r is too *small* (no local bias), we may quickly get close to the target; but then we'll have to use grid links to finish
  - effective search requires a delicate *mixture* of link distances
- The result (informally): as N becomes large:
  - r = 2 is the *only value* that permits rapid navigation (~log(N) steps)
  - a "knife's edge" result; very sensitive
- Note: *locality of information* crucial to this argument
  - At r <= 2, will have small diameter, but local algorithms can't find the short paths

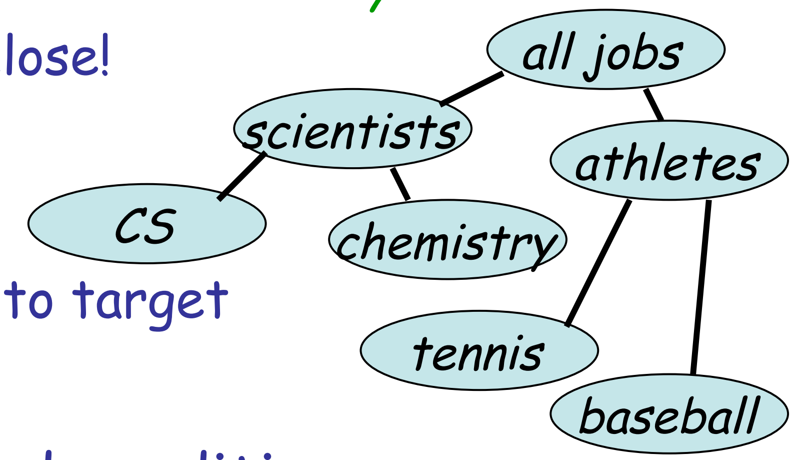




From Brockmann, Hufnagel, Geisel (2006) *Best-fit value of r = 1.59*

Navigation via Identity

- Watts et al.:
  - we don't navigate social networks by purely "geographic" information
  - we don't use any *single* criterion; recall Dodds et al. on Columbia SW
  - different criteria used at different points in the chain
- Represent individuals by a *vector* of attributes
  - profession, religion, hobbies, education, background, etc…
  - attribute values have distances between them (tree-structured)
  - distance between individuals: minimum distance in *any* attribute
  - only need *one thing in common* to be close!
- Algorithm:
  - given attribute vector of target
  - forward message to neighbor closest to target
- Let's look a bit at the paper
- Permits fast navigation under broad conditions
  - not as sensitive as Kleinberg's model



Summary

- Efficient navigation has both structural and algorithmic requirements
- Kleinberg's model and question captures both
- Result predicts delicate mixture of connectivity for success
- Not too far from reality? (Where's George? data)
- Watts et al. provide more "sociological" model
- More complex, but less sensitive