

MIDTERM EXAMINATION
Networked Life (NETS 112)
November 21, 2013
Prof. Michael Kearns

This is a closed-book exam. You should have no material on your desk other than the exam itself and a pencil or pen. If you run out of room on a page, you may use the back, but be sure to indicate you have done so.

Name: _____

Penn ID: _____

Problem 1: _____/10

Problem 2: _____/20

Problem 3: _____/16

Problem 4: _____/16

Problem 5: _____/20

Problem 6: _____/18

TOTAL: _____/100

Problem 1 (10 points) Answer “True” or “False” for each of the following assertions.

a. If we plot a heavy-tailed distribution on a log-log scale, the line will be strongly curved.

False – The line will be straight.

b. The edge density of a network G is equal to the probability that a random pair of vertices in G are connected by an edge.

True – If you pick a random pair of vertices, the probability that there is an edge between them is the number of edges divided by number of total possible edges, which equals the edge density.

c. Nash proved that every game has a pure strategy equilibrium.

False – He proved that every game has a mixed strategy equilibrium. Note that mixed equilibria include pure strategy equilibria.

d. In the ring-rewiring network formation model, if $q = 1$, the network will exhibit low clustering.

True – At high q , we rewired lots of edges, and therefore have low clustering.

e. The page rank of a vertex depends only on the in- and out-degrees of the vertex and its neighbors.

False – Also depends on rank of the incoming neighbors.

f. The network property of having an average shortest path distance of length at most k exhibits a threshold phenomenon in the Erdos-Renyi model.

True – This is a monotone property. Adding more edges can only decrease the average shortest path distance.

g. In Schelling's segregation model, the payoff of a vertex depends only on the choices of the vertex and its neighbors.

True – Payoff only depends of the color choices of your immediate neighborhood.

h. There are equilibria of the consensus game in which not every player chooses the same color.

True - We can have equilibria where players are “stuck” playing different colors.

i. In the coloring game behavioral experiments, the easiest network for human subjects to solve is a 2-colorable cycle.

False – This is (surprisingly) the hardest network to solve.

j. In the network trading model, a given network may have more than one set of equilibrium prices and trades.

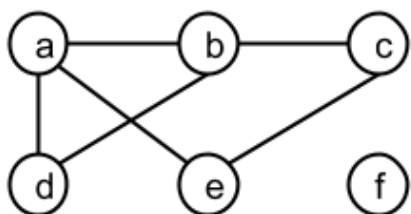
False – Equilibrium is completely determined by the network structure.

Problem 2 (20 points)

a) (8 points) Fill in the following chart by putting a checkmark in a box if the network formation model creates networks that exhibit the property.

	Giant Component	Heavy Tailed Distribution	Small Diameter	High Clustering
Erdos-Renyi	X		X	
Alpha model	(both accepted)		X	X
Ring rewiring	X		X	X
Preferential Attachment	X	X	X	

Suppose we are generating a network according to one of the formation models, and at some intermediate point we have the network below.



b) (4 points) Suppose the formation model is Erdos-Renyi. What is the probability that we add the a-f edge in the next step? (Consider the version of Erdos-Renyi in which we connect random pairs of unconnected vertices.)

There are 9 remaining edges, and each is equally likely to be added, so $1/9$.

Half credit was given for the following answers:

- Some interpreted this question as in part c), where the only candidate edges are those involving f. In this case, there are 5 possible edges, and each is equally likely to be added, so $1/5$.
- We specified that this is the first (and main) version of Erdos-Renyi, but if you used the other version, the probability is equal to the edge density, which is $6/15$.

c) (4 points) Suppose the formation model is preferential attachment. What is the probability that we add the a-f edge in the next step? (Assume the only candidate edges are those involving the “new” vertex f.)

The probability is the degree of a divided by the sum of all vertex degrees, or $3/12 = 1/4$.

d) (4 points) Suppose the formation model is the alpha model. List the following four edges in decreasing order of their probability of being added in the next step: a-f, b-f, d-e, b-e. Clearly indicate if there are ties.

b-e have 2 neighbors in common (highest probability)

d-e have 1 neighbor in common (second highest probability)

a-f and b-f have 0 neighbors in common (tied for lowest probability)

Problem 3 (16 points)

a) (8 points) Recall the PageRank algorithm discussed in lecture. Write the equation that we use to update the rank of each vertex. Explain what each variable represents.

$$R(p) = \sum_{q \in \text{POINTS}(p)} R(q) / \text{out}(q)$$

b) (4 points) Describe the algorithm's termination condition.

When the updates no longer change *any* values. In other words, we run a complete iteration, and *none* of the values change.

c) (4 points) Suggest a strategy that an owner of a webpage might use to increase his rank.

Pay some pages to link to yours, etc.

Incorrect answers: link to other influential pages, or link to fewer pages. Neither of these affect your page rank.

My favorite answer: get the New York Times to replace its entire site with a link to your page. Not sure how this would be accomplished, but it would certainly be effective.

Problem 4 (16 points)

a) (4 points) Consider the following payoff matrix, in which the variables $a, b, c,$ etc. represent the numerical payoffs of the two players. Describe the necessary and sufficient conditions for (Action 1, Action 1) to be an equilibrium. Your conditions should be expressed in terms of the payoff variables.

	Action 1	Action 2
Action 1	(a, b)	(c, d)
Action 2	(e, f)	(g, h)

$$a > e$$

$$b > d$$

These conditions are both necessary and sufficient. If $a > e$, the row player will not want to change, given that the column player is staying in the first column. If $b > d$, the column player will not want to change, given that the row player is staying in the first row.

b) (4 points) Create a payoff matrix in which the equilibrium does not lead to optimal social welfare.

Lots of possibilities; most commonly cited was some version of Prisoner's Dilemma:

	Action 1	Action 2
Action 1	(-8, -8)	(0, -10)
Action 2	(-10, 0)	(-1, -1)

(Action 1, Action 1) is an equilibrium, but the social welfare is -16, whereas (Action 2, Action 2) has a better social welfare of -2.

c) (4 points) Explain what it means for a game to have a mixed strategy equilibrium.

Each player maintains a probability distribution over actions, and in each round, he privately “flips coins” to choose an action from this distribution. This is an equilibrium if no player wants to change his distribution, fixing the distributions of the other players.

d) (4 points) Explain how a correlated equilibrium is different from a mixed equilibrium, and describe a potential advantage of a correlated equilibrium.

There is an element of “shared randomization”. Based on this “public signal”, players choose their action. (Note that they do *not* choose actions together, or choose their action based on each other's choices.) Correlated equilibrium can lead us to more socially optimal solutions (e.g. traffic light).

Problem 5 (20 points)

(8 points) For a) the Beauty Contest Game and b) the Ultimatum Game, describe the theoretical equilibrium of the game, and describe the observed behavior of human subjects. If the human subject behavior is different from the equilibrium prediction, suggest an explanation.

a) Beauty Contest: Theoretical equilibrium is for everyone to choose 0. But because of “bounded rationality”, players cannot perform infinite rounds of iterated reasoning, and instead choose higher numbers (around 20 or so).

b) Ultimatum Game: Theoretical equilibrium is for Player A to offer the smallest possible amount to Player B, and for Player B to accept. Due to “inequality aversion”, Player A will usually offer between 20-50%, and Player B will refuse offers that are lower than 20%.

c) (4 points) Describe the effect of the rewiring probability q on behavioral performance in the coloring and consensus games. In general, which game was easier for subjects to solve?

As we increase q , consensus becomes easier, and coloring becomes harder. But in general, coloring was easier to solve (easier to get people to disagree than to agree).

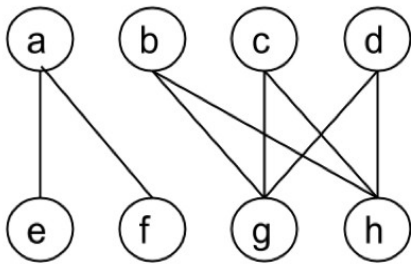
d. (4 points) Describe the network used in the minority power biased voting experiments. How is this network generated, and how are the payoffs for the players chosen?

This network was generated by preferential attachment – we have a few high-degree connector vertices, and then the remainder are of low-degree. The high-degree minority was given preference for one color, and the majority was given preference for the other color.

f. (4 points) Did the minority power experiments result in optimal social welfare? Why or why not?

No. Most times, these experiments converged to the color preference of the minority. But this is not socially optimal, since the majority receives lower payoff than possible.

Problem 6 (18 points)



a) (4 points) Find the set S that contributes to the greatest wealth inequality at equilibrium. State the members of S and $N(S)$.

$$S = \{e, f\}$$
$$N(S) = \{a\}$$

$v(S) = 2/1 = 2$, which is the biggest $v(S)$ you can get in this graph

b) (6 points) Calculate the equilibrium wealth of each vertex.

$$a = 2$$
$$b = 2/3$$
$$c = 2/3$$
$$d = 2/3$$
$$e = 1/2$$
$$f = 1/2$$
$$g = 3/2$$
$$h = 3/2$$

c) (4 points) In the network trading behavioral experiments, what was the easiest type of network for human subjects to solve, and why?

A pair network, in which each vertex is connected to one vertex of the opposite type. Note that having a perfect matching is not enough, because it may be hard to a vertex to know *which* of the vertices he is connected to is the “right” one. And being bipartite is certainly not enough – just because a graph is bipartite does not mean it has a perfect matching.

d) (4 points) In the network trading behavioral experiments, what is the best model for behavioral wealths? Suggest an explanation.

The answer we were looking for was : $0.25 * \text{uniform distribution} + 0.75 * \text{equilibrium prediction}$ – this is due to inequality aversion (see slide 31, or Coursera video for a further explanation).

Full credit was also given for answers that discussed how networks with low equilibrium variation led to better behavioral performance.