Contagion in Networks

Networked Life
NETS 112
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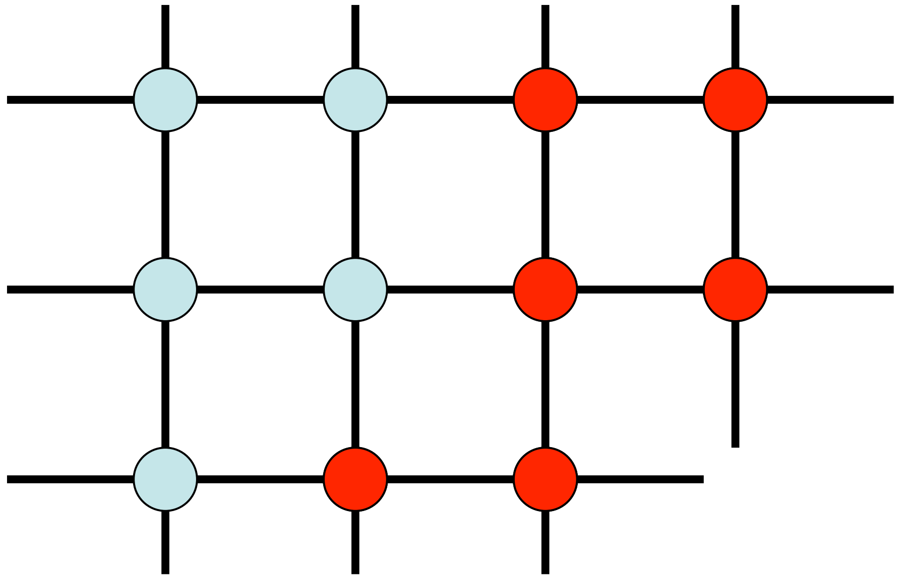
"Epidemos"

- Forest fire simulation:
  - grid of forest and vacant cells
  - fire always spreads to adjacent four cells
    - "perfect" stickiness or infectiousness
  - *connectivity* parameter:
    - probability of forest
  - fire will spread to all of *connected component* of source
  - tip when forest ~ 0.6
  - clean mathematical formalization (e.g. fraction burned)
- Viral spread simulation:
  - population on a grid network, each with four neighbors
  - *stickiness* parameter:
    - probability of passing disease
  - *connectivity* parameter:
    - probability of rewiring local connections to random long-distance
  - no long distance connections: tip at stickiness ~ 0.3
  - at rewiring = 0.5, often tip at stickiness ~ 0.2

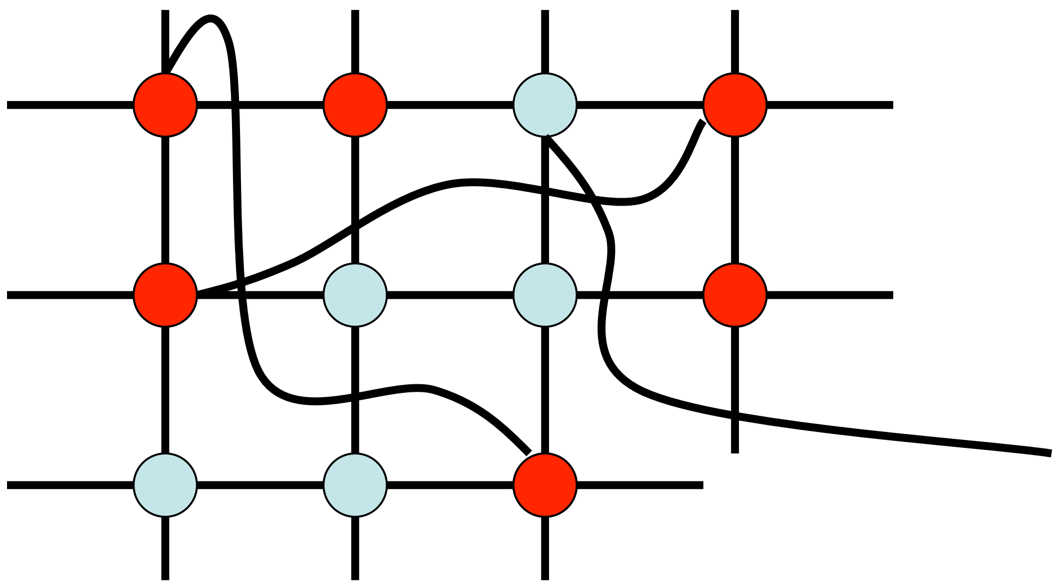
"Mathematizing" the Forest Fire

- Start with a regular 2-dimensional grid network
  - this represents a complete forest
- Delete each vertex (and its edges) with probability p (independently)
  - this represents random "clear-cutting" or natural fire breaks
- Choose a random remaining vertex v
  - this is my campsite
- Q: What is the expected size of v's *connected component?*
  - this is how much of the forest is going to burn



"Mathematizing" the Epidemic

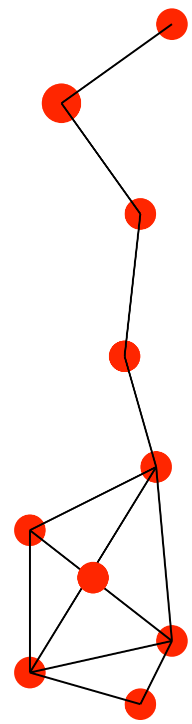
- Start with a regular 2-dimensional grid network
  - this represents a dense population with "local" connections (neighbors)
- *Rewire* each edge with probability p to a *random* destination
  - this represents "long-distance" connections (chance meetings)
- Choose a random remaining vertex v
  - this is an infection; spreads *probabilistically* to each of v's neighbors
- Fraction killed more complex:
  - depends on both size and *structure* of v's connected component
- Important theme:
  - mixing regular, local structure with random, long-distance connections



Some Remarks on the Demos

- Connectivity patterns were either *local* or *random*
  - will eventually formalize such models
  - what about other/more realistic structure?
- Tipping was inherently a *statistical* phenomenon
  - probabilistic nature of connectivity patterns
  - probabilistic nature of disease spread
  - model *likely* properties of a large *set* of possible outcomes
  - can model either inherent randomness or variability
- Formalizing tipping in the forest fire demo:
  - might let grid size N → infinity, look at fixed values of p
  - is there a threshold value q:
    - $p < q$ → expected fraction burned < 1/10
    - $p > q$ → expected fraction burned > 9/10

Structure and Dynamics Case Study: A "Contagion" Model of Economic Exchange



- Imagine an undirected, connected network of individuals
  - no model of network formation
- Start each individual off with some amount of currency
- At each time step:
  - each vertex divides their current cash equally among their neighbors
  - (or chooses a random neighbor to give it all to)
  - each vertex thus also *receives* some cash *from* its neighbors
  - repeat
- A *transmission* model of economic exchange --- no "rationality"
- Q: How does network structure influence outcome?
- A: As time goes to infinity:
  - vertex i will have fraction deg(i)/D of the wealth; D = sum of deg(i)
  - degree distribution *entirely* determines outcome!
  - "connectors" are the wealthiest
  - not obvious: consider two degree = 2 vertices…
- How does this outcome change when we consider more "realistic" dynamics?
  - e.g. we each have goods available for trade/sale, preferred goods, etc.
- What other processes have similar dynamics?
  - looking ahead: models for web surfing behavior