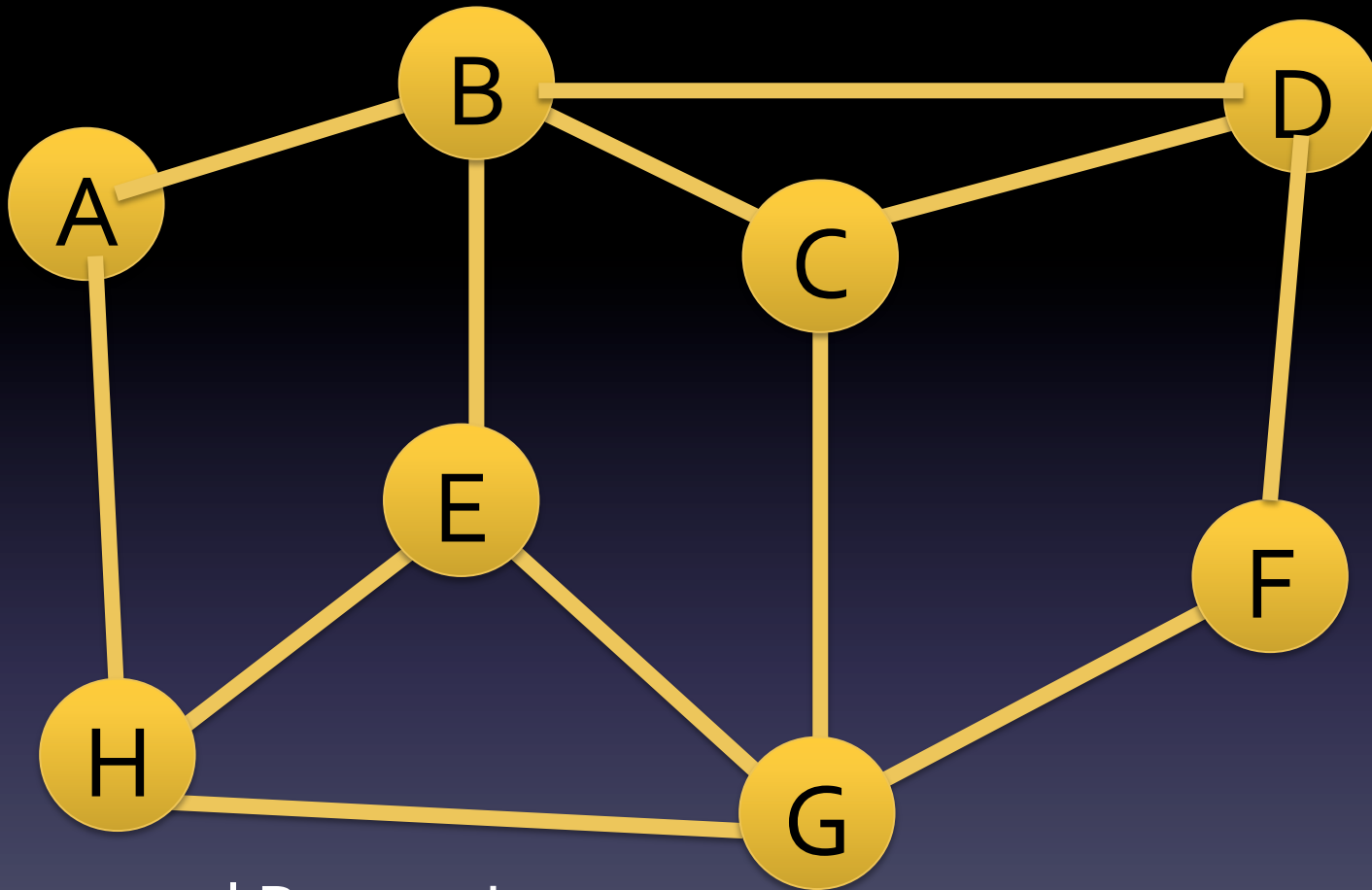


NETS 112 Final Review

December 9, 2014

Shahin Jabbari and Ryan Rogers

Networks



Structural Properties:

- Degree, Degree Distribution, Distance, Diameter, Clustering Coefficient.

Examples from Class

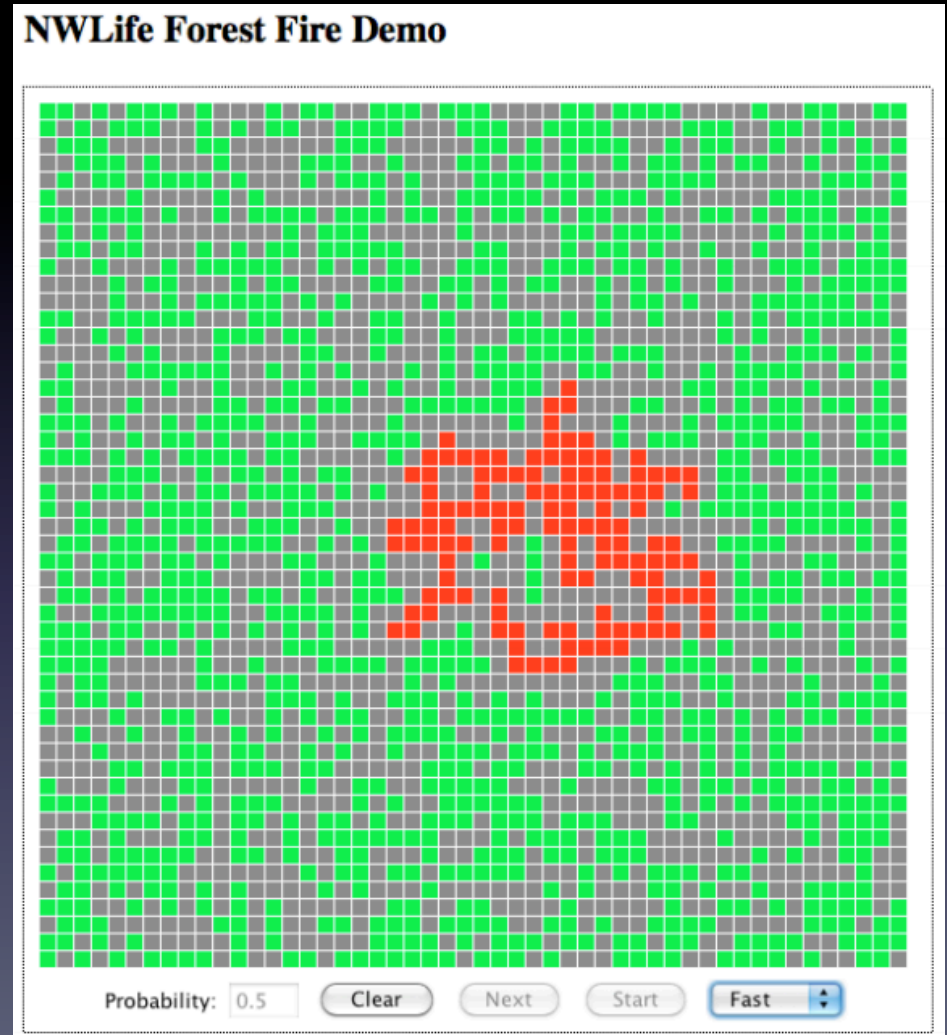
- Erdos Number Project
- Last Name Experiment
- Toss a ball from one person in the class to another to reach a specific person (local vs global info)

Dynamics in Networks

- Network Structure (static) definitely impacts a process (dynamics) in a network
- 2 Demos showing **Contagion in Networks:**
 - Forest Fire
 - Average Degree

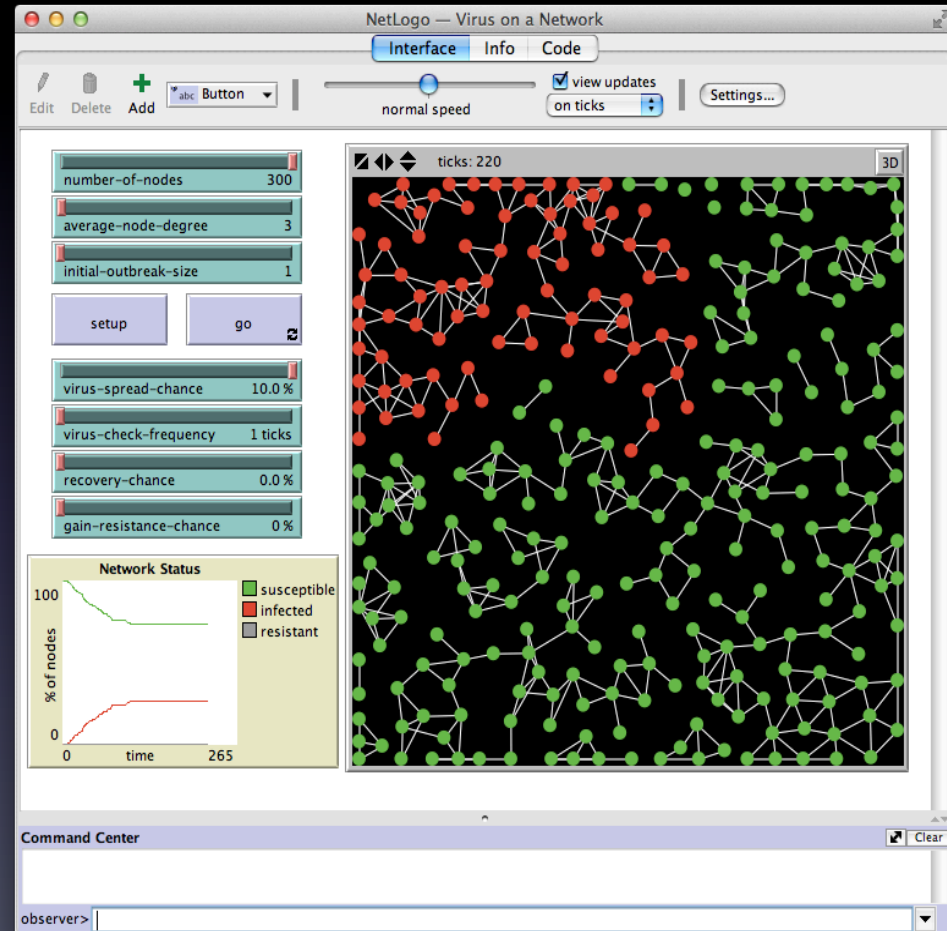
Forest Fire Demo

- What Network did this use?
 - Lattice and then deleted some nodes
- What parameter can we adjust?
 - Probability of keeping a node
- Describe what is meant by a tipping point at $p = 0.6$
 - There is a sharp increase in the number of infected nodes from when p is slightly below 0.6 and then when $p = 0.6$.



Average Degree (Viral Spread) Demo

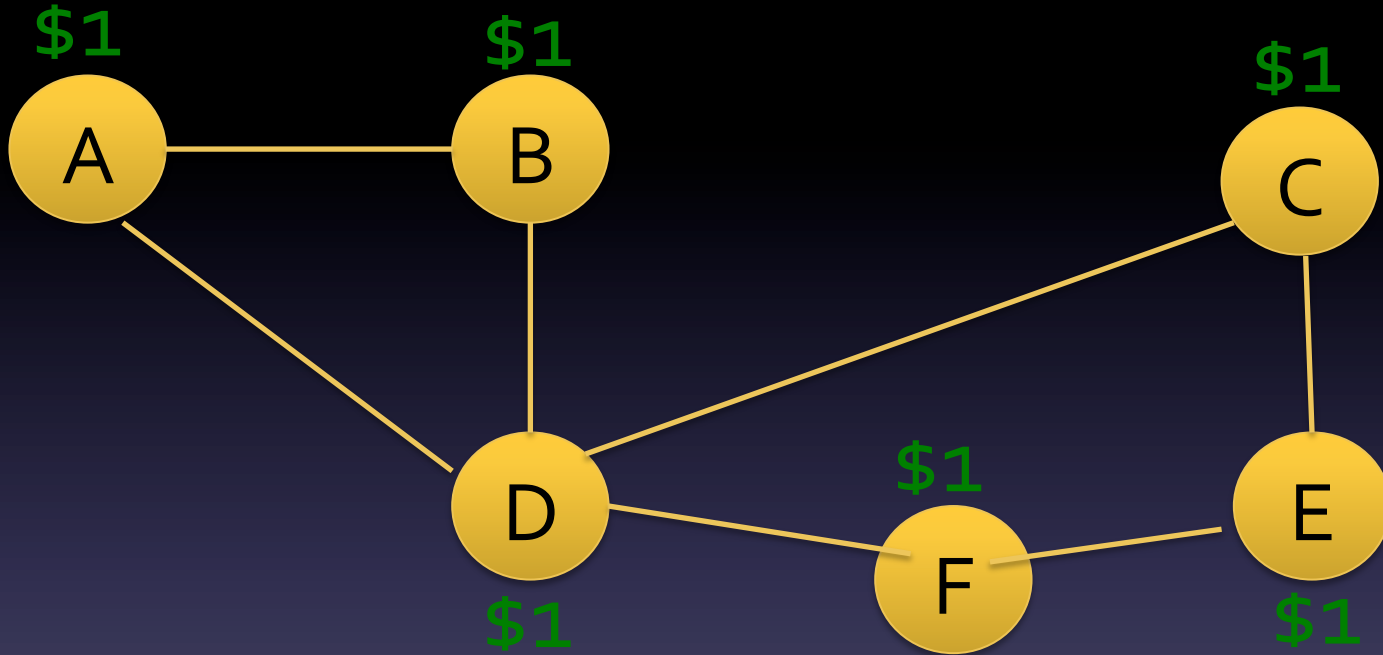
- What Network did this use?
 - N nodes with no edges and then add edges to random pairs until there are $E = d \times N/2$ edges
- What parameter can we adjust?
 - Average node degree d
- Describe what is meant by a tipping point at $d=3$
 - There is a sharp increase in the number of infected nodes when we select a random node to infect



Contagion in Economic Exchange

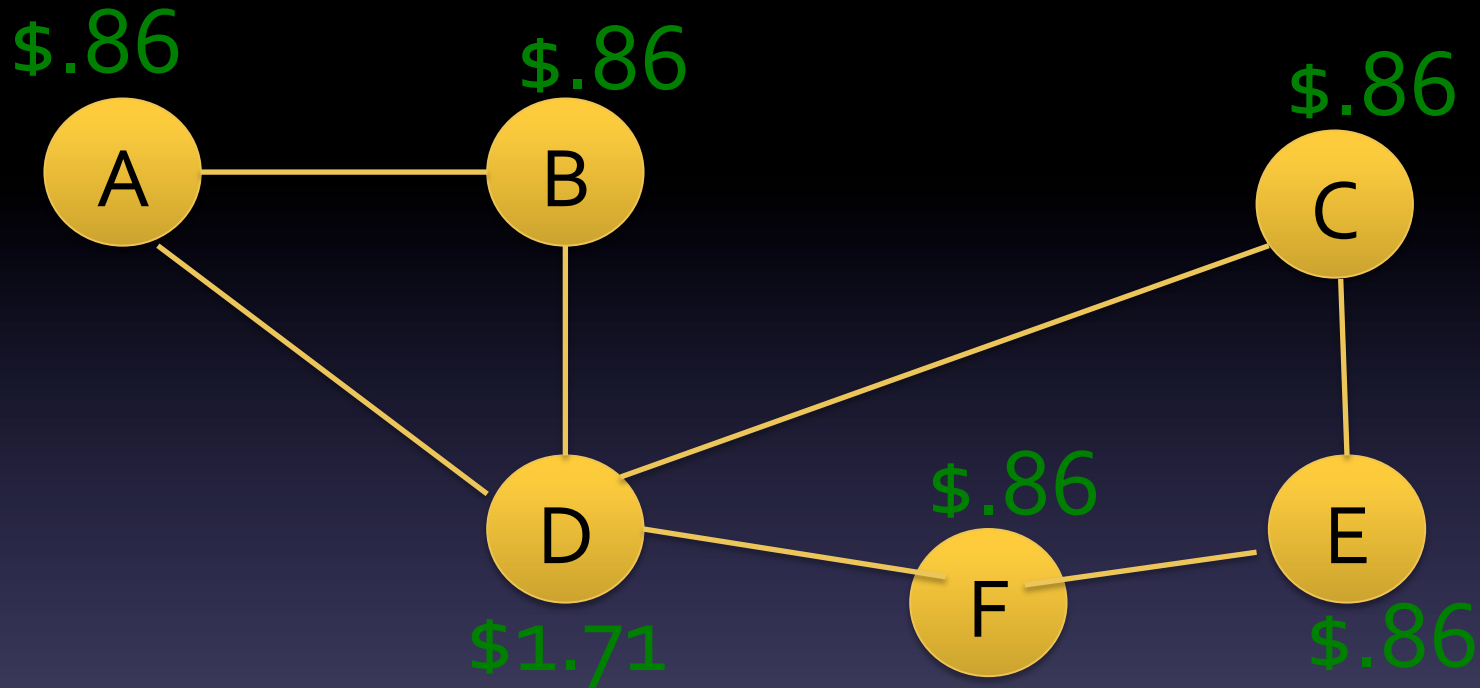
- Given a Graph, we let each node start with \$1.
- Each vertex divides their current cash equally among their neighbors.
- Repeat.
- Equilibrium when players give away as much as they receive

Example



- Equilibrium Wealth for each node u : $w(u)$
- $w(u) =$
(Total Wealth) \times $\text{deg}(u) / (\text{Sum of all Degrees})$

Example



- Equilibrium Wealth for each node u : $w(u)$
- $w(u) =$
(Total Wealth) \times $\text{deg}(u) / (\text{Sum of all Degrees})$

Navigation in Networks

- Travers and Milgram, Dodds et al.
- To find short paths in a graph:
 - Graph must have a small diameter
 - Players must be able to find the short paths with only local info.

Kleinberg's Model

- Begin with a grid network
- Add a an edge from a node to another node that is a distance d away with probability

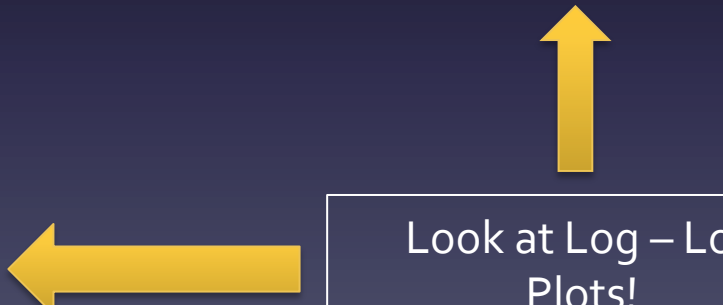
$$p(d) = \text{constant} \times (1/d)^r$$

What happens for large r and for small r ?

Kleinberg's Model

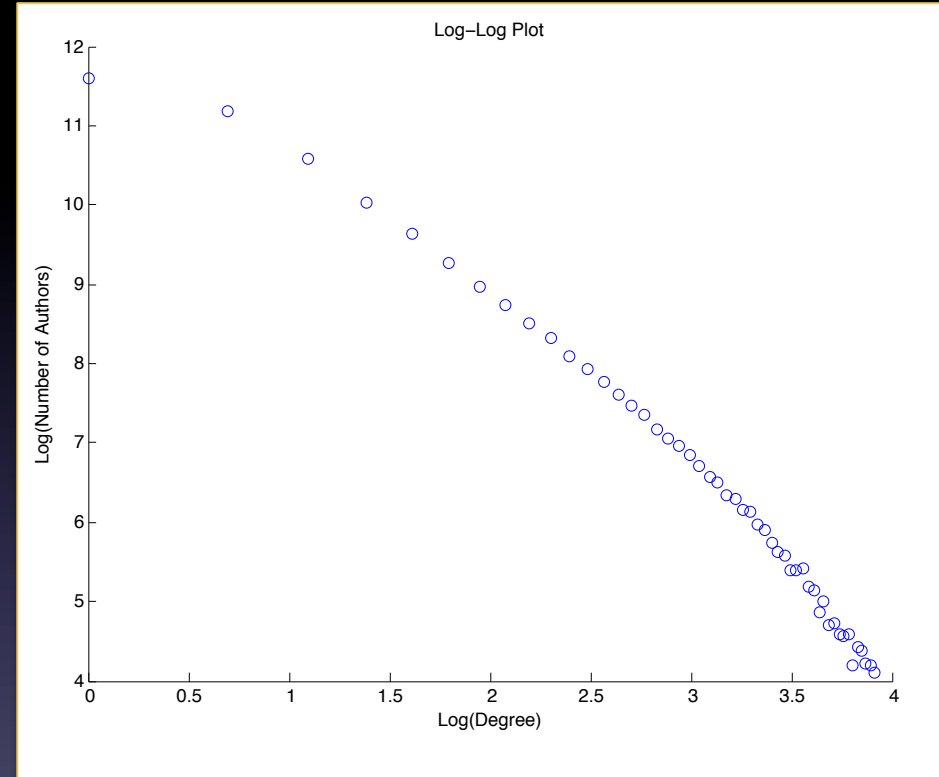
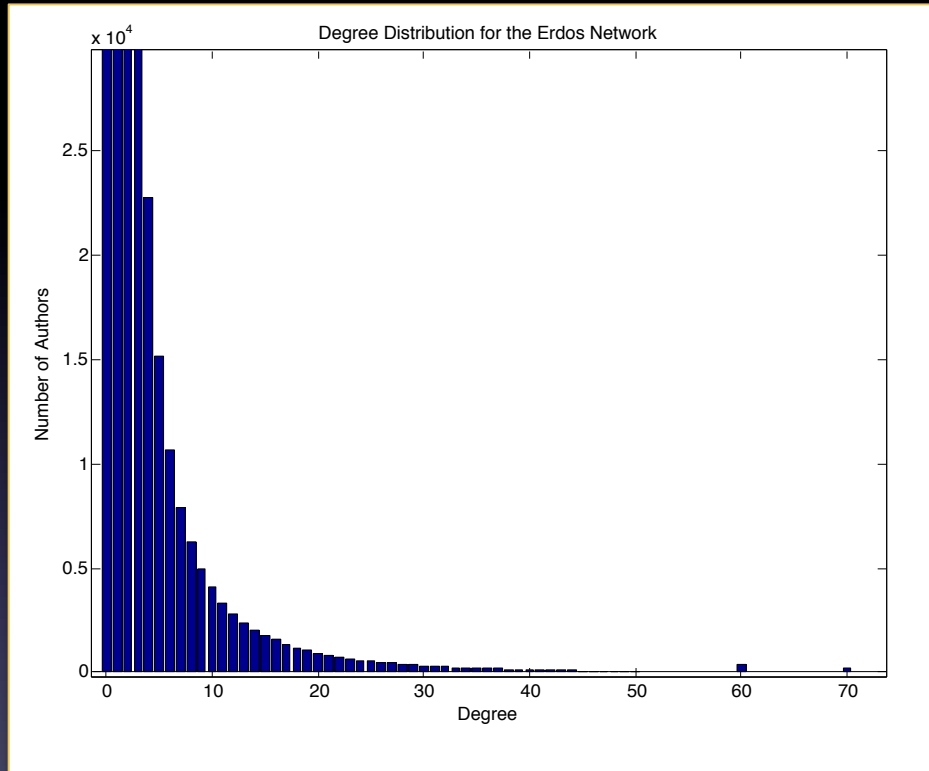
- How do player's move in the network?
 - Players know the address of their destination and their neighbors
 - Players only forward to the neighbor who is closest to the destination
- What happens when $r = 2$?
 - Efficient navigation $\sim \text{Log}(N)$ steps to any node using the procedure.

The “Look” of Real Networks

- Heavy Tailed Distribution: $\text{Prob}(x) \sim (1/x)^B$
 - Power Law $(1/x)^B \Rightarrow \text{Log}((1/x)^B) = -B \text{Log}(x)$
 - Normal Distribution
 - $\text{Log}(\text{Prob}) = \text{Curved!}$
- 

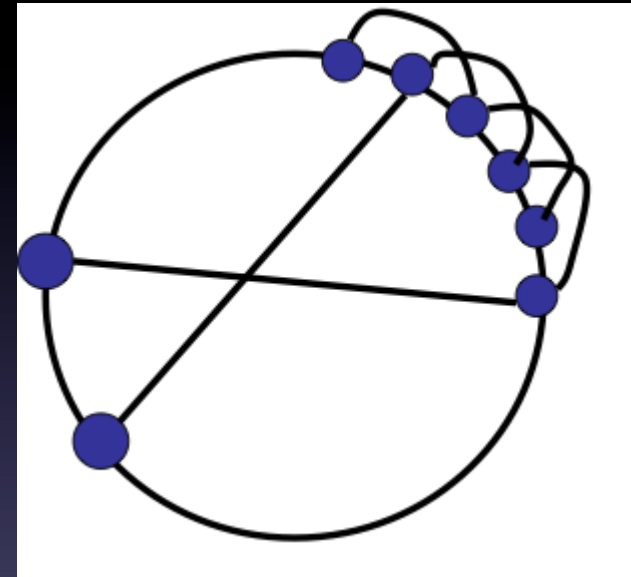
Look at Log - Log
Plots!

Example



Models of Network Formation

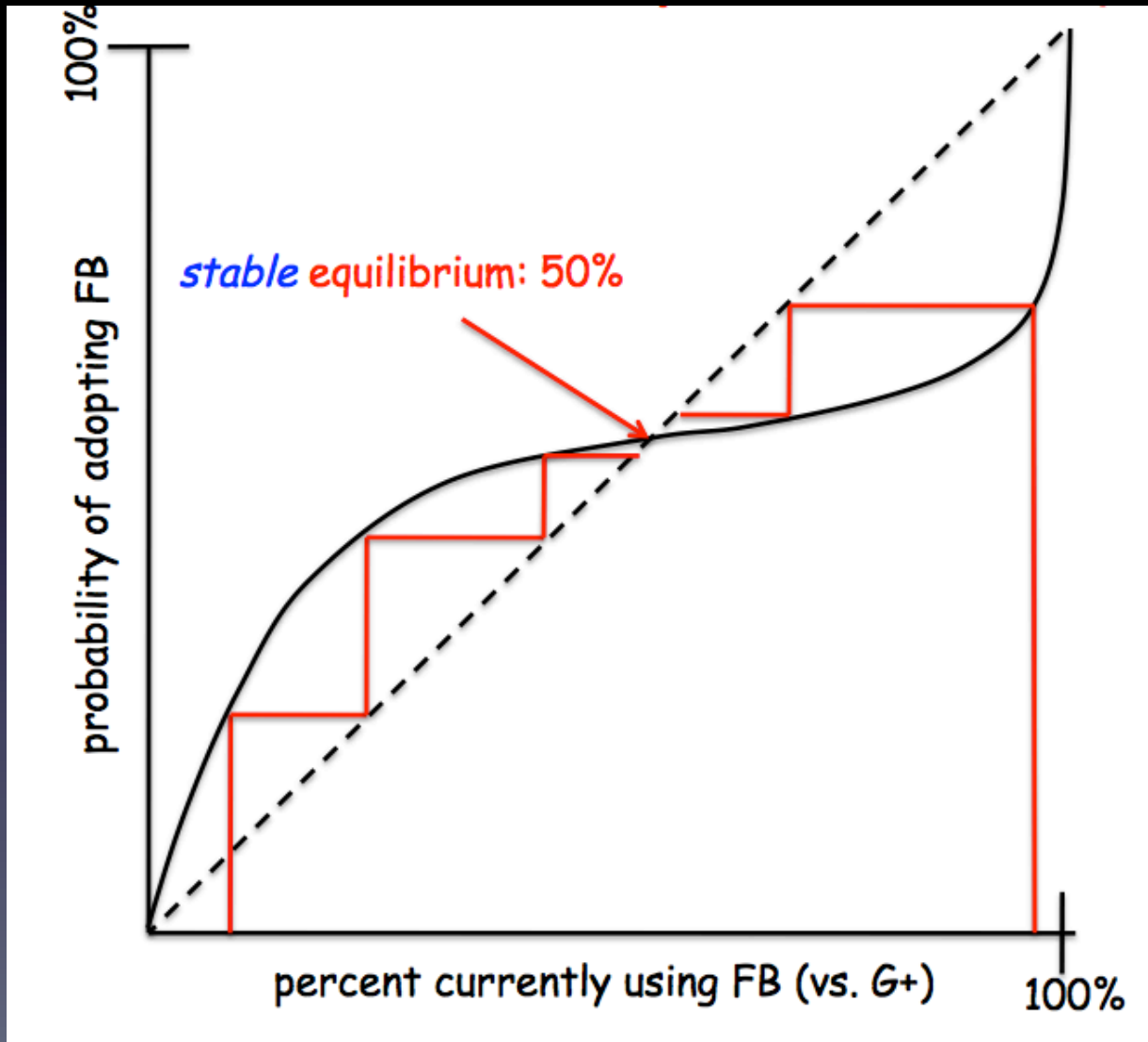
- Erdos-Renyi
 - Nodes are fixed, add edges
- Small World
 - e.g., rewiring model
- Preferential Attachment
 - Start with two connected nodes, add a node, add an edge, continue;



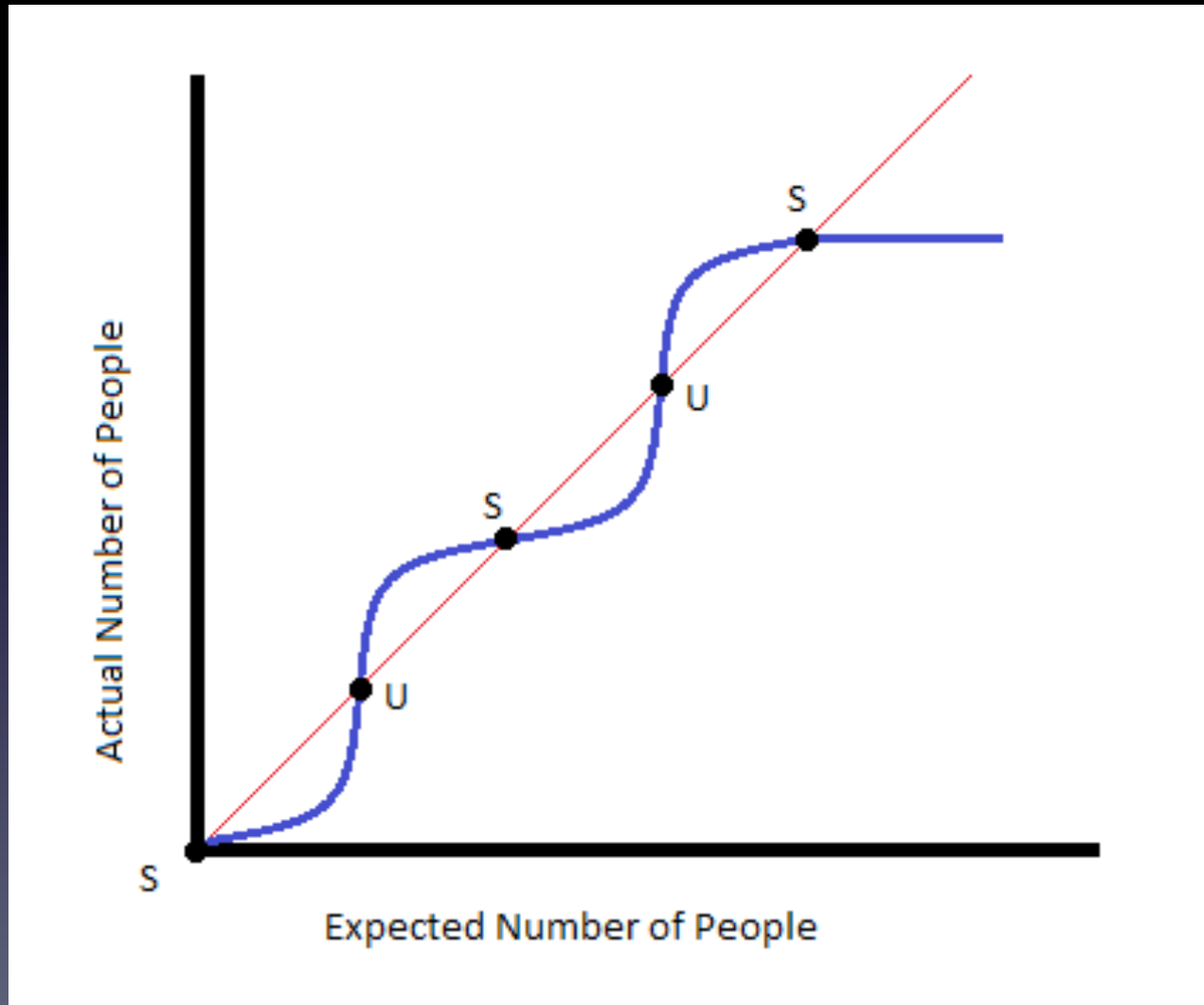
Models of Network Formation

Model	Small Diameter	Giant Component	High Clustering Coefficient	Heavy Tail
Erdos-Renyi	Yes	Yes	No	No
Small World	Yes	Yes	Yes	No
Preferential Attachment	Yes	Yes	No	Yes

Attendance Dynamic

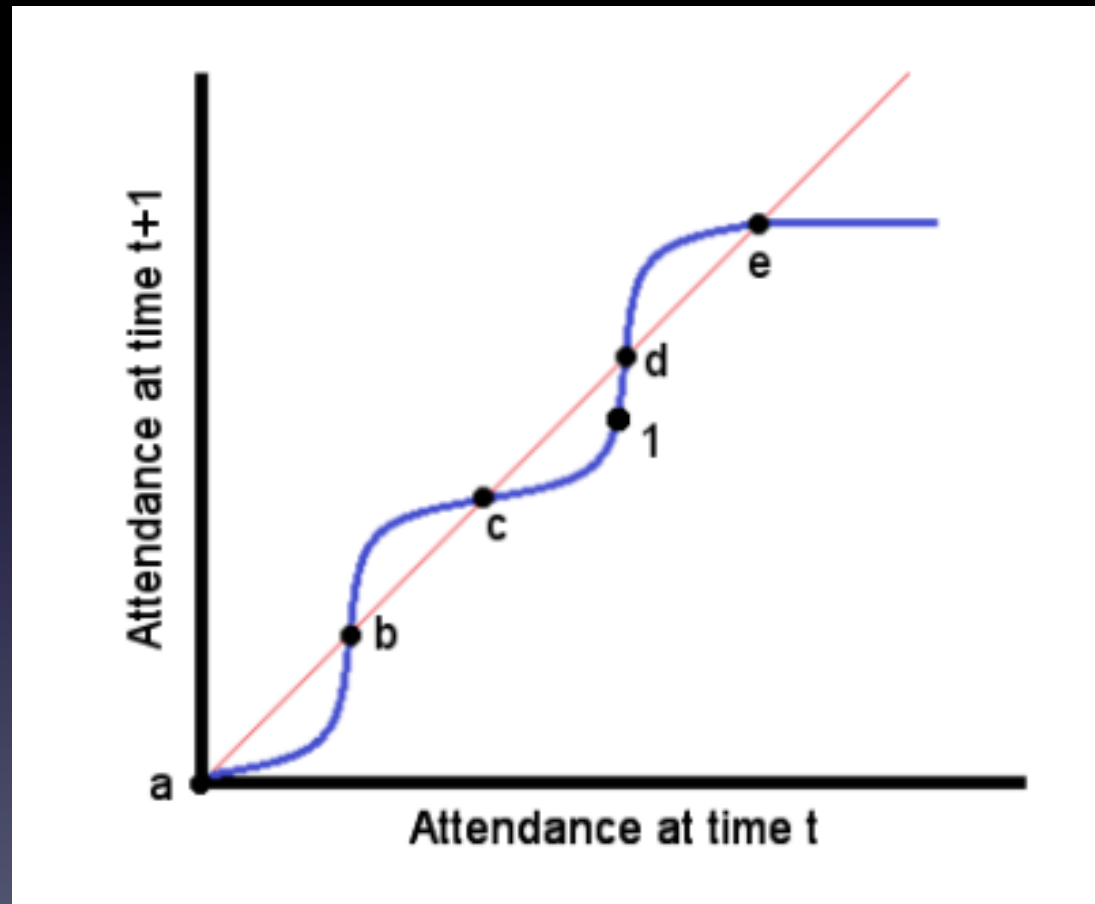


Attendance Dynamic

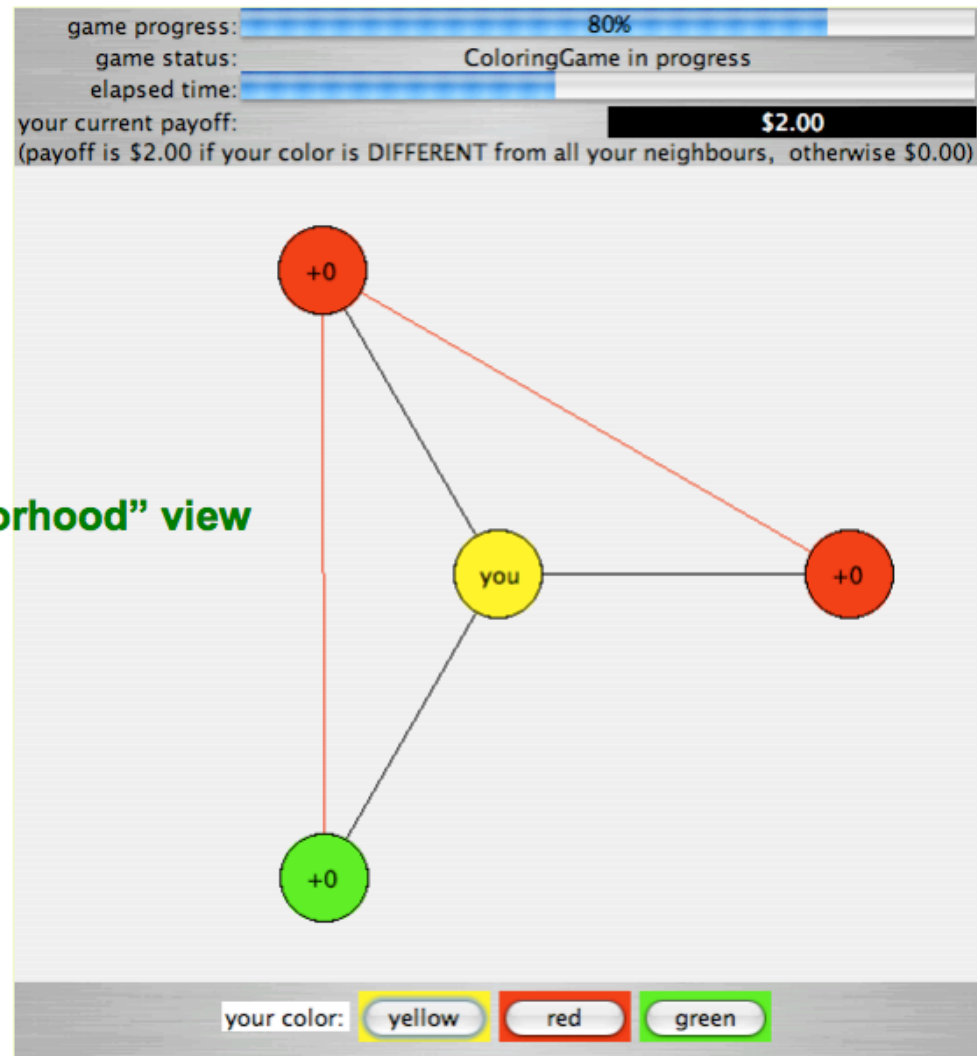


Attendance Dynamic

- What will happen if we start at 1?
- Unstable vs. Stable

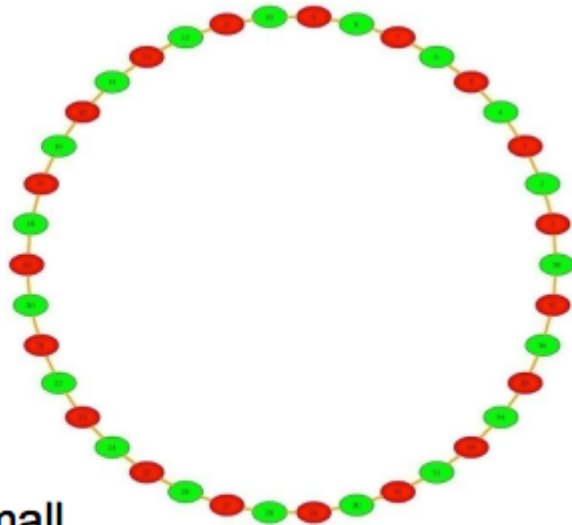


Networked Games



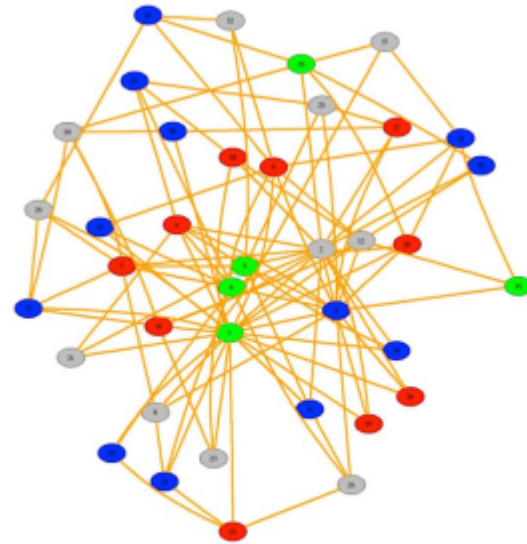
“first neighborhood” view

Coloring



Small
Worlds
Family

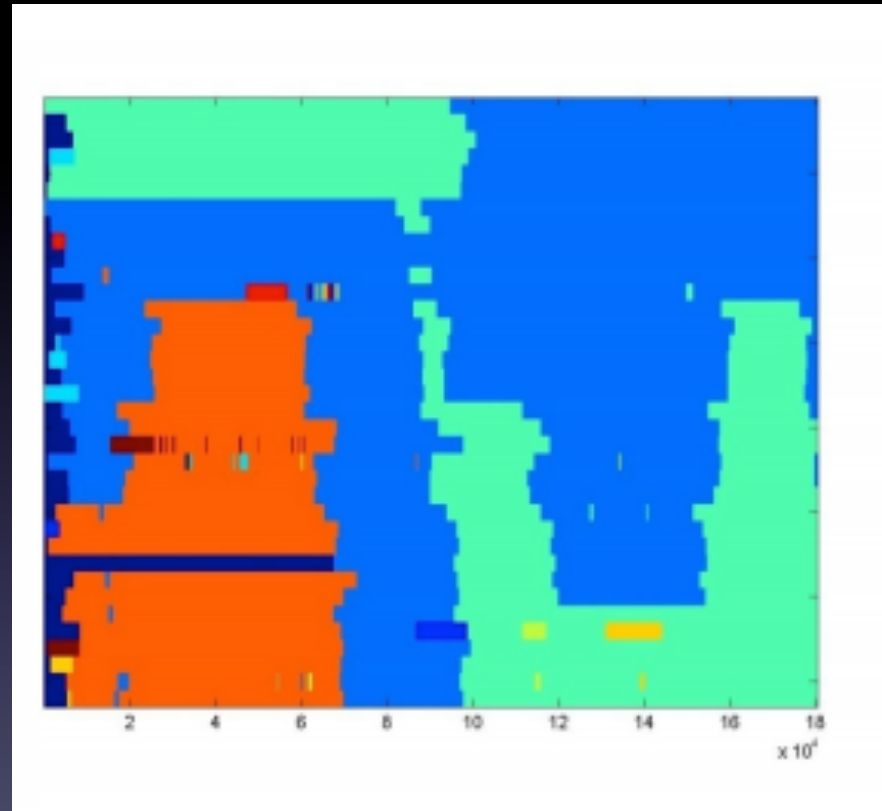
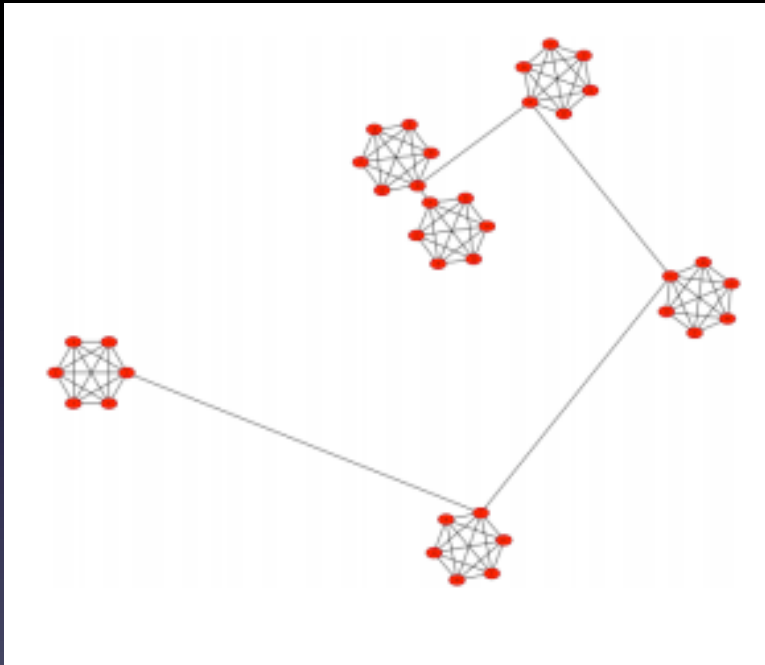
Simple Cycle



Preferential Attachment,

$\nu = 3$

Consensus



Game Theory

- What is different from a Pure Strategy Nash Equilibrium and a Mixed Strategy Nash Equilibrium?
 - Players can randomize over actions. The probability distribution is public, but the realization, i.e. what action is actually chosen, is private. Think Rock-Paper-Scissors!
- (Mixed) Nash Equilibrium **always** exist in games with finite number of players and actions.

Mixed Strategy Equilibrium

Actions	A	B
A	-3, -3	-1, 2
B	2, -1	-10, -10

Mixed Strategy Equilibrium

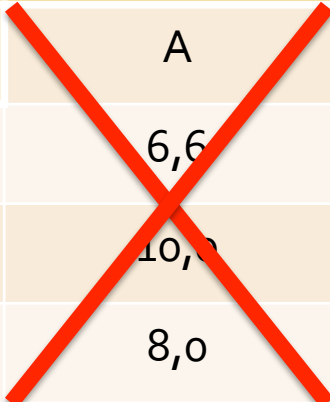
Actions	A	B
A	2, 4	1, 3
B	2, 1	1, -2

Game Theory

		Column Player Actions		
		A	B	C
Row Player Actions	A	6,6	8,20	0,8
	B	10,0	5,5	2,8
	C	8,0	20,0	4,4

Game Theory

		Column Player Actions		
		A	B	C
Row Player Actions	A	6,6	8,20	0,8
	B	10,0	5,5	2,8
	C	8,0	20,0	4,4

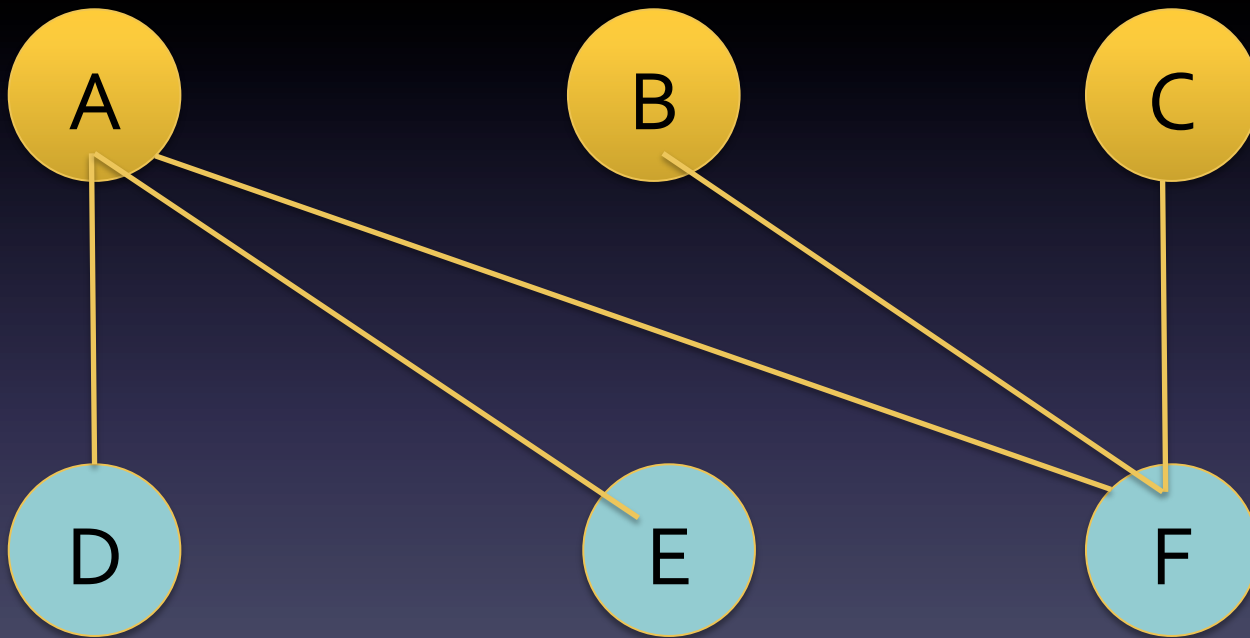


Game Theory

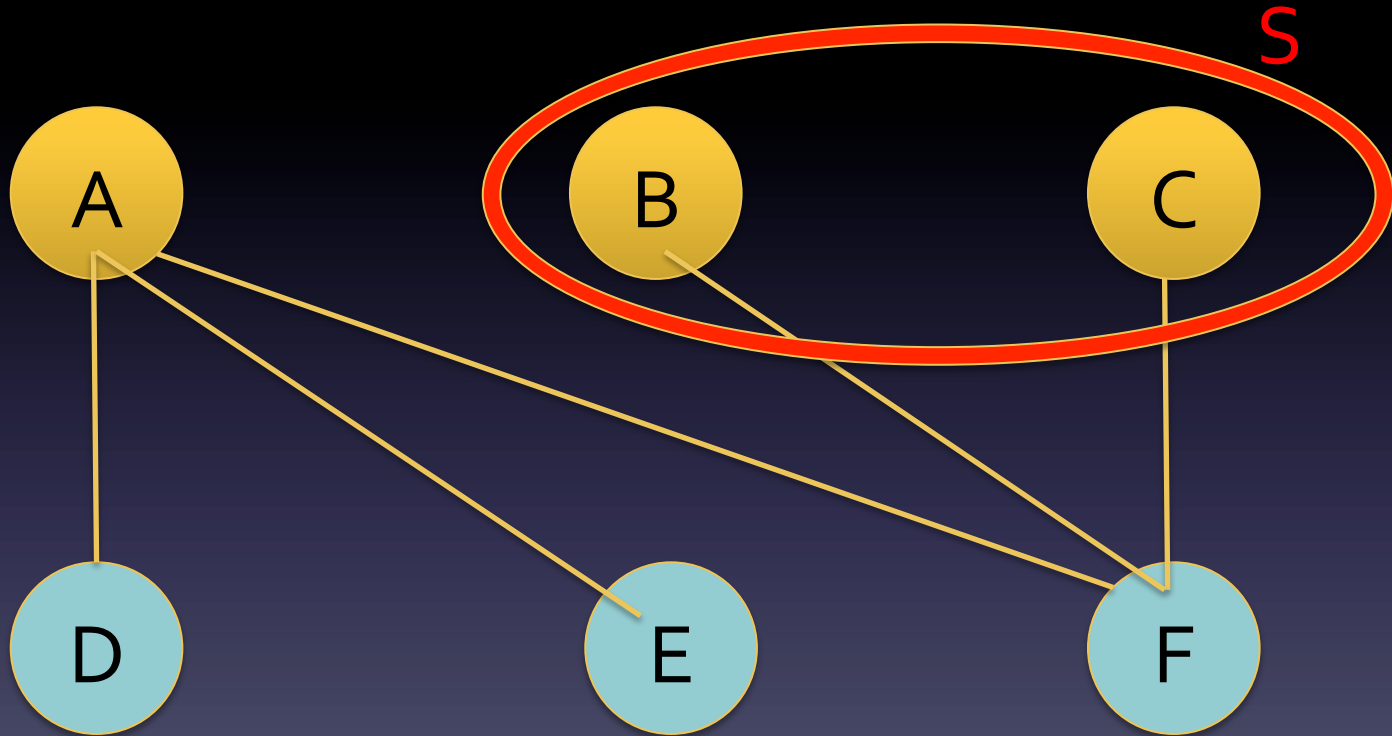
		Column Player Actions		
		A	B	C
Row Player Actions	A	6,6	8,20	0,8
	B	10,0	5,5	2,8
	C	8,0	20,0	4,4

- Nash Equilibrium at (C,C) for a payoff of 8.
- Optimal Strategy (A,B) gives payoff of 28.
- $POA = 28/8 > 1$

Network Trading

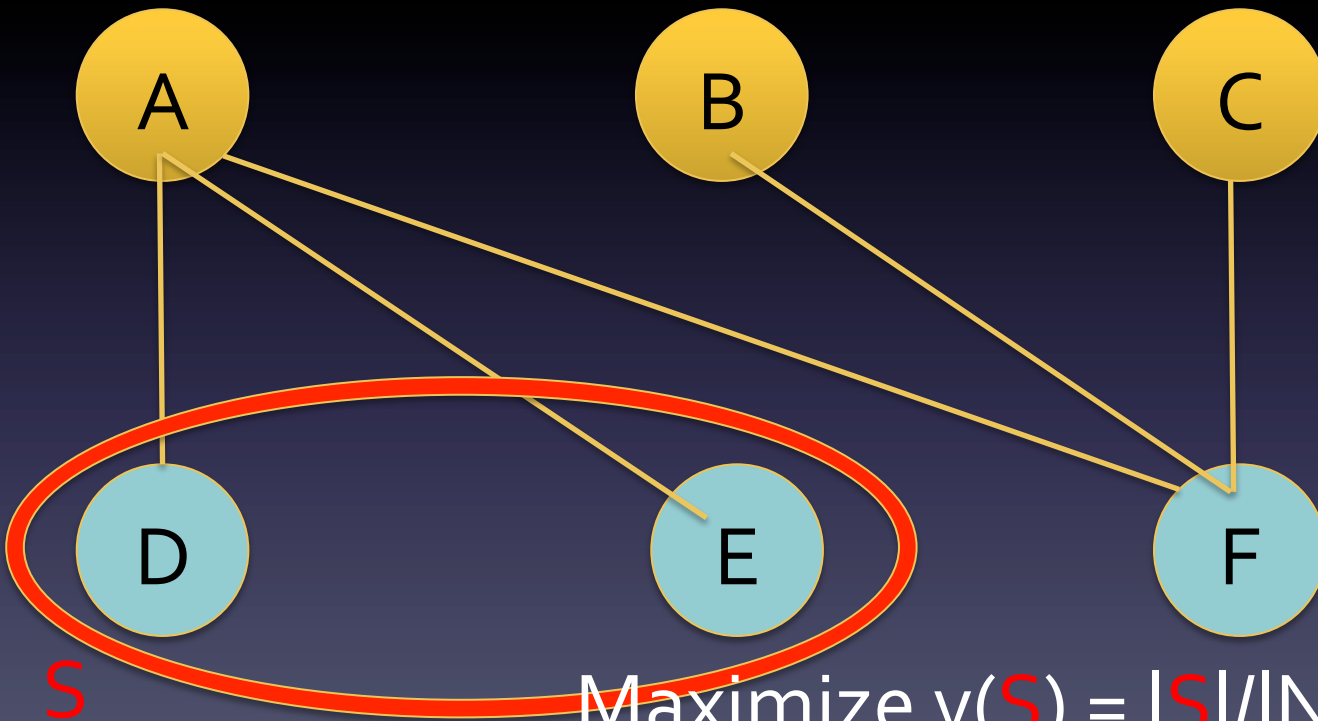


Network Trading



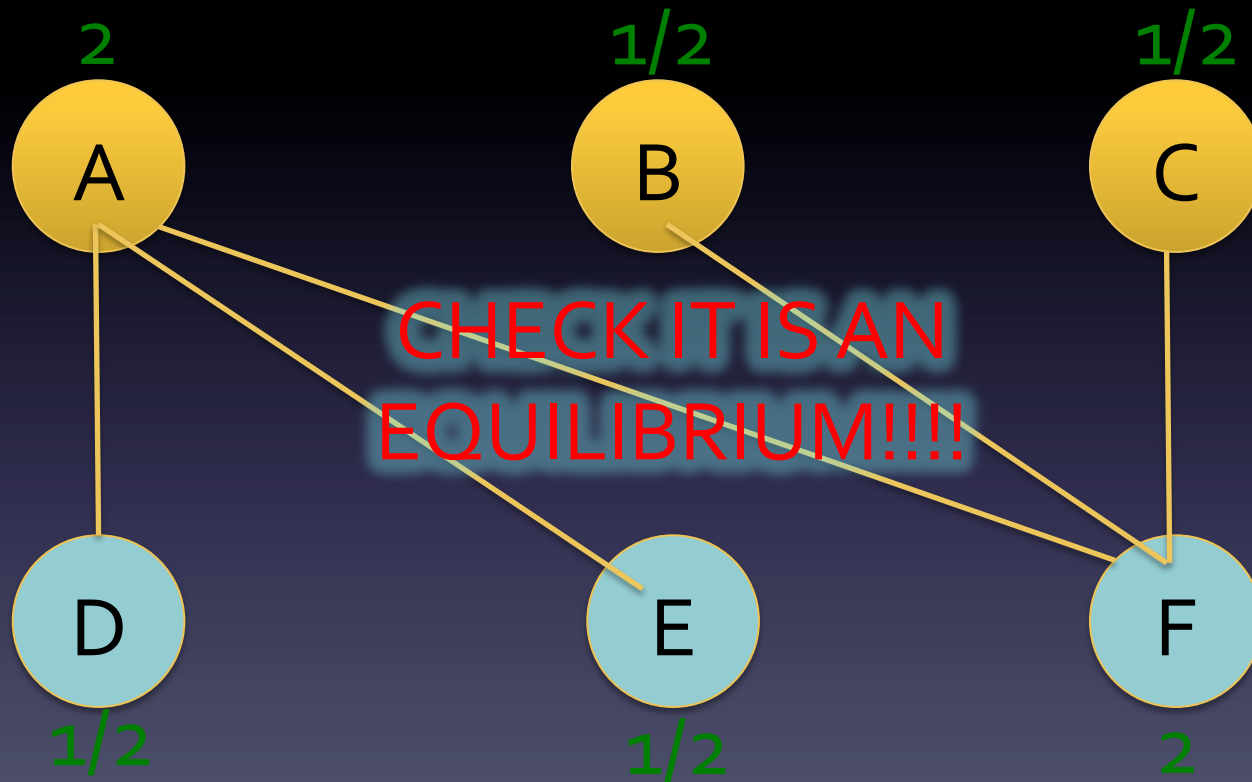
Maximize $v(S) = |S|/|N(S)| = 2$
 \Rightarrow B, C can get at most $\frac{1}{2}$ and F gets at least 2.

Network Trading



Maximize $v(S) = |S|/|N(S)| = 2$
 \Rightarrow D, E can get at most $\frac{1}{2}$ and A gets at least 2.

Network Trading



Trades: D and E give all their good to A, and A gives $\frac{1}{2}$ of her good each to D and E. Similarly, B and C give all their good to F, and F gives $\frac{1}{2}$ of his good to B and C.

Network Trading (in-class experiment)

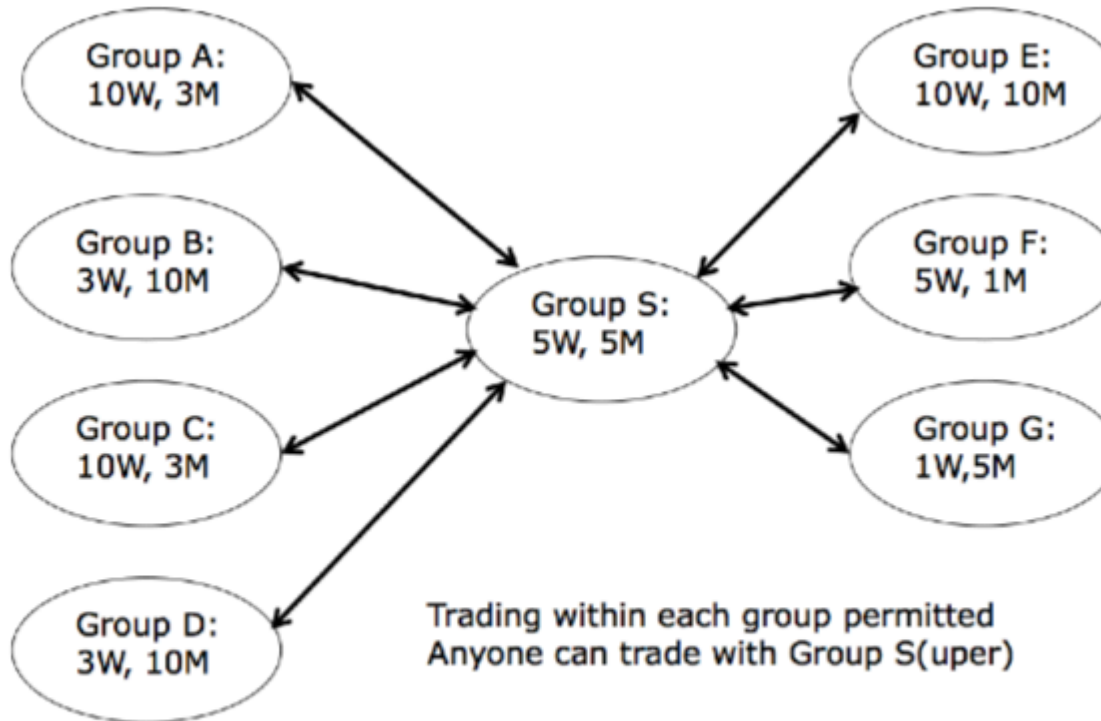
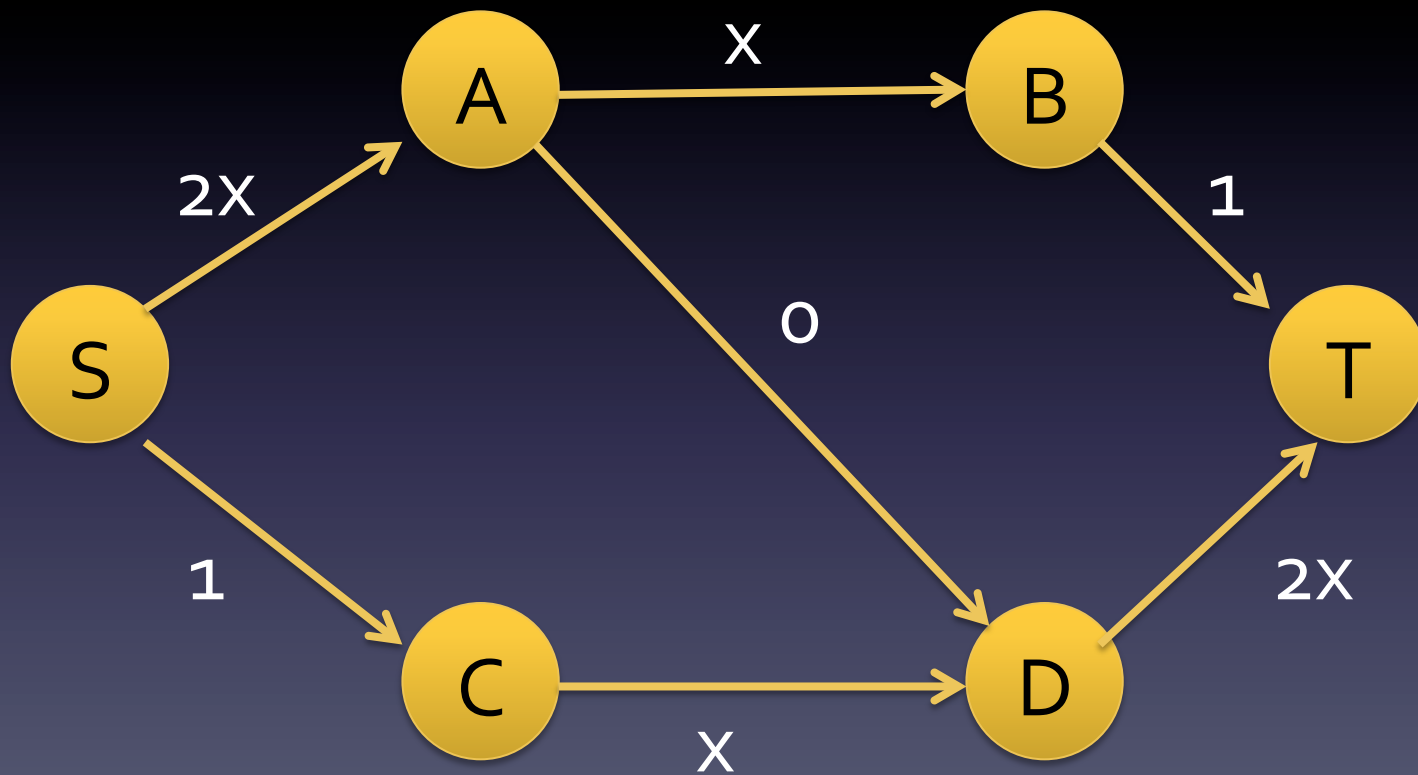
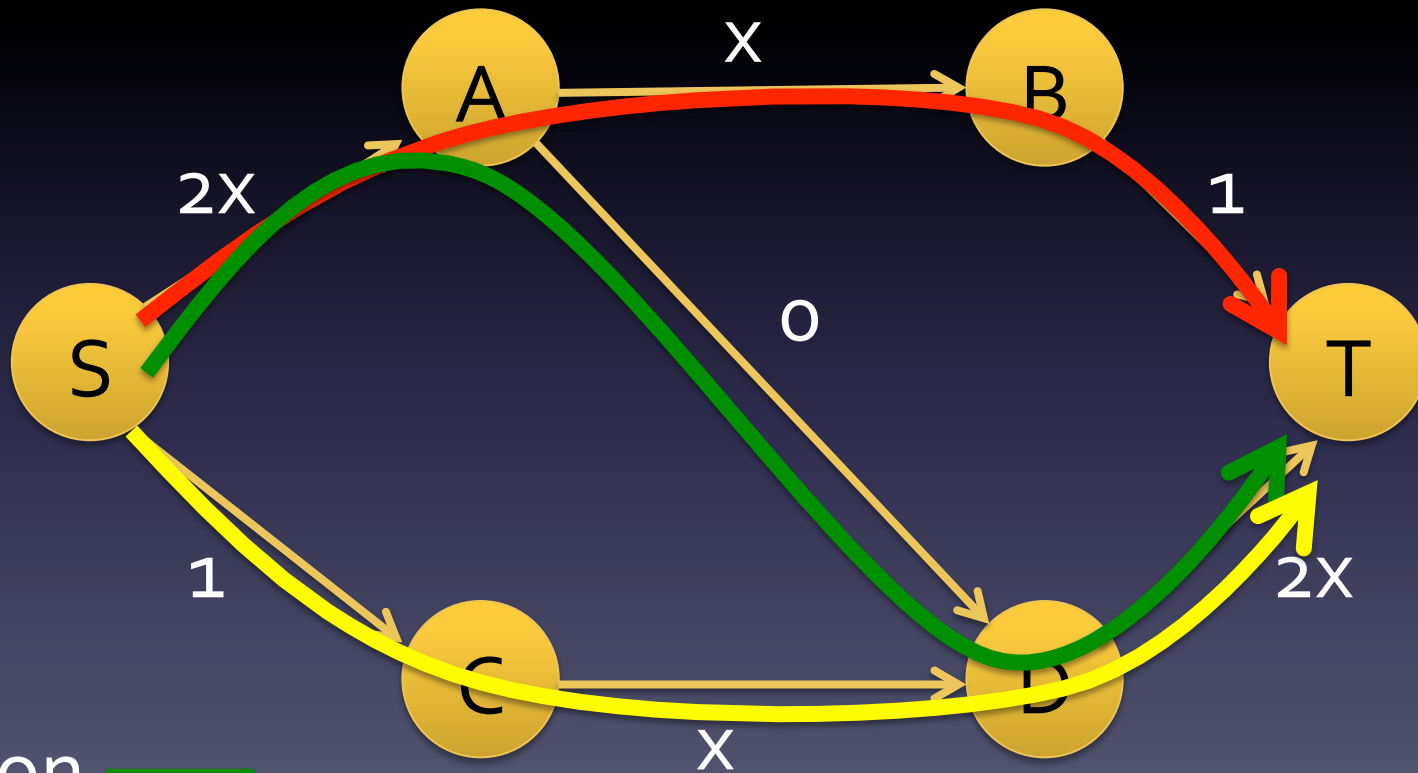


Figure 1: Network Trading - In Class Example

Selfish Routing



Selfish Routing

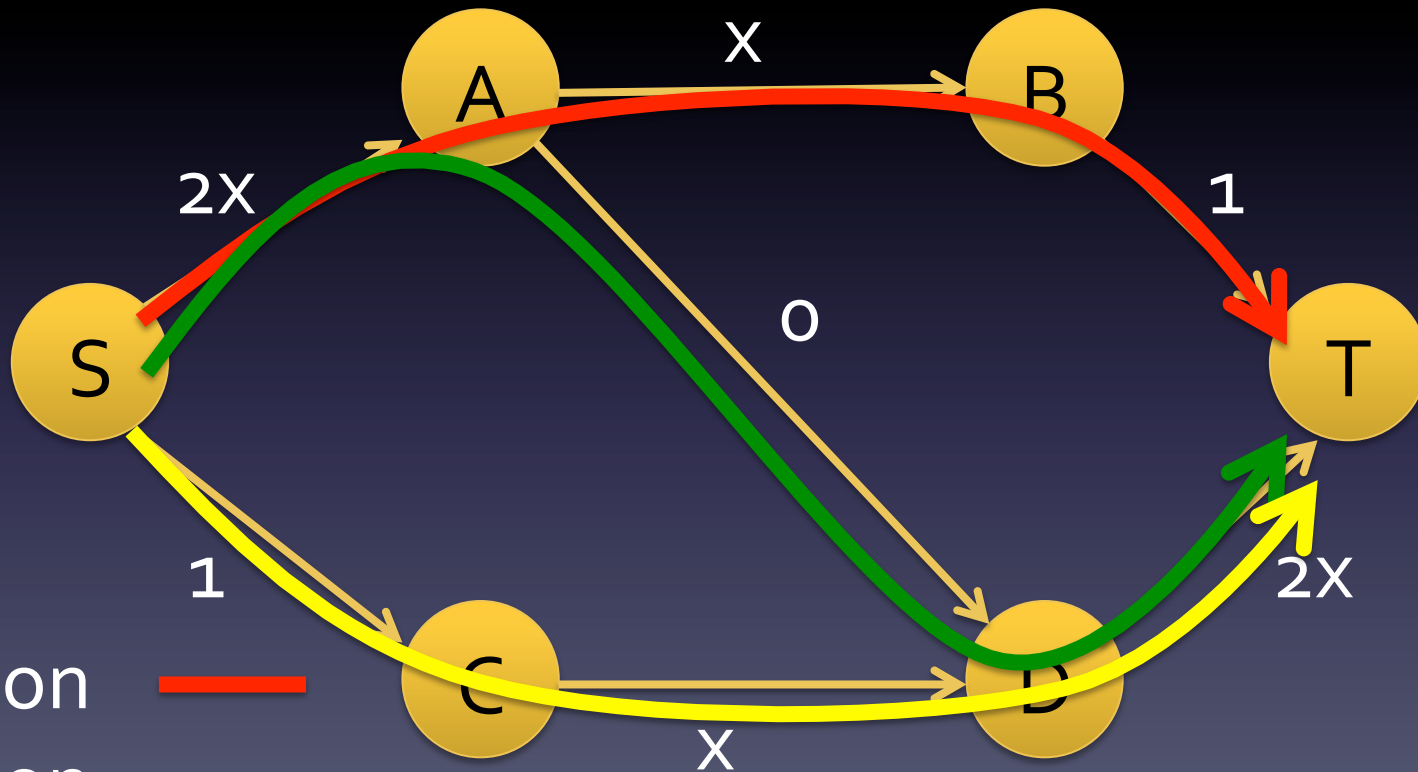


All on 

Cost = 4

Equilibrium? **NO!**

Selfish Routing



1/3 on



1/3 on



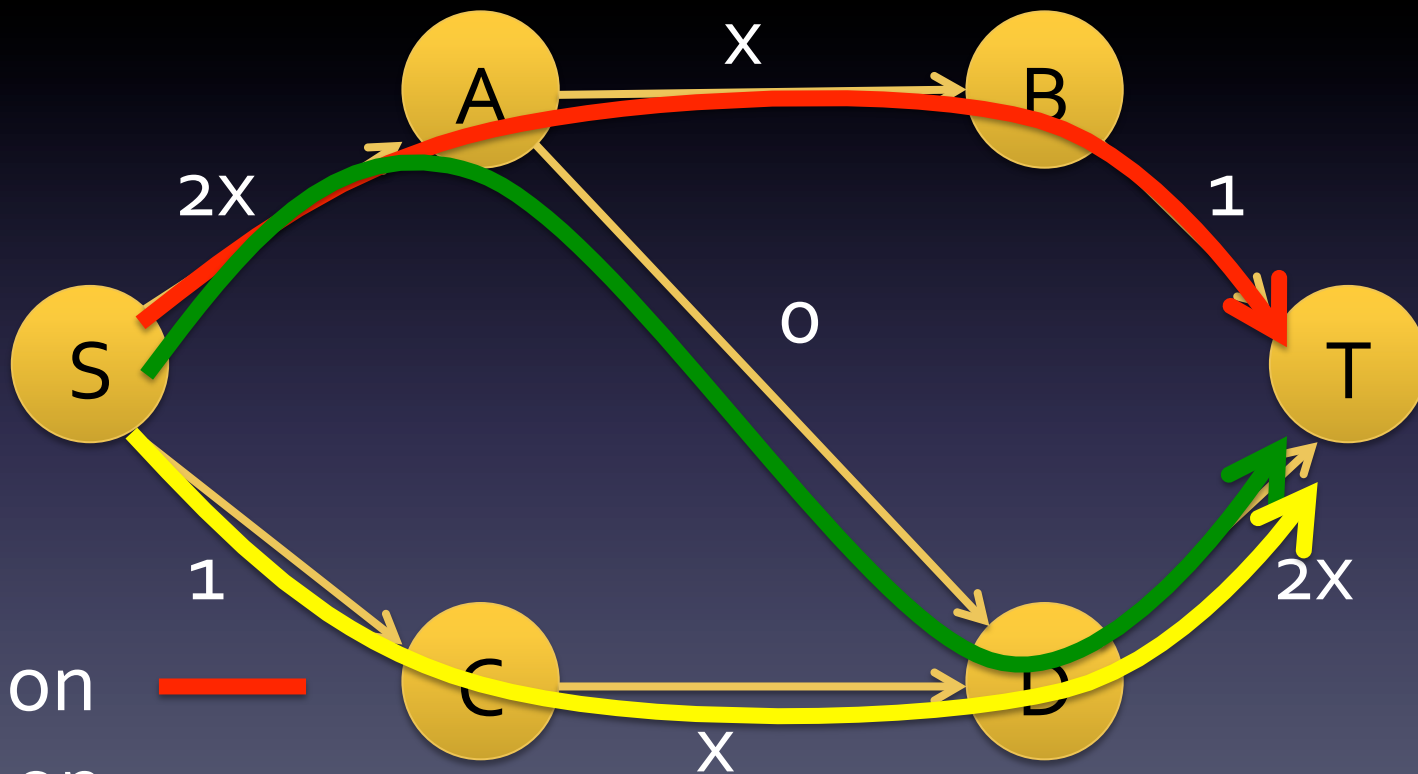
1/3 on



Cost = $8/3 = 2.6667$

Equilibrium? **YES!**

Selfish Routing



1/2 on 
0 on 
1/2 on 

Cost = 2.5 = **OPT**
Equilibrium? **NO!**

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