NETS 112 Final Review

December 9, 2014

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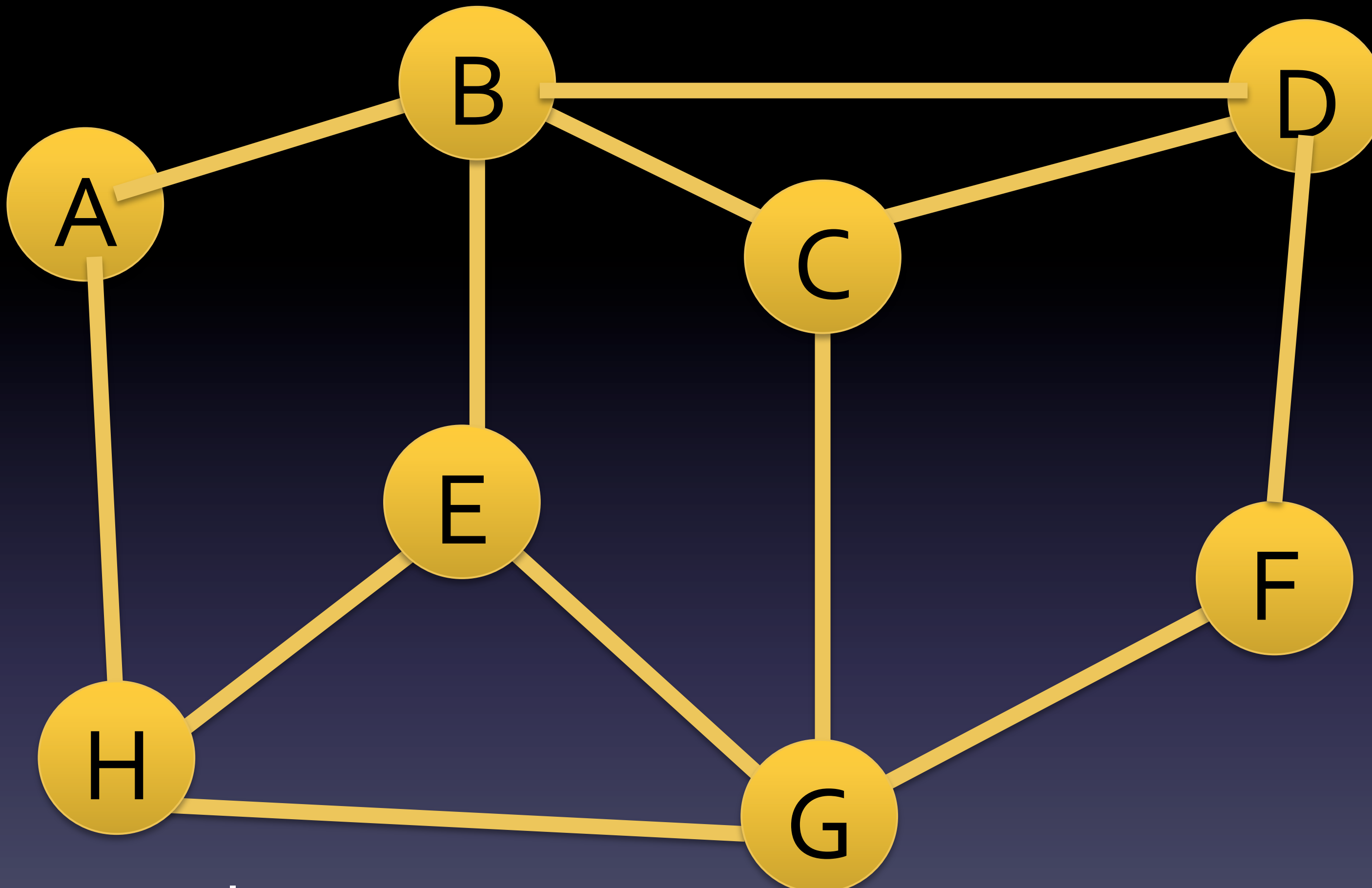
Networks



Structural Properties:

- Degree, Degree Distribution, Distance, Diameter, Clustering Coefficient.

Examples from Class

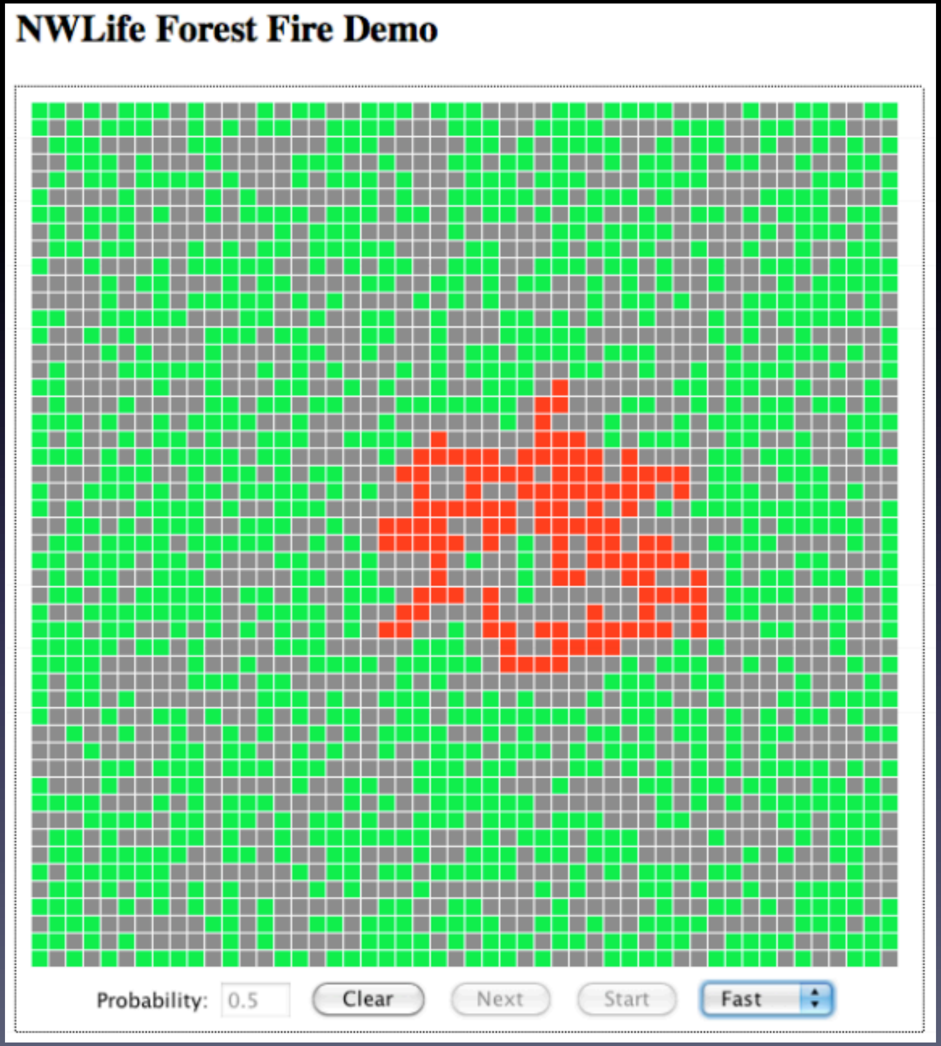
- Erdos Number Project
- Last Name Experiment
- Toss a ball from one person in the class to another to reach a specific person (local vs global info)

Dynamics in Networks

- Network Structure (static) definitely impacts a process (dynamics) in a network
- 2 Demos showing **Contagion in Networks:**
  - **Forest Fire**
  - **Average Degree**

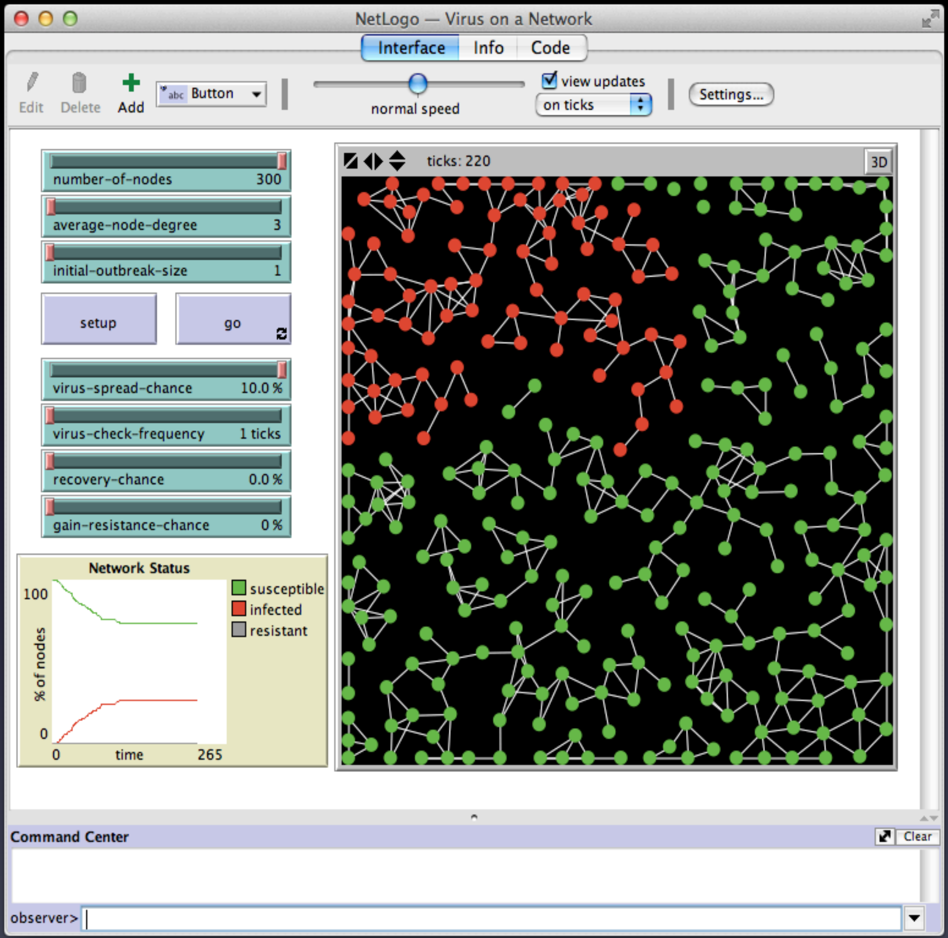
Forest Fire Demo

- What Network did this use?
  - Lattice and then deleted some nodes
- What parameter can we adjust?
  - Probability of keeping a node
- Describe what is meant by a tipping point at p = 0.6
  - There is a sharp increase in the number of infected nodes from when p is slightly below 0.6 and then when p = 0.6.



Average Degree (Viral Spread) Demo

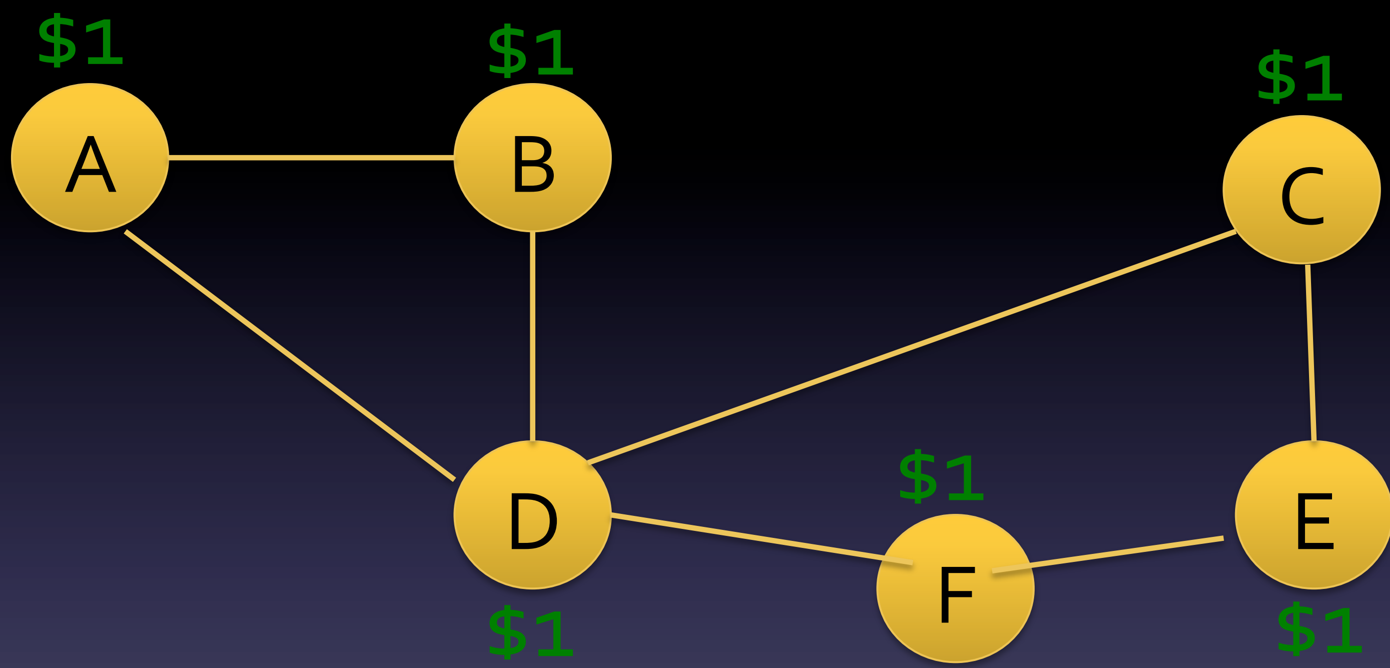
- What Network did this use?
  - N nodes with no edges and then add edges to random pairs until there are E = d x N/2 edges
- What parameter can we adjust?
  - Average node degree d
- Describe what is meant by a tipping point at d=3
  - There is a sharp increase in the number of infected nodes when we select a random node to infect



Contagion in Economic Exchange

- Given a Graph, we let each node start with $1.
- Each vertex divides their current cash equally among their neighbors.
- Repeat.
- Equilibrium when players give away as much as they receive

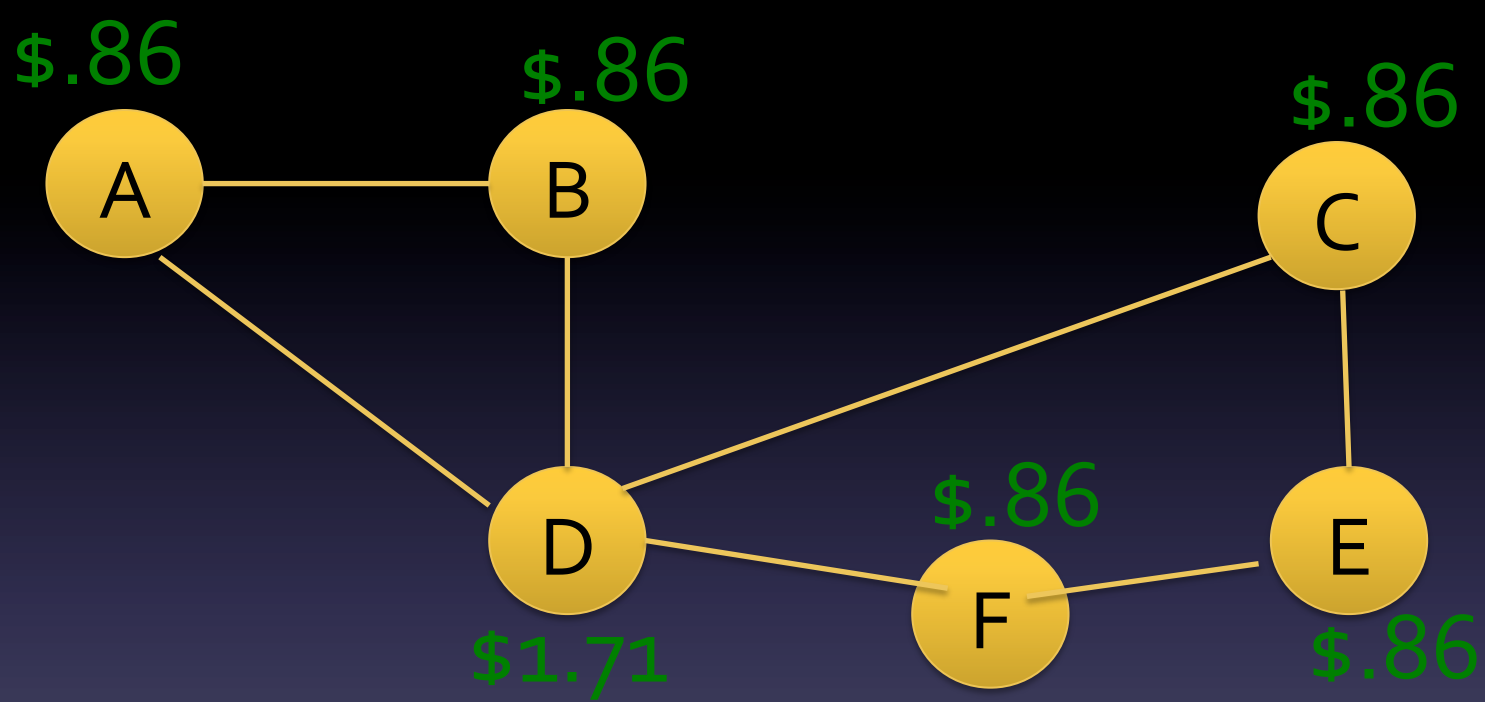
Example



- Equilibrium Wealth for each node u: w(u)
- w(u) =

  (Total Wealth) x deg(u)/(Sum of all Degrees)

Example



- Equilibrium Wealth for each node u: w(u)
- w(u) =

  (Total Wealth) x deg(u)/(Sum of all Degrees)

Navigation in Networks

- Travers and Milgram, Dodds et al.
- To find short paths in a graph:
  - Graph must have a small diameter
  - Players must be able to find the short paths with only local info.

Kleinberg's Model

- Begin with a grid network
- Add a an edge from a node to another node that is a distance d away with probability

$$p(d) = \text{constant} \times (1/d)^r$$

What happens for large r and for small r?

Kleinberg’s Model

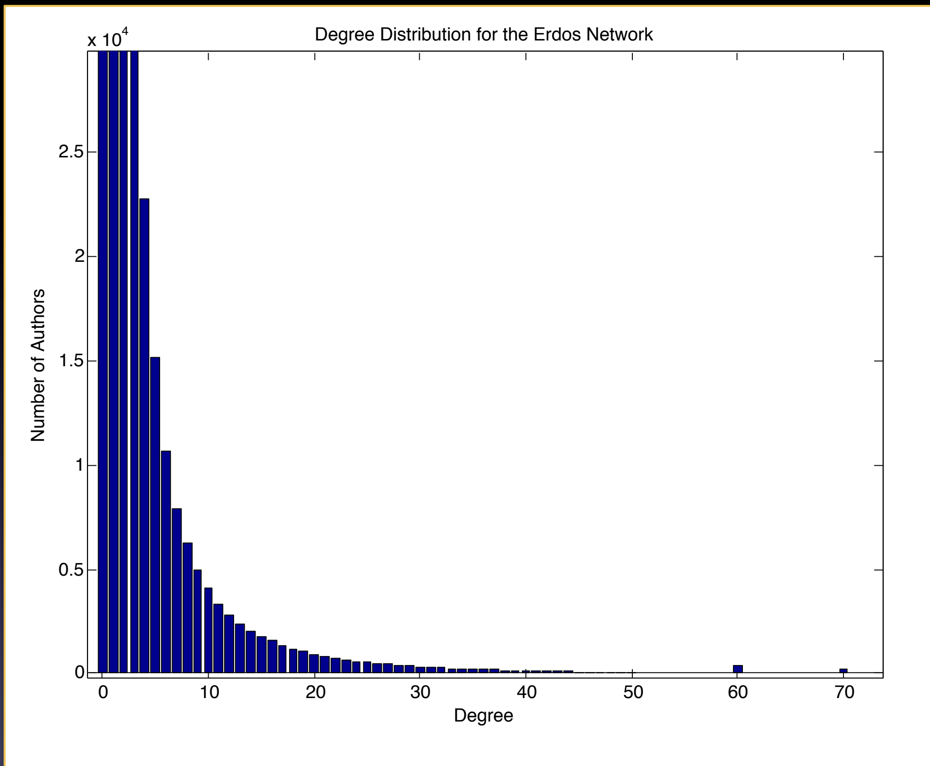
- How do player’s move in the network?
  - Players know the address of their destination and their neighbors
  - Players only forward to the neighbor who is closest to the destination
- What happens when r = 2?
  - Efficient navigation ~ Log(N) steps to any node using the procedure.

The "Look" of Real Networks

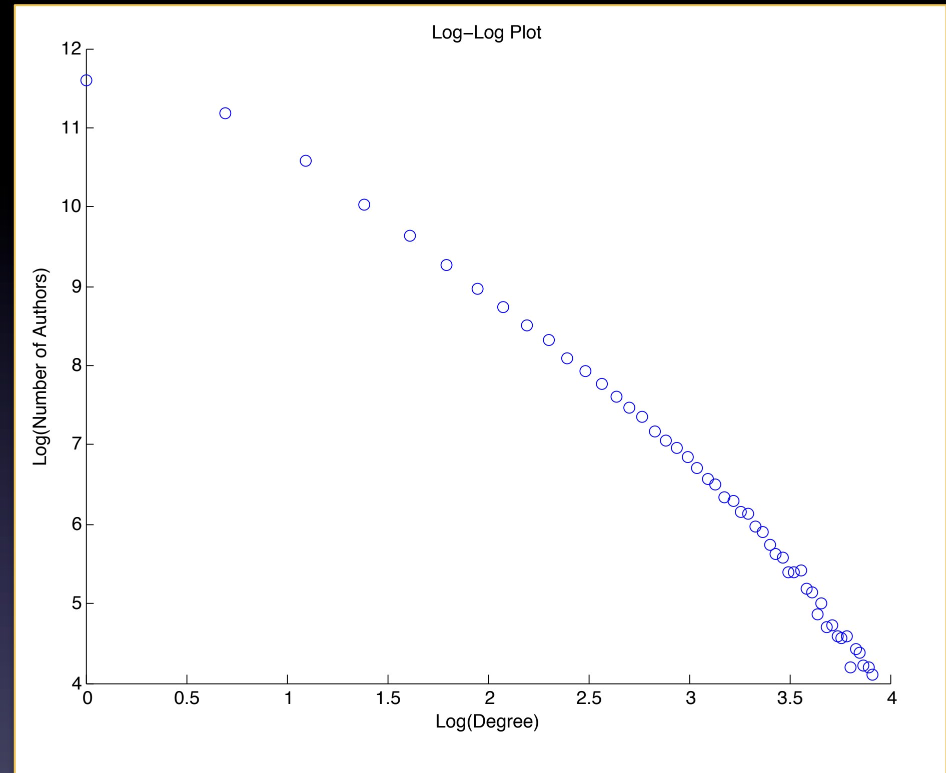
- Heavy Tailed Distribution: Prob(x) ~ $(1/x)^B$
  - Power Law $(1/x)^B$ => $\text{Log}((1/x)^B) = -B\,\text{Log}(x)$
- Normal Distribution
  - Log(Prob) = Curved!

Look at Log – Log Plots!

Example



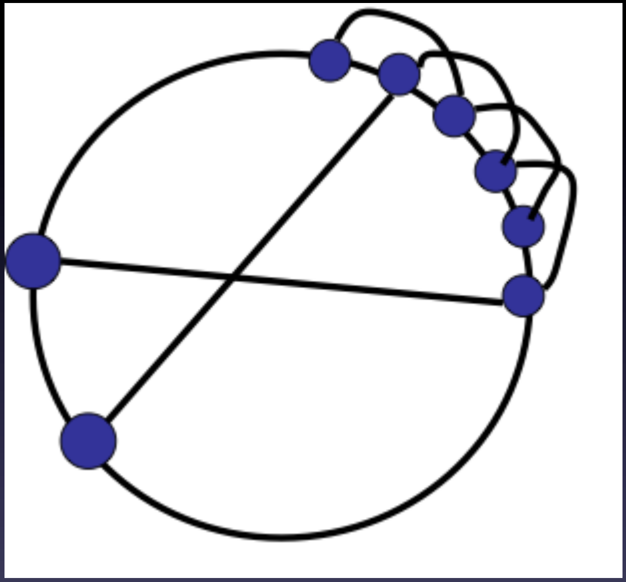
Degree Distribution for the Erdos Network
x 10^4
2.5
2
1.5
1
0.5
0
Number of Authors
0
10
20
30
40
50
60
70
Degree



Log–Log Plot
12
11
10
9
8
7
6
5
4
Log(Number of Authors)
0
0.5
1
1.5
2
2.5
3
3.5
4
Log(Degree)

Models of Network Formation

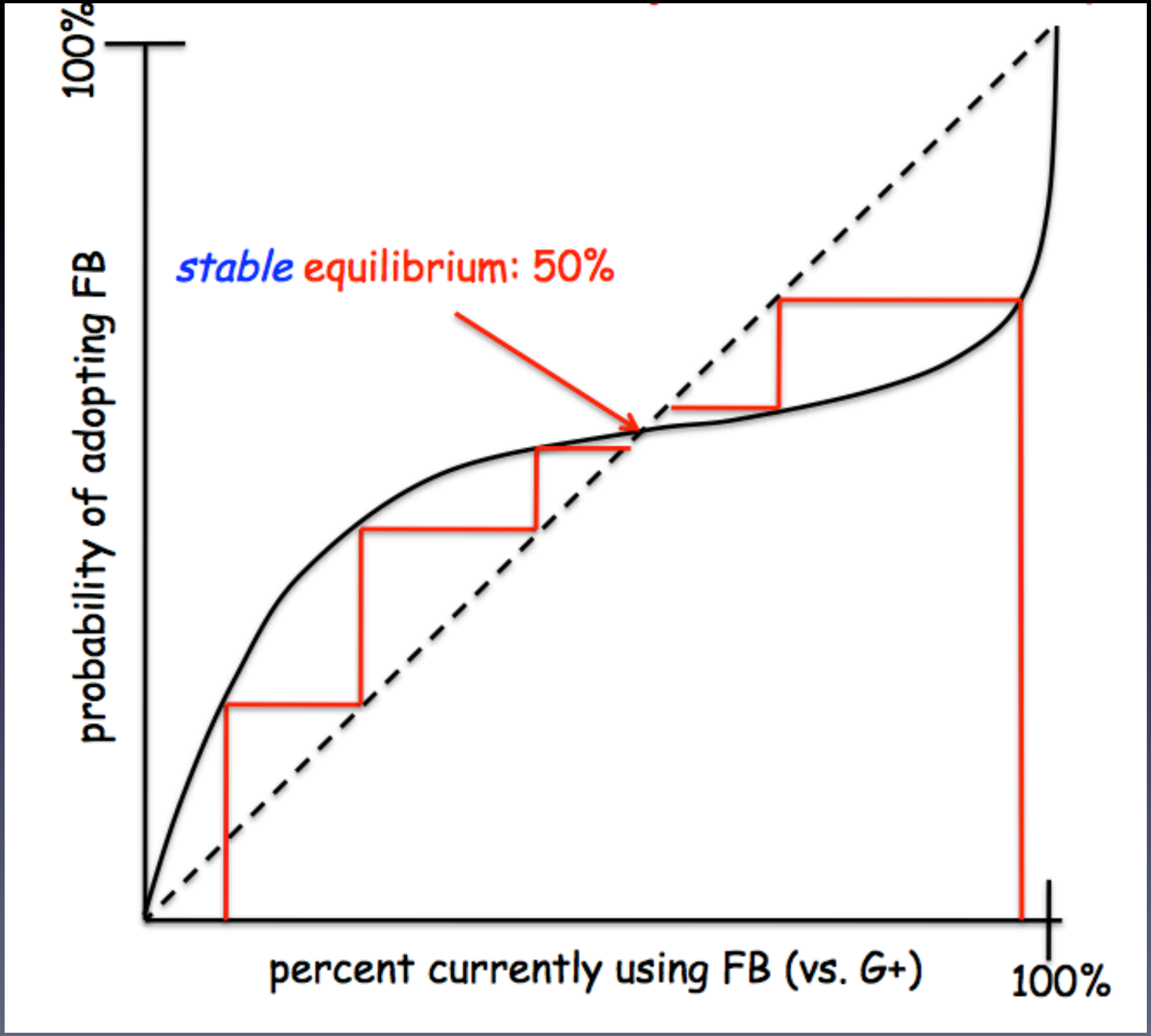
- Erdos-Renyi
  - Nodes are fixed, add edges
- Small World
  - e.g., rewiring model
- Preferential Attachment
  - Start with two connected nodes, add a node, add an edge, continue;



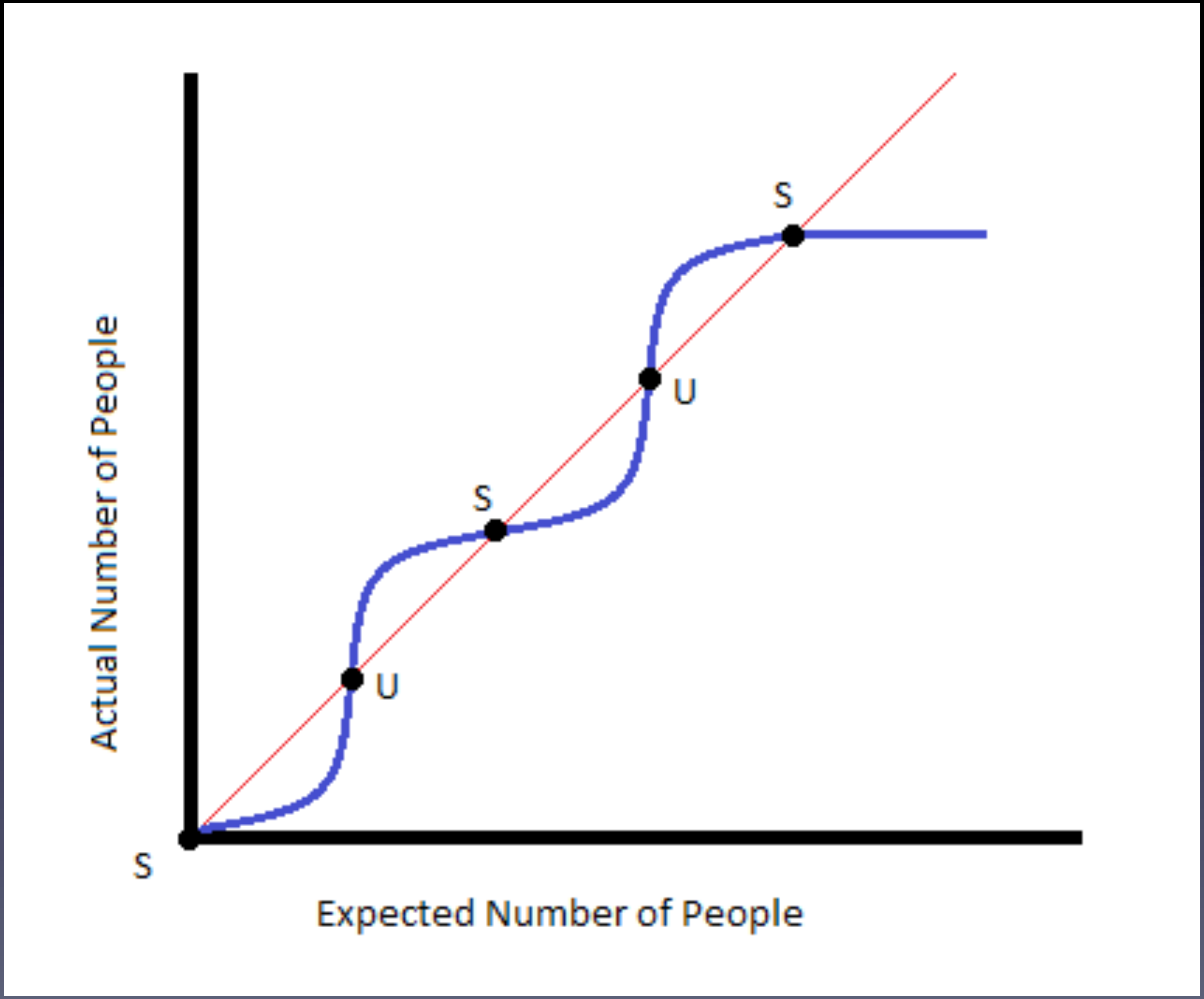
Models of Network Formation

Model	Small Diameter	Giant Component	High Clustering Coefficient	Heavy Tail
Erdos-Renyi	Yes	Yes	No	No
Small World	Yes	Yes	Yes	No
Preferential Attachment	Yes	Yes	No	Yes

Attendance Dynamic

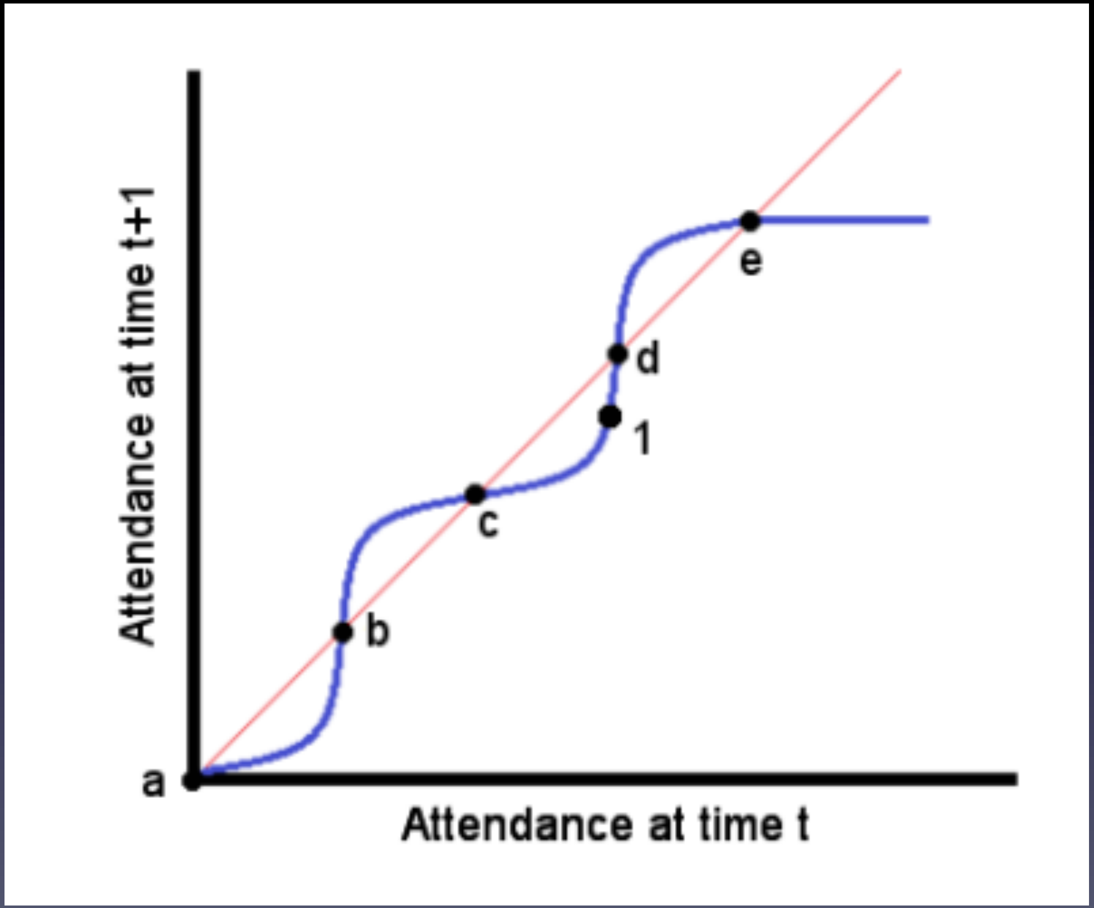


Attendance Dynamic

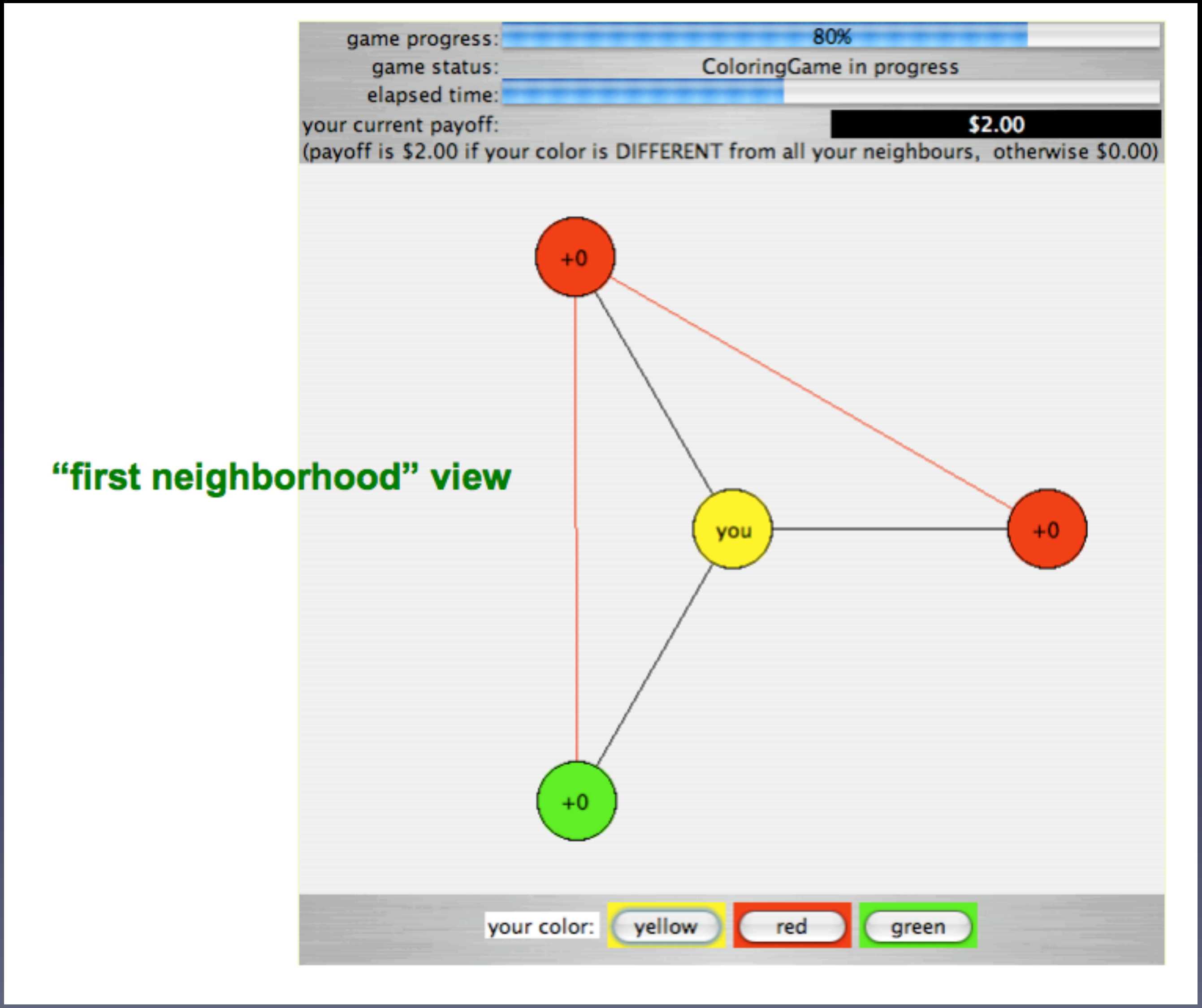


Attendance Dynamic

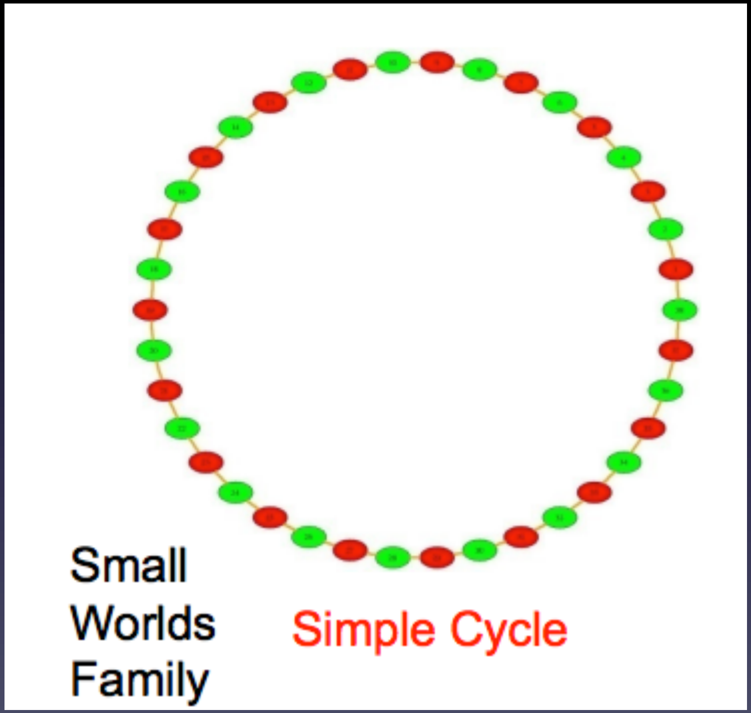
- What will happen if we start at 1?
- Unstable vs. Stable

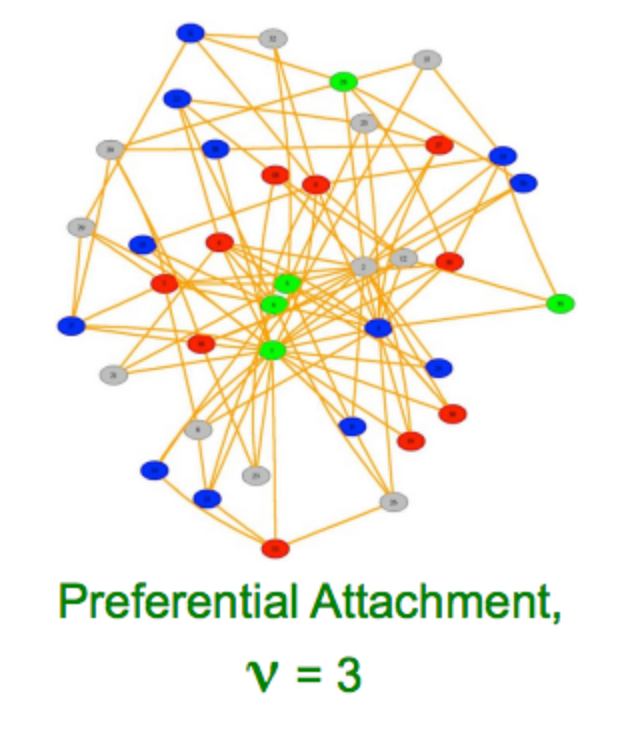


Networked Games

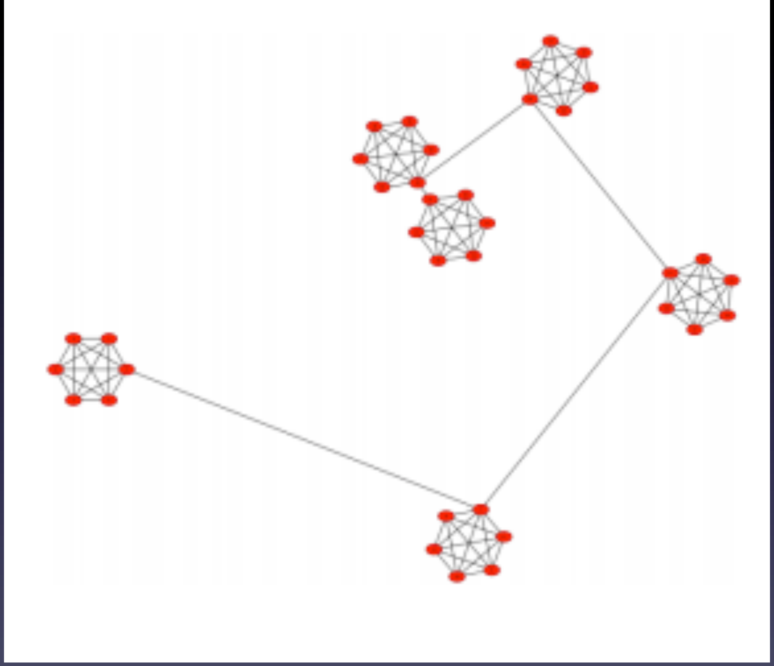


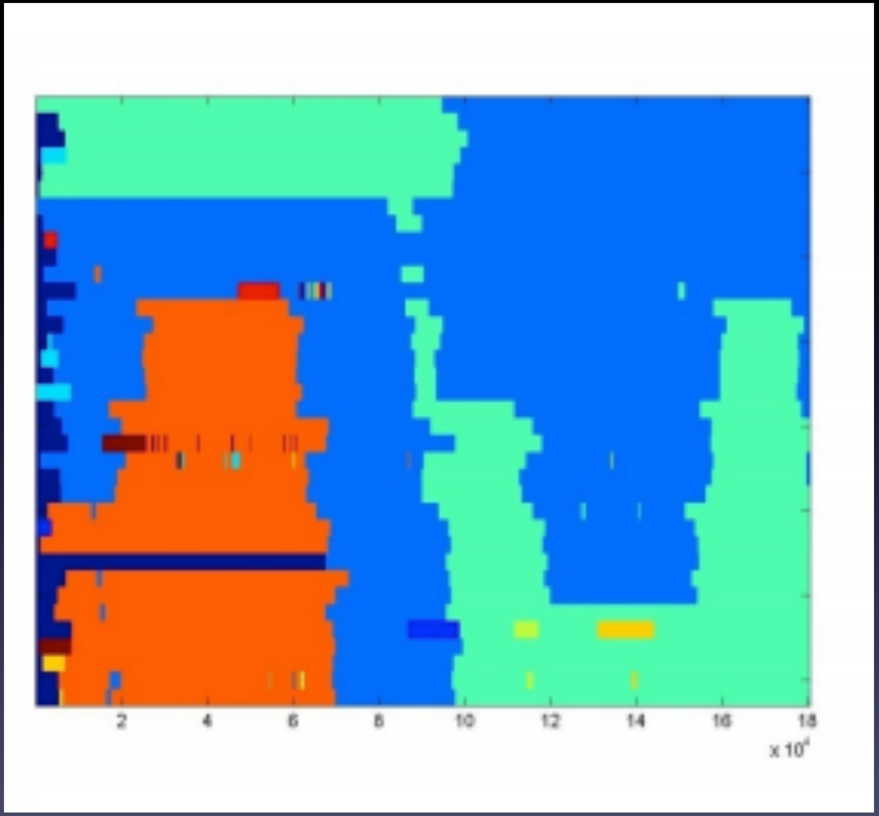
Coloring


Small
Worlds
Family
Simple Cycle


Preferential Attachment,
ν = 3

Consensus





Game Theory

- What is different from a Pure Strategy Nash Equilibrium and a Mixed Strategy Nash Equilibrium?
  - Players can randomize over actions. The probability distribution is public, but the realization, i.e. what action is actually chosen, is private. Think Rock-Paper-Scissors!
- (Mixed) Nash Equilibrium **always** exist in games with finite number of players and actions.

Mixed Strategy Equilibrium

Actions	A	B
A	-3, -3	-1, 2
B	2, -1	-10, -10

Mixed Strategy Equilibrium

Actions	A	B
A	2, 4	1, 3
B	2, 1	1, -2

Game Theory

		Column Player Actions		
		A	B	C
Row Player Actions	A	6,6	8,20	0,8
	B	10,0	5,5	2,8
	C	8,0	20,0	4,4

Game Theory

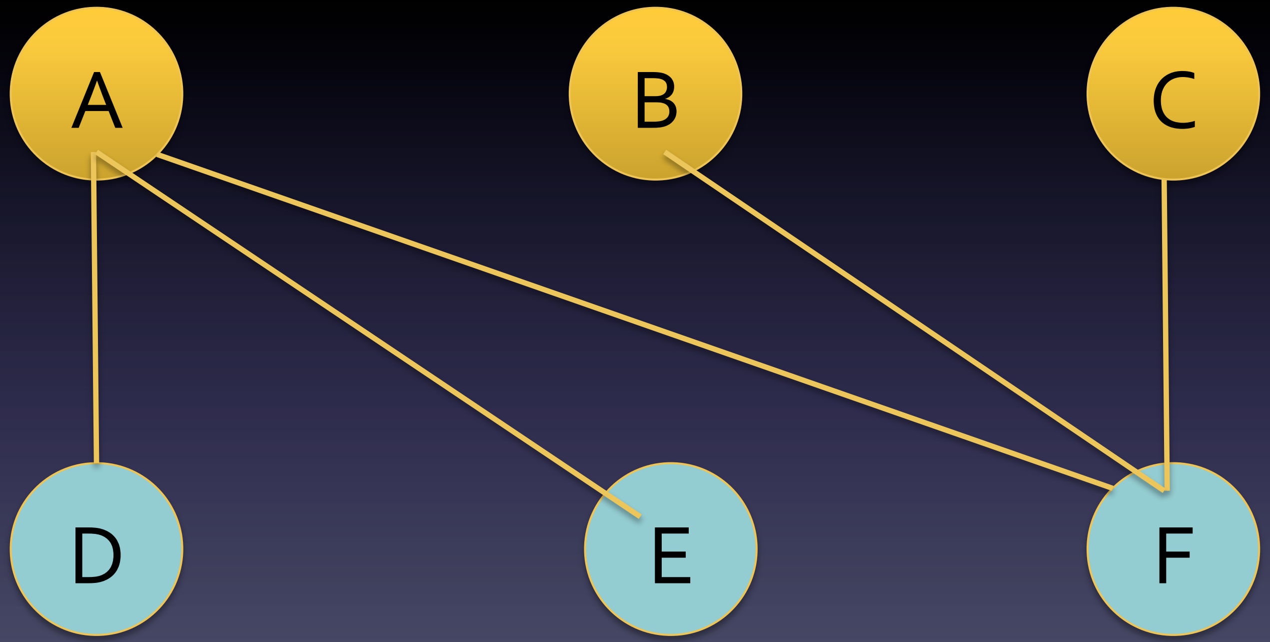
		Column Player Actions		
		~~A~~	B	C
Row Player Actions	A	~~6,6~~	8,20	0,8
	B	~~10,0~~	5,5	2,8
	C	~~8,0~~	20,0	4,4

Game Theory

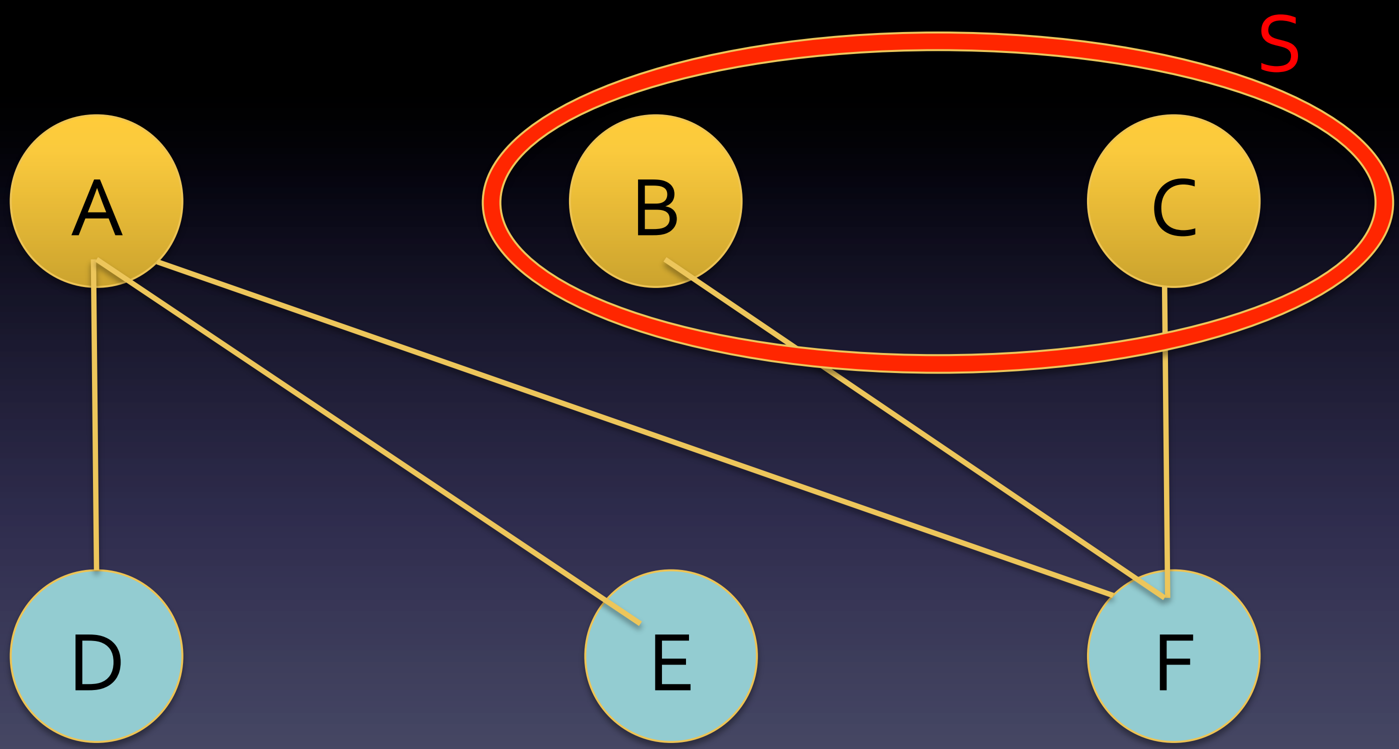
		Column Player Actions		
		~~A~~	B	C
Row Player Actions	~~A~~	~~6,6~~	~~8,20~~	~~0,8~~
	~~B~~	~~10,0~~	~~5,5~~	~~2,8~~
	C	~~8,0~~	20,0	4,4

- Nash Equilibrium at (C,C) for a payoff of 8.
- Optimal Strategy (A,B) gives payoff of 28.
- POA = 28/8 > 1

Network Trading

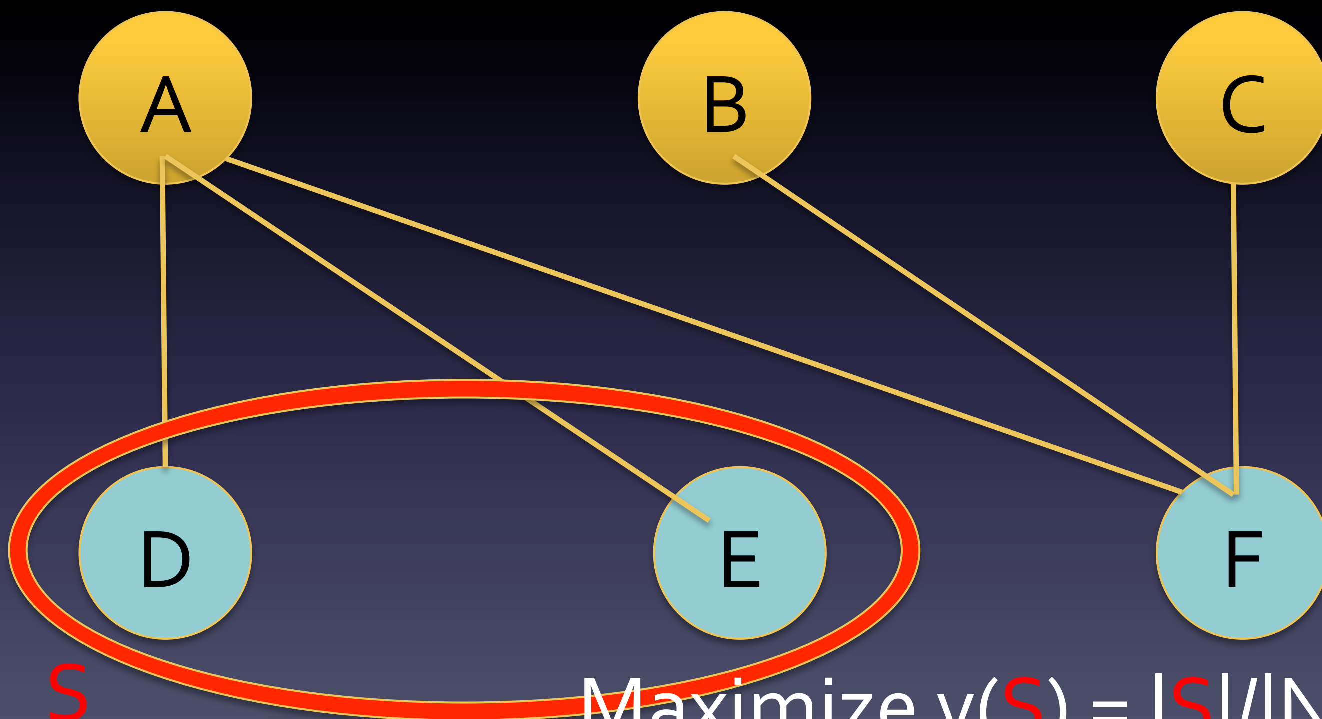


Network Trading



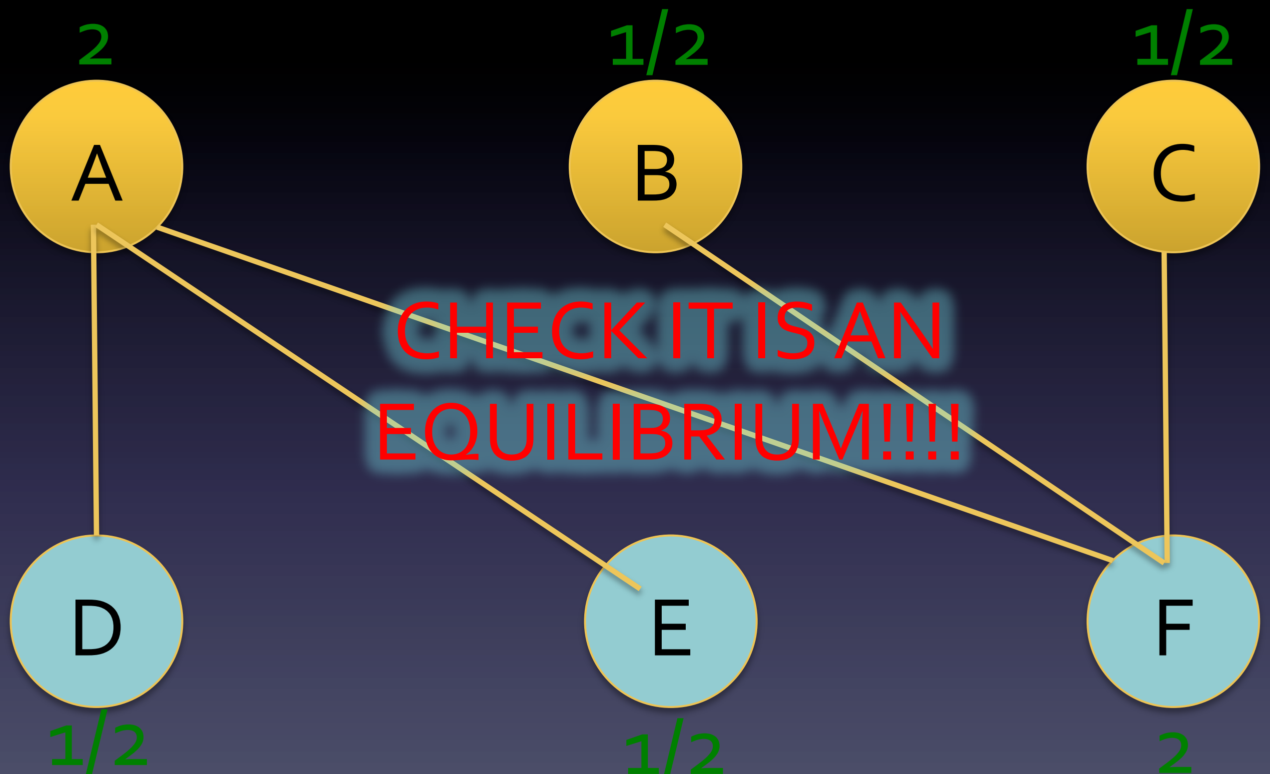
Maximize $v(S) = |S|/|N(S)| = 2$
=> B,C can get at most ½ and F gets at least 2.

Network Trading



Maximize v(S) = |S|/|N(S)| = 2
=> D,E can get at most ½ and A gets at least 2.

Network Trading



Trades: D and E give all their good to A, and A gives ½ of her good each to D and E. Similarly, B and C give all their good to F, and F gives ½ of his good to B and C.

Network Trading (in-class experiment)

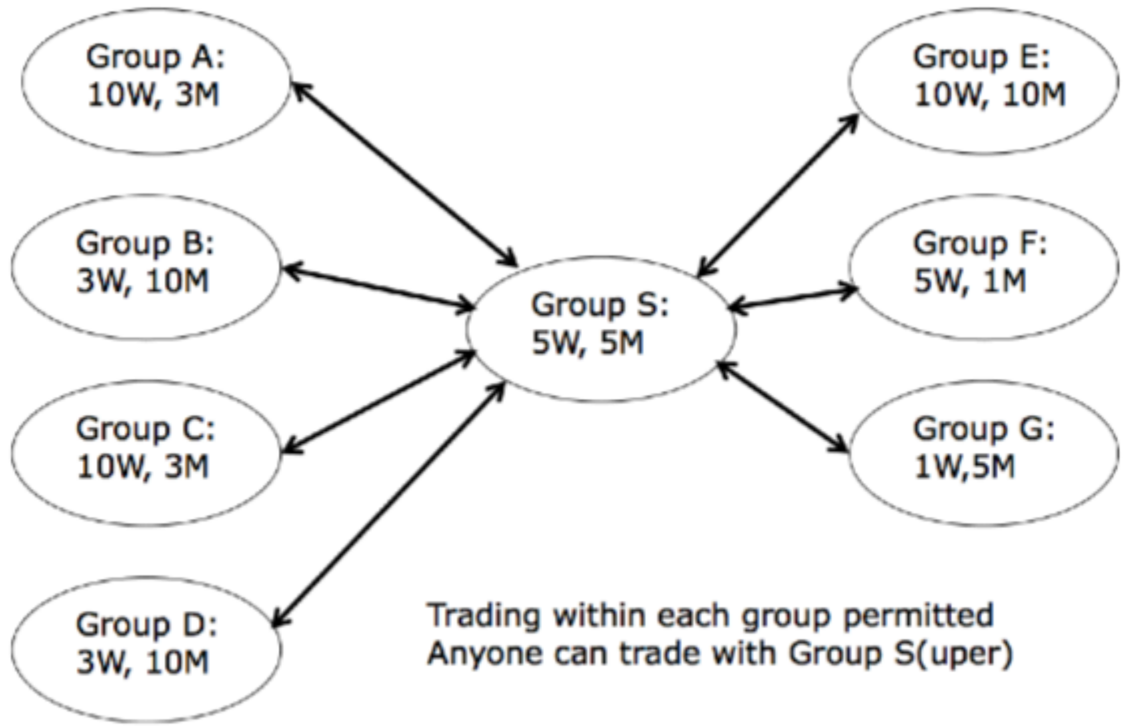
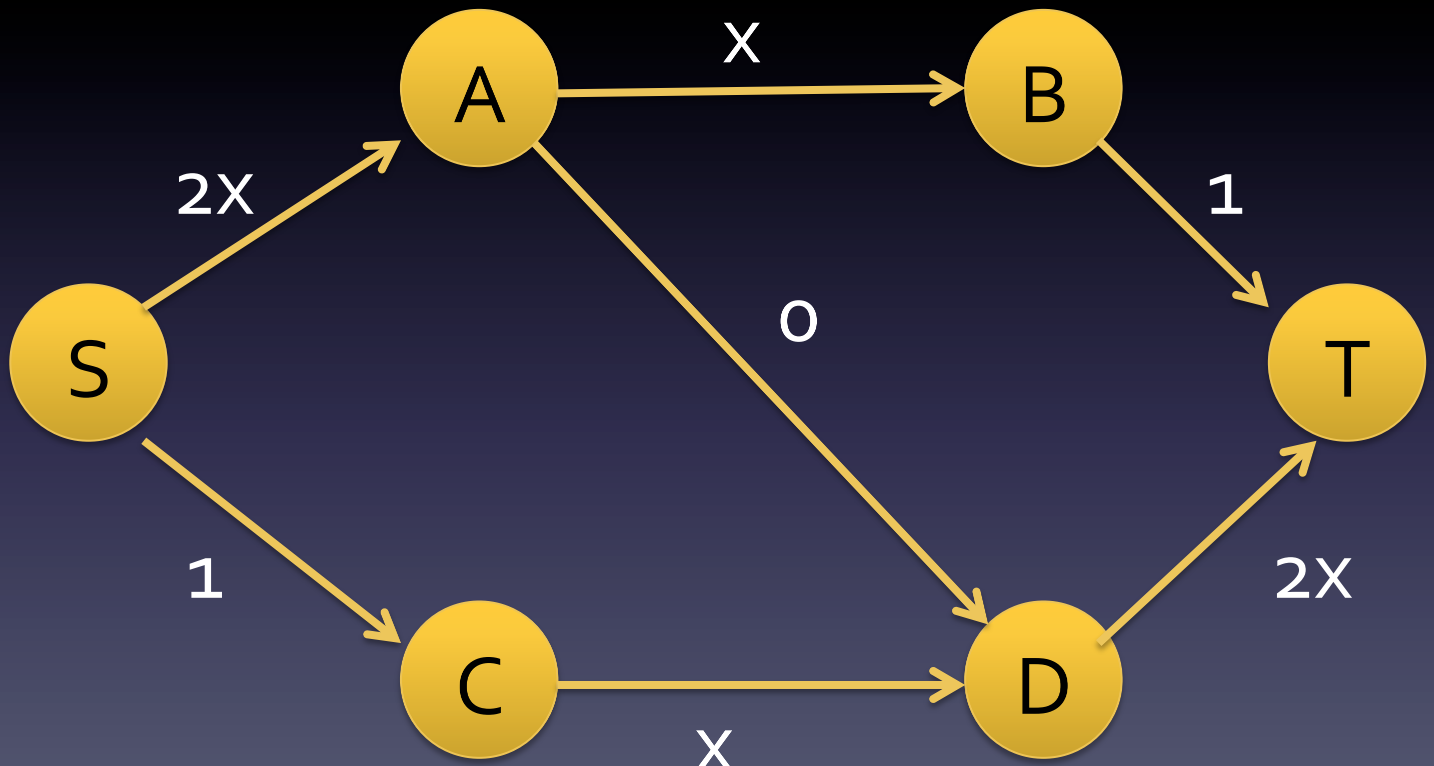
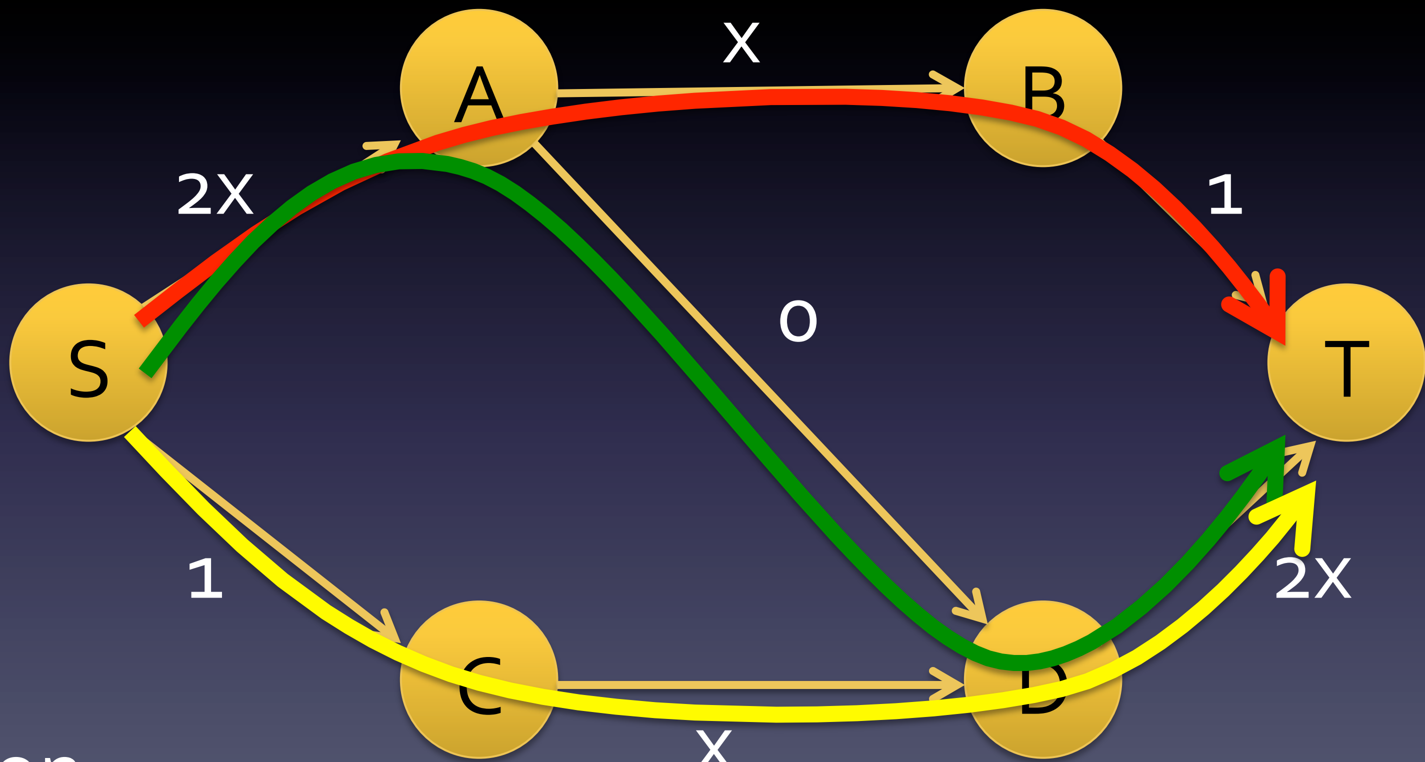


Figure 1: Network Trading - In Class Example

Selfish Routing



Selfish Routing

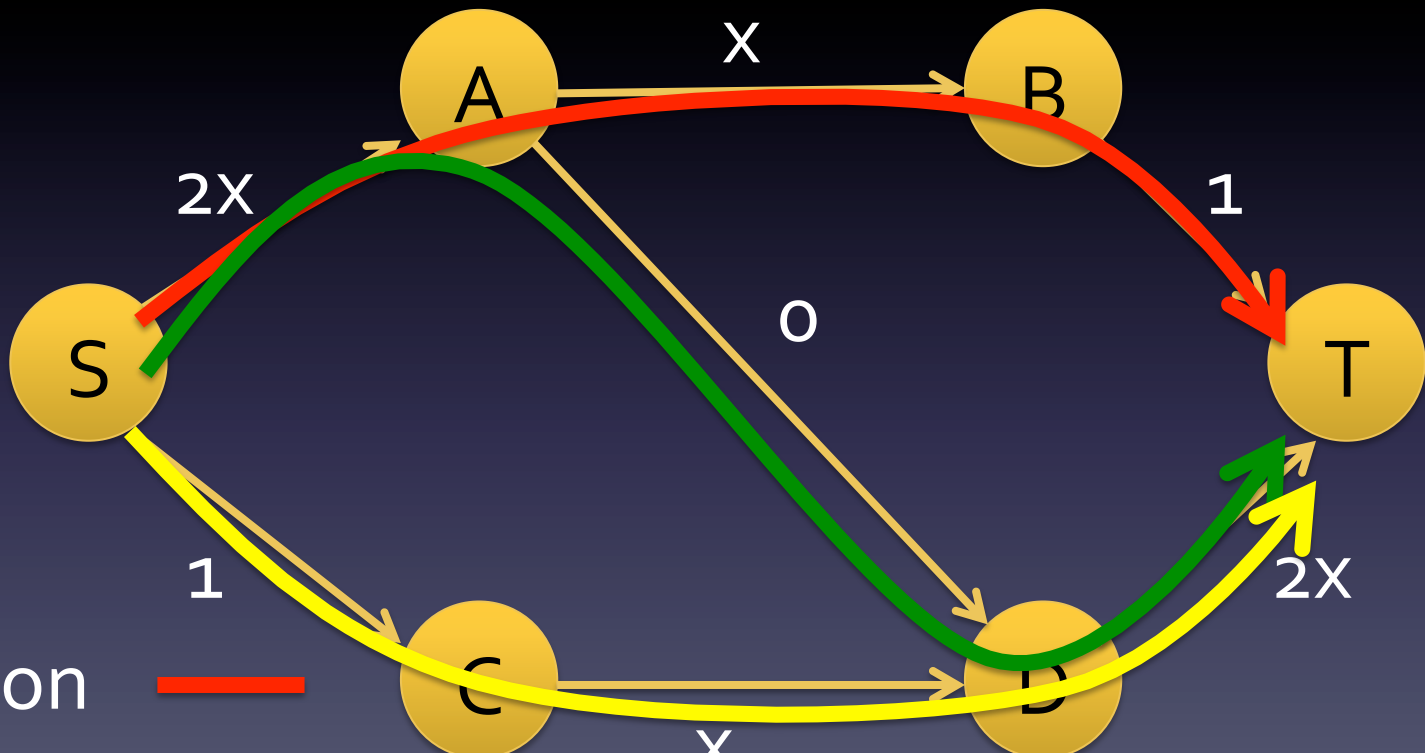


All on

Cost = 4

Equilibrium? NO!

Selfish Routing

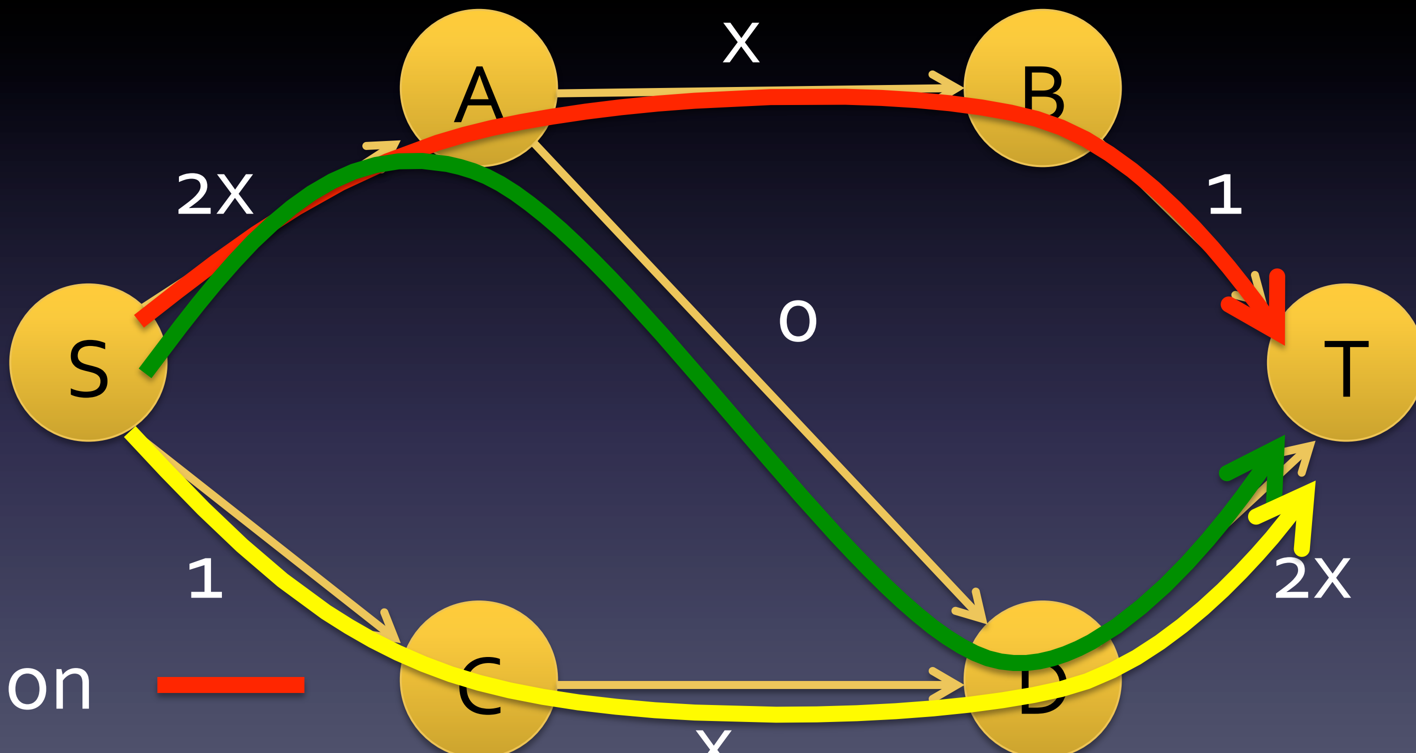


1/3 on
1/3 on
1/3 on

Cost =8/3=2.6667

Equilibrium? YES!

Selfish Routing

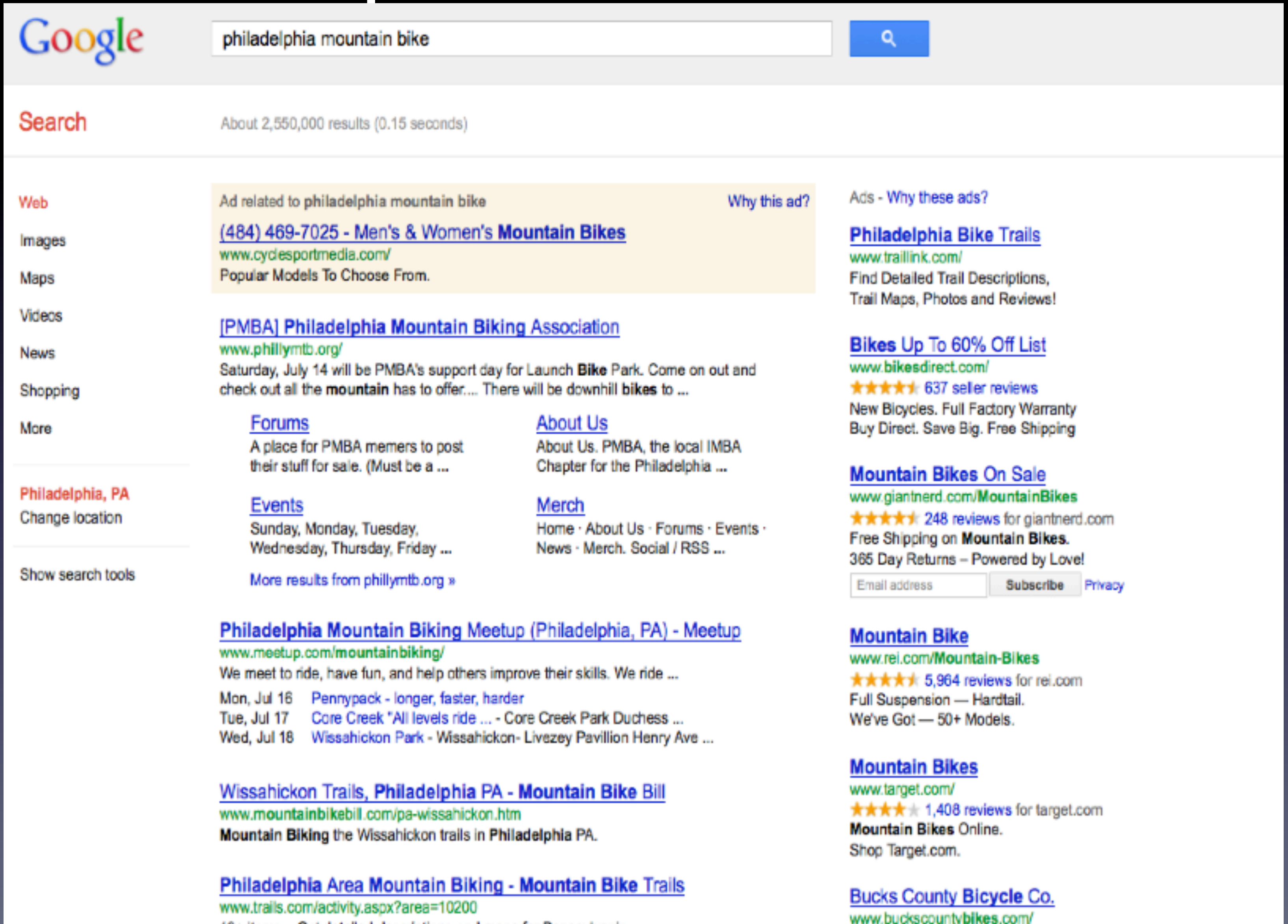


1/2 on
0 on
1/2 on

Cost =2.5 = OPT

Equilibrium? NO!

Sponsored Search



Sponsored Search

- Sponsored web search: a market for ad placement
- Advertiser submit their bids
- The winners get their ads displayed
- Advertisers pay only if their ads get clicked
- Second price auction

Sponsored Search