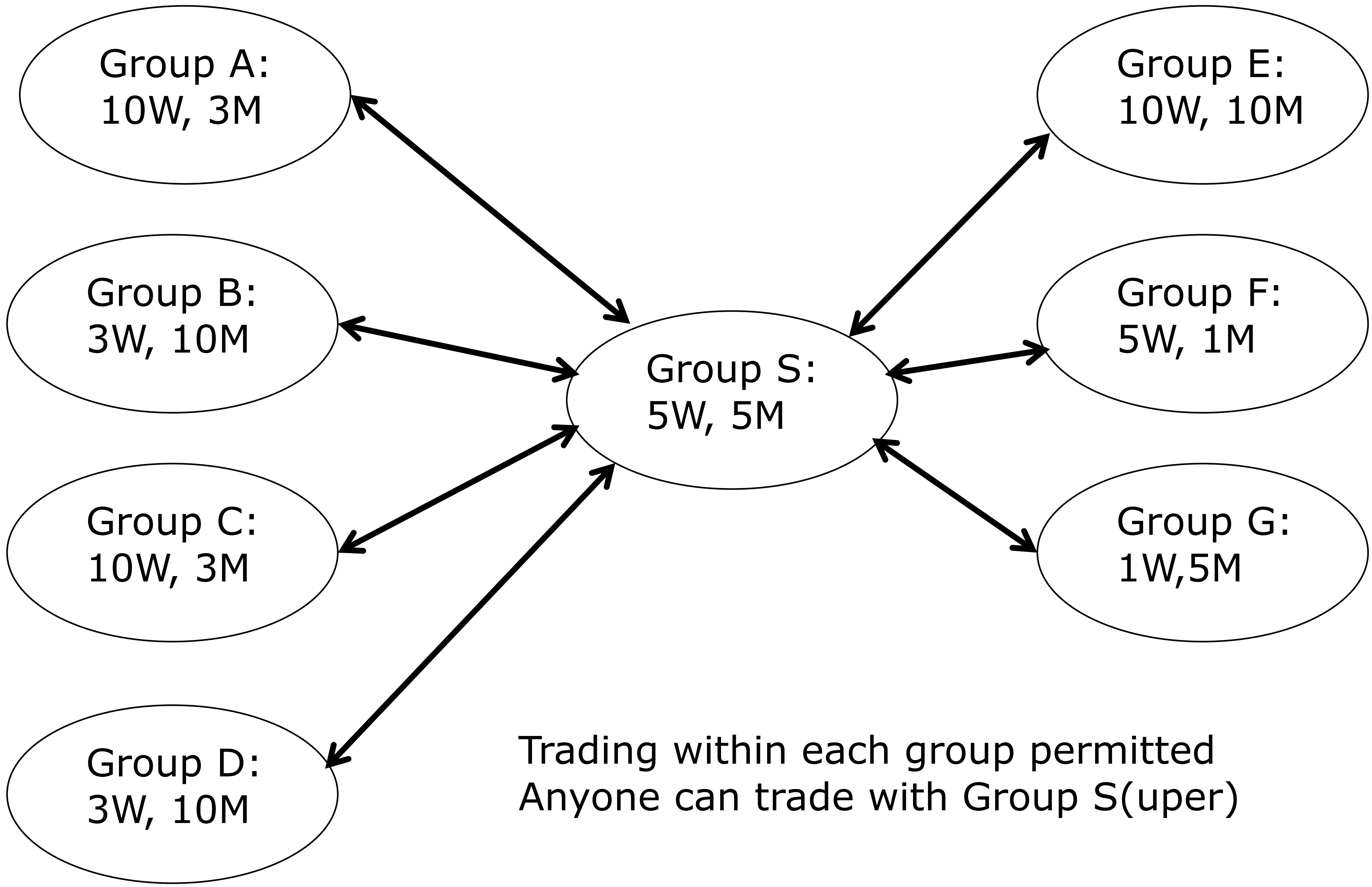
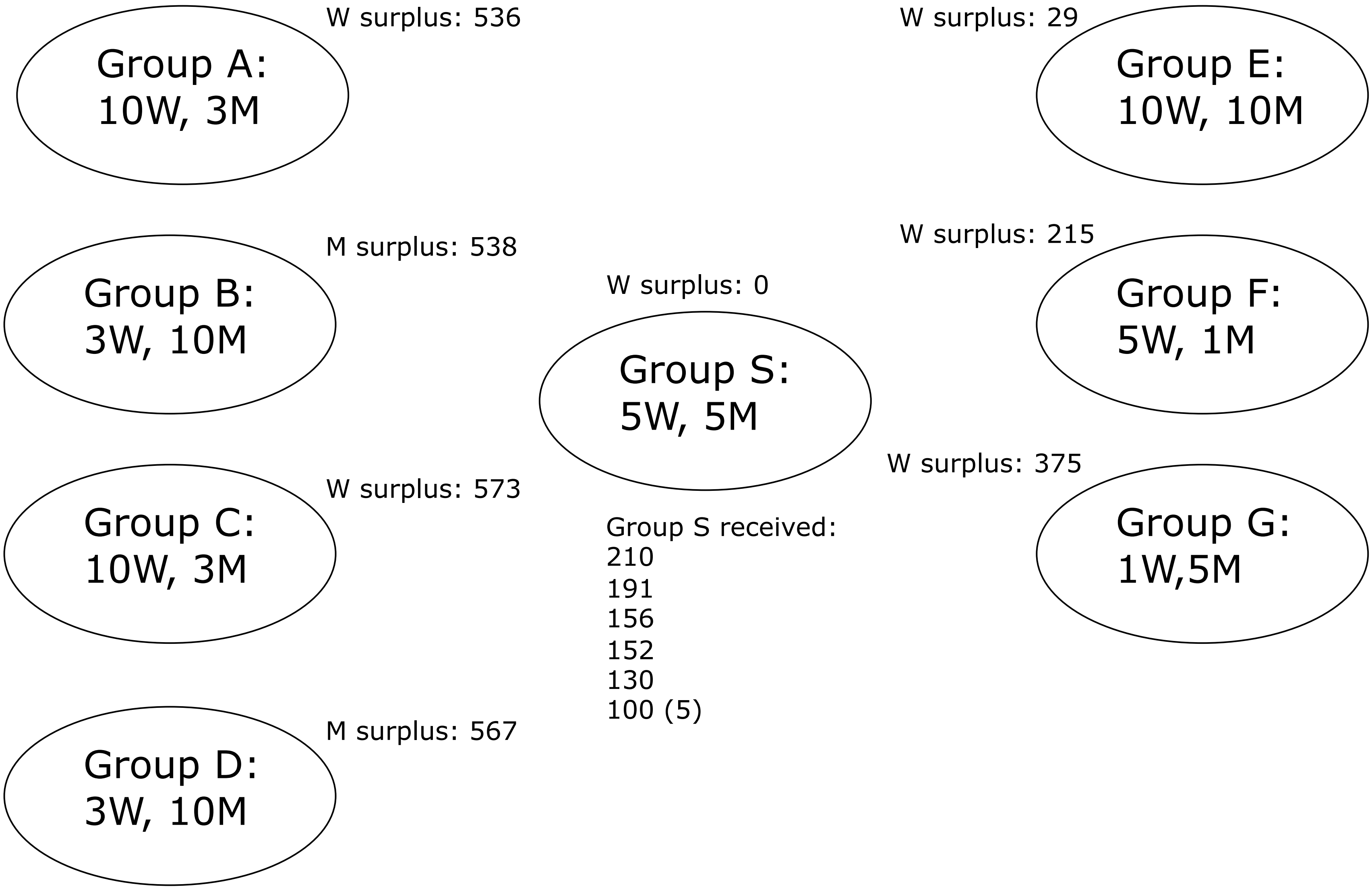

Group A:
10W, 3M
Group B:
3W, 10M
Group C:
10W, 3M
Group D:
3W, 10M
Group S:
5W, 5M
Group E:
10W, 10M
Group F:
5W, 1M
Group G:
1W,5M
Trading within each group permitted
Anyone can trade with Group S(uper)


W surplus: 536
Group A:
10W, 3M
M surplus: 538
Group B:
3W, 10M
W surplus: 573
Group C:
10W, 3M
M surplus: 567
Group D:
3W, 10M
W surplus: 0
Group S:
5W, 5M
Group S received:
210
191
156
152
130
100 (5)
W surplus: 29
Group E:
10W, 10M
W surplus: 215
Group F:
5W, 1M
W surplus: 375
Group G:
1W,5M

Group A:
10W, 3M

Group B:
3W, 10M

Group C:
10W, 3M

Group D:
3W, 10M

Group S:
5W, 5M

Group E:
10W, 10M

Group F:
5W, 1M

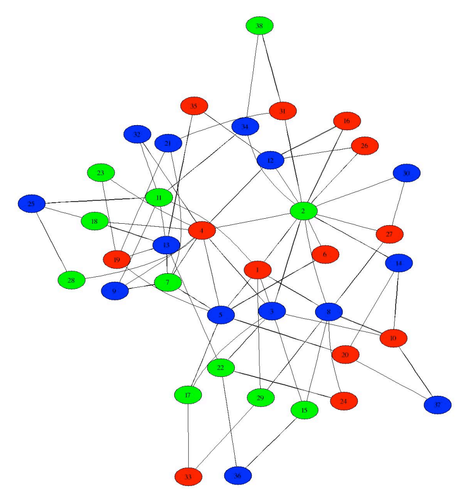
Group G:
1W,5M

What “should” have happened?

- Group E trades internally 1-for-1
- Group A,C,F Ws trade with A,C,F,S Ms
  Exchange rate: 25/12 W for 1 M
- Group B,D,G Ms trade with B,D,G,S Ws
  Exchange rate: 25/12 M for 1 W

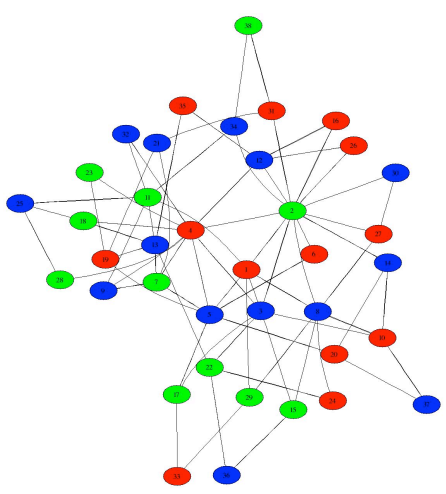
# Roadmap

- Networked trading motivation
- A simple model and its equilibrium
- A detailed example



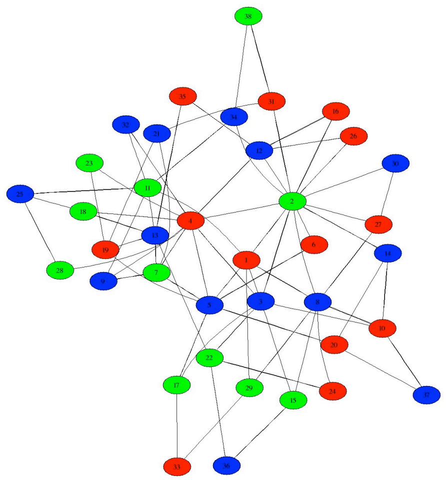
# Trading in Networks:
# I. Model

Prof. Michael Kearns
Networked Life
NETS 112
Fall 2014



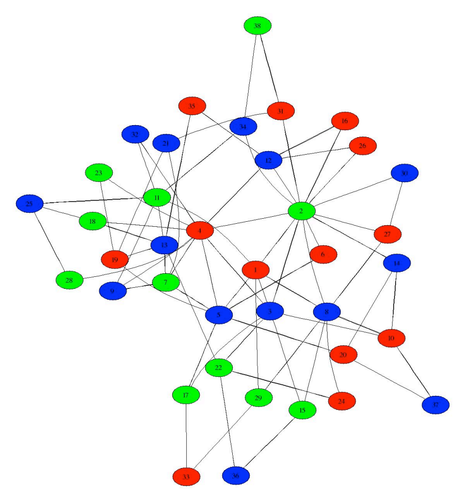
# Roadmap

- Networked trading motivation
- A simple model and its equilibrium
- A detailed example



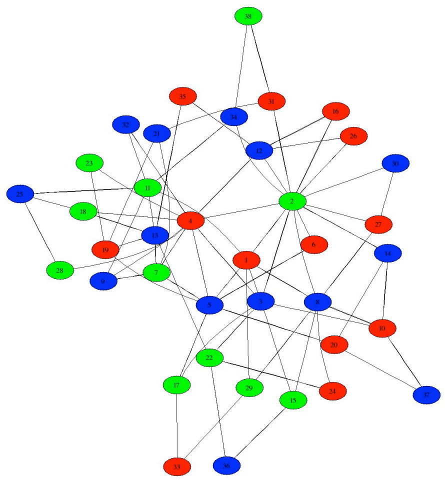
# Networked Games vs. Trading

- Models and experiments so far (coloring, consensus, biased voting):
  - simple coordination games
  - extremely simple actions (pick a color)
  - "trivial" equilibrium theories ("good" equilibrium or "trapped" players)
  - no equilibrium predictions about network structure and individual wealth
- Networked trading:
  - a "financial" game
  - complex action space (set of trades with neighbors)
  - nontrivial equilibrium theory
  - detailed predictions about network structure and individual wealth



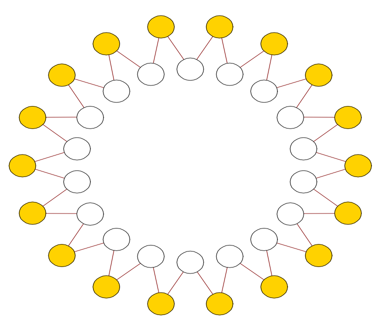
# Networked Trading: Motivation

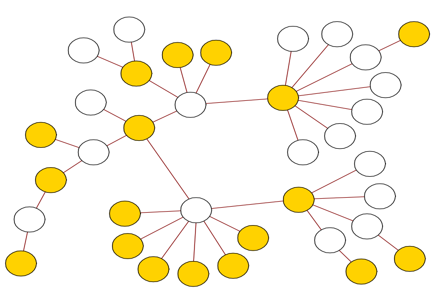
- Settings where there are restrictions on who can trade with whom
- International trade: restrictions, embargos and boycotts
- Financial markets: some transactions are forbidden
  - e.g. trades between brokerage and proprietary trading in investment banks
- Geographic constraints: must find a local housecleaning service
- Natural to model by a network:
  - vertices representing trading parties
  - presence of edge between u and v: trading permitted between parties
  - absence of edge: trading forbidden



# A Simple Model of Networked Trading

- Imagine a world with only two goods or commodities for trading
  - let's call them Milk and Wheat
- Two types of traders:
  - Milk traders: start game with 1 unit (fully divisible) of Milk, but only value Wheat
  - Wheat traders: start game with 1 unit of Wheat, but only value Milk
  - trader's payoff = amount of the "other" good they obtain through trades
  - "mutual interest in trade"
  - equal number of each type → same total amount of Milk and Wheat
- Only consider *bipartite* networks:
  - all edges connect a Milk trader to a Wheat trader
  - can only trade with your network neighbors!
  - all trades are irrevocable
  - no resale or arbitrage allowed

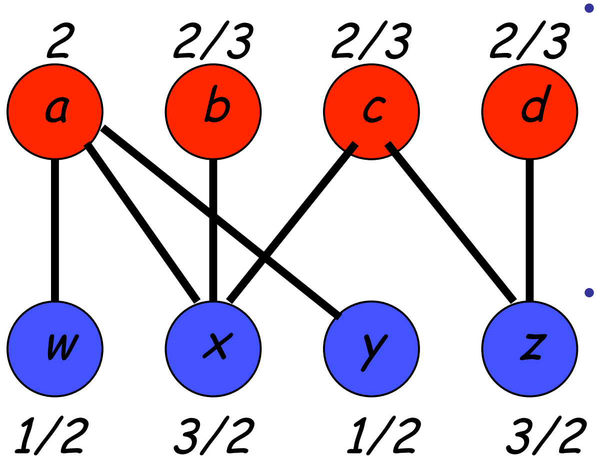




# Equilibrium Concept

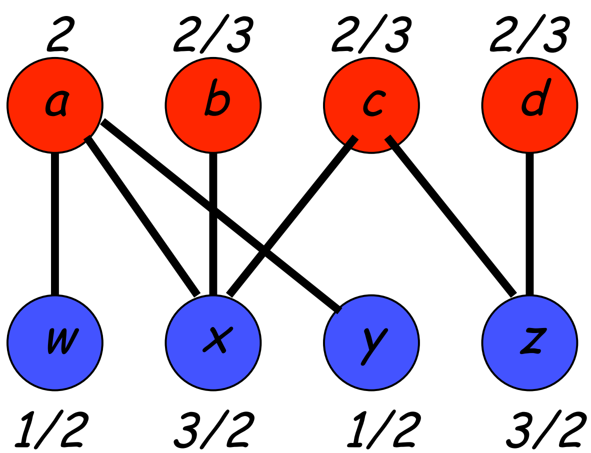
- Imagine we assigned a price or exchange rate to each vertex/trader
  - e.g. "I offer my 1 unit of Milk for 1.7 units of Wheat"
  - e.g. "I offer my 1 unit of Wheat for 0.8 units of Milk"
  - note: "market" sets the prices, not traders ("invisible hand")
  - unlike a traditional game --- traders just react to prices
- Equilibrium = set of prices + trades such that:
  - 1. market *clears:* everyone trades away their initial allocation
  - 2. rationality (best responses): a trader only trades with best prices in neighborhood
  - e.g. if a Milk trader's 4 neighbors offer 0.5, 1.0, 1.5, 1.5 units Wheat, they can trade only with those offering 1.5
  - note: set of trades must ensure supply = demand at every vertex
- Simplest example: complete bipartite network
  - every pair of Milk and Wheat traders connected by an edge
  - equilibrium prices: everyone offers their initial 1 unit for 1 unit of the other good
  - equilibrium trades: pair each trader with a unique partner of other type
  - market clears: everyone engages in 1-for-1 trade with their partner
  - rationality: all prices are equal, so everyone trading with best neighborhood prices

# A More Complex Example

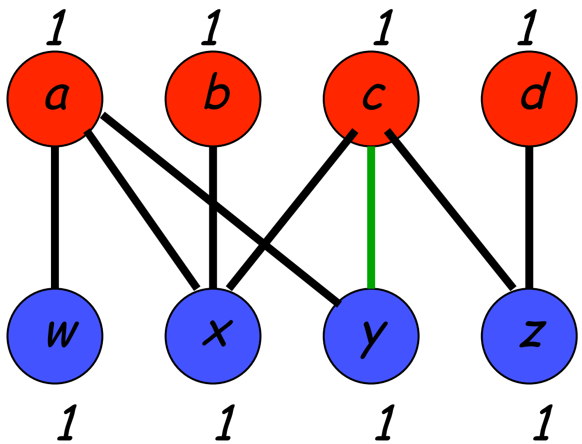



- equilibrium prices as shown (amount of the other good demanded)
- equilibrium trades:
  - a: sends ½ unit each to w and y, gets 1 from each
  - b: sends 1 unit to x, gets 2/3 from x
  - c: sends ½ unit each to x and z, gets 1/3 from each
  - d: sends 1 unit to z, gets 2/3 from z
- equilibrium check, blue side:
  - w: traded with a, sent 1 unit
  - x: traded with b and c, sent 1 unit
  - y: traded with a, sent 1 unit
  - z: traded with c and d, sent 1 unit

# Remarks



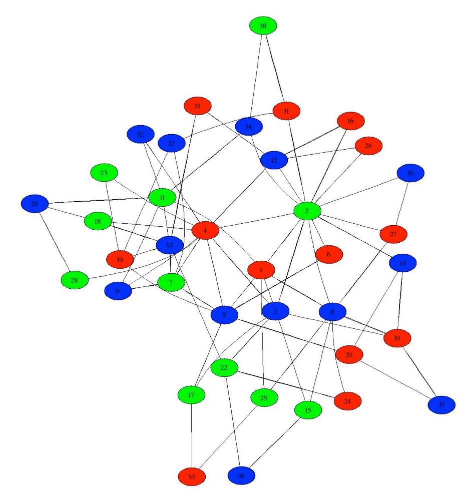

- How did I figure this out? Not easy in general
- Some edges unused by equilibrium
- Trader wealth = equilibrium price at their vertex
- If two traders trade, their wealths are reciprocal (w and 1/w)
- Equilibrium *prices (wealths)* are always unique
- Network structure led to *variation* in wealth




- Suppose we add the single green edge
- Now equilibrium has no wealth variation!

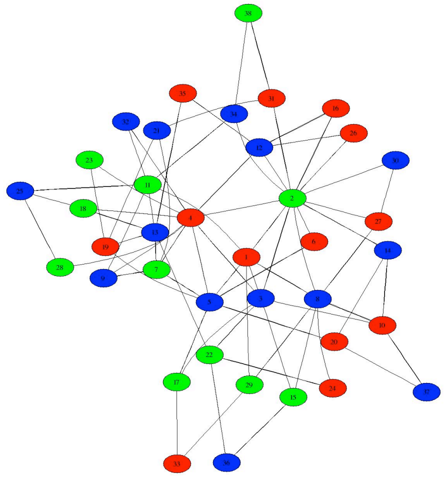
# Summary

- (Relatively) simple networked trading model
- Equilibrium = prices + trades such that market clears, traders rational
- Some networks don't have wealth variation at equilibrium, some do
- Next: What is the general relationship between structure and prices?



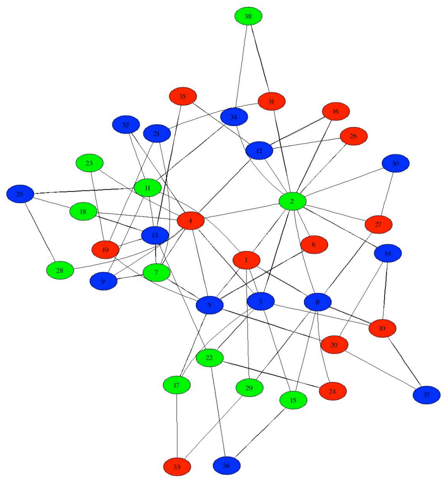
# Trading in Networks:
# II. Network Structure and Equilibrium

## Networked Life
## Prof. Michael Kearns



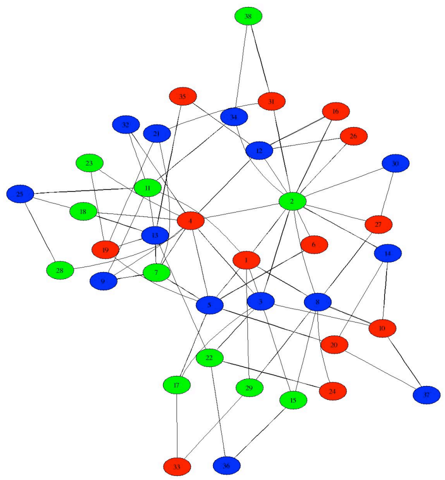
# Roadmap

- Perfect matchings and equilibrium equality
- Characterizing wealth inequality at equilibrium
- Economic fairness of Erdös-Renyi and Preferential Attachment

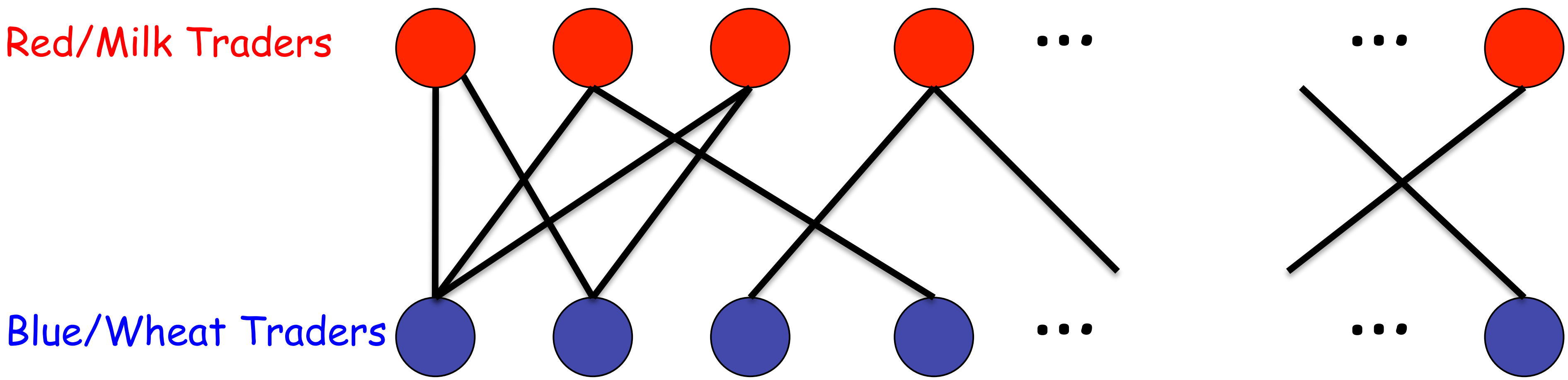


# Trading Model Review

- Bipartite network, equal number of Milk and Wheat traders
- Each type values only the other good
- Equilibrium = prices + trades such that market clears, traders rational

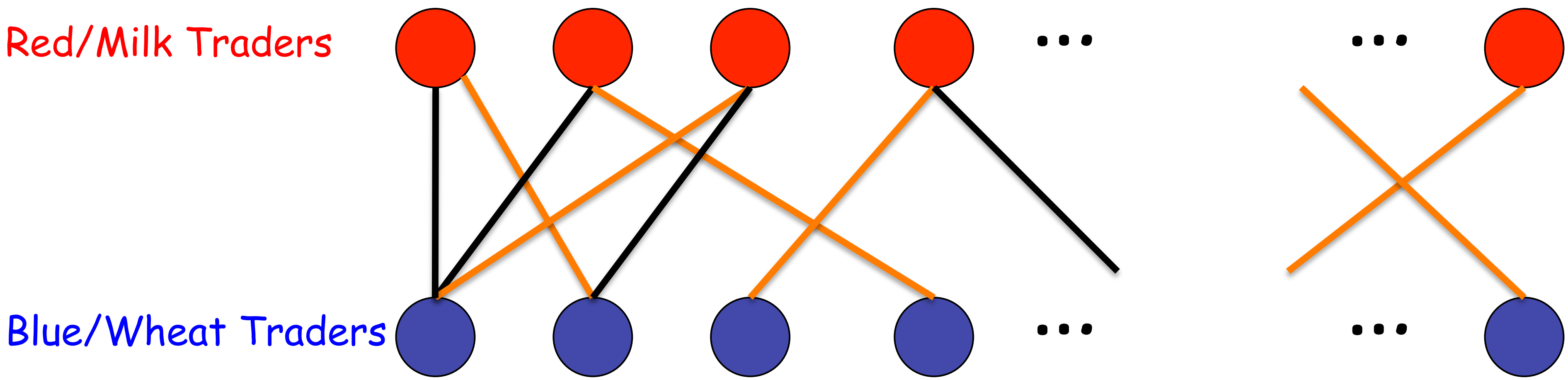


# Perfect Matchings



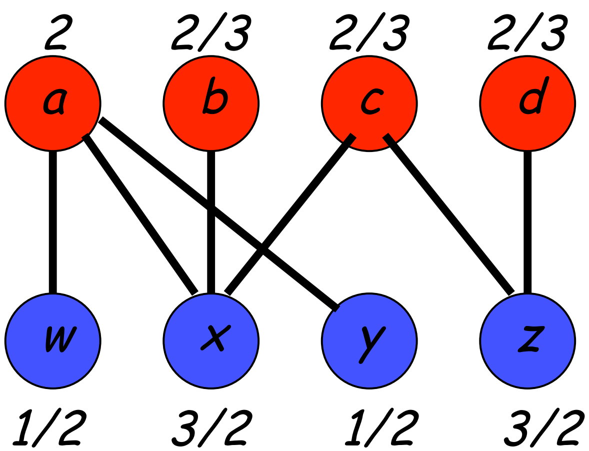

- A pairing of reds and blues so everyone has *exactly one partner*
- So really a subset of the edges with each vertex in exactly one edge
- Some networks may have many different perfect matchings
- Some networks may have no perfect matchings

# Perfect Matchings

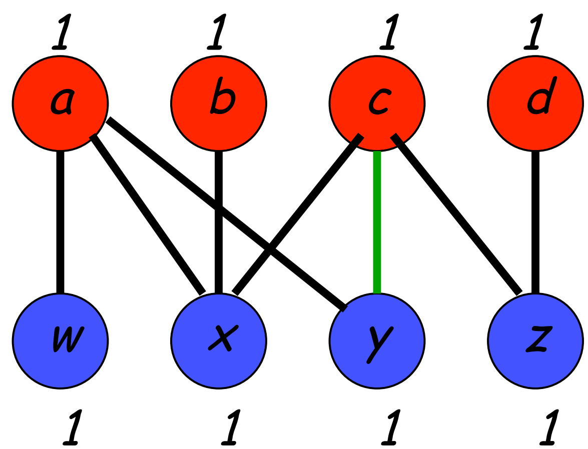



- A pairing of reds and blues so everyone has *exactly one partner*
- So really a subset of the edges with each vertex in exactly one edge
- Some networks may have many different perfect matchings
- Some networks may have no perfect matchings

# Examples



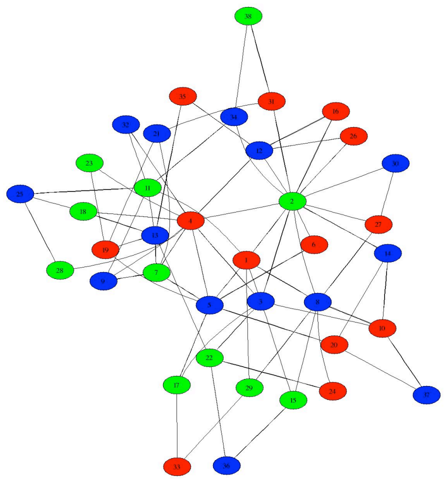

Has no perfect matching



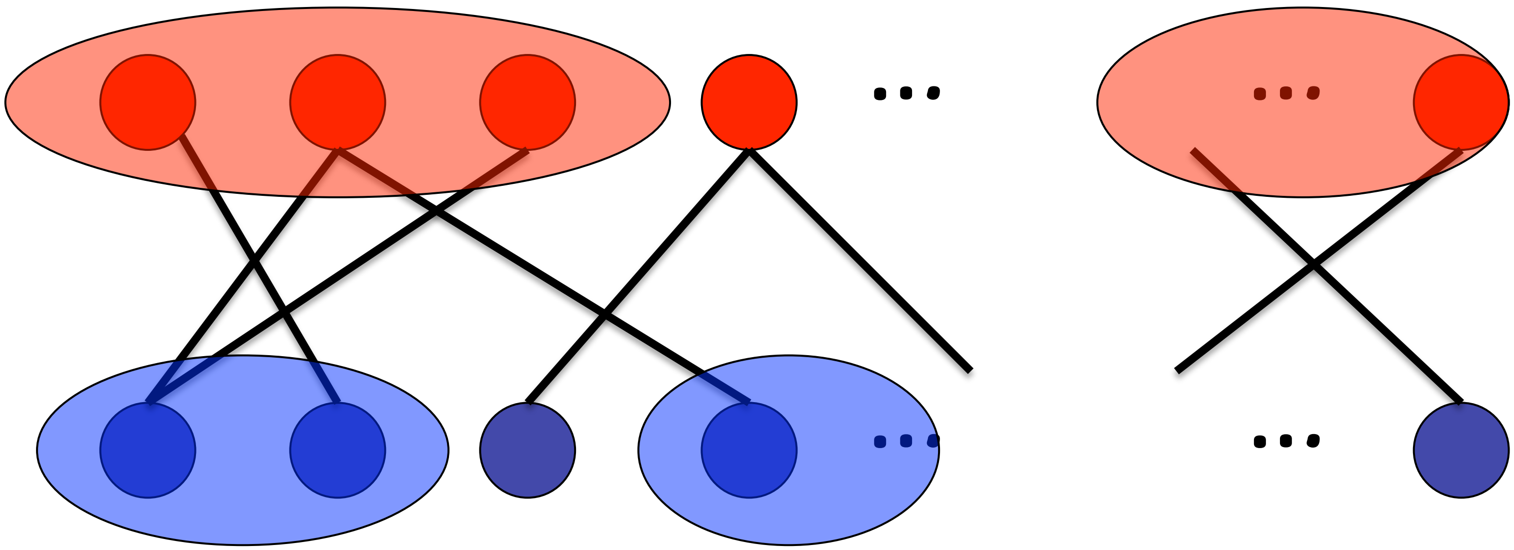

Has a perfect matching

# Perfect Matchings and Equality

- Theorem: There will be *no wealth variation* at equilibrium (all exchange rates = 1) if and only if the bipartite trading network contains a perfect matching.
- Characterizes sufficient "trading opportunities" for fairness
- What if there is no perfect matching?



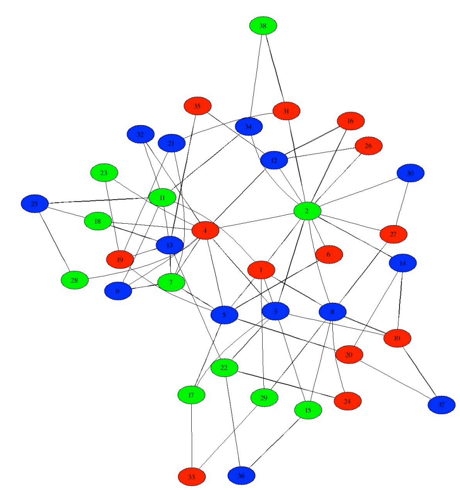
# Neighbor Sets



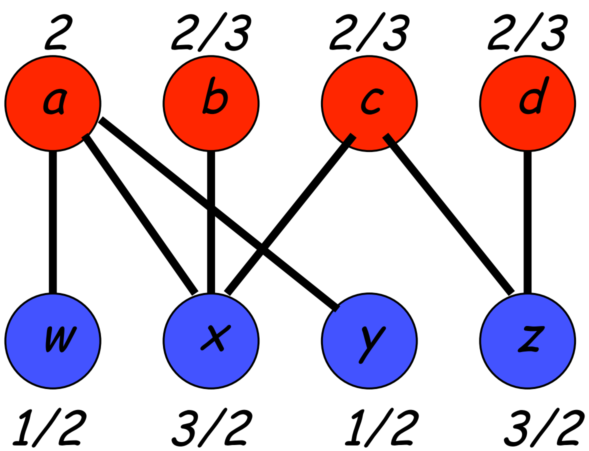
- Let S be any set of traders on one side
- Let N(S) be the set of traders on the other side connected to any trader in S; these are the only trading partners for S collectively
- Intuition: if N(S) is much smaller than S, S may be in trouble
- S are "captives" of N(S)
- Note: If there is a perfect matching, N(S) always *at least as large* as S

# Characterizing Inequality

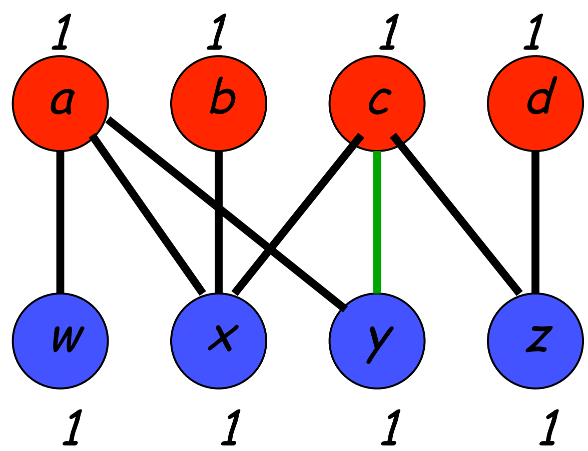
- For any set S, let v(S) denote the ratio (size of S)/(size of N(S))
- Theorem: If there is a set S such that v(S) > 1, then at equilibrium the traders in S will have wealth at most 1/v(S), and the traders in N(S) will have wealth at least v(S).
- Example: v(S) = 10/3 → S gets at most 3/10, N(S) at least 10/3
- Greatest inequality: find S maximizing v(S)
- Can iterate to find all equilibrium wealths
- Corollary: adding edges can only *reduce* inequality
- Network structure completely determines equilibrium wealths
- Note: trader/vertex degree not directly related to equilibrium wealth



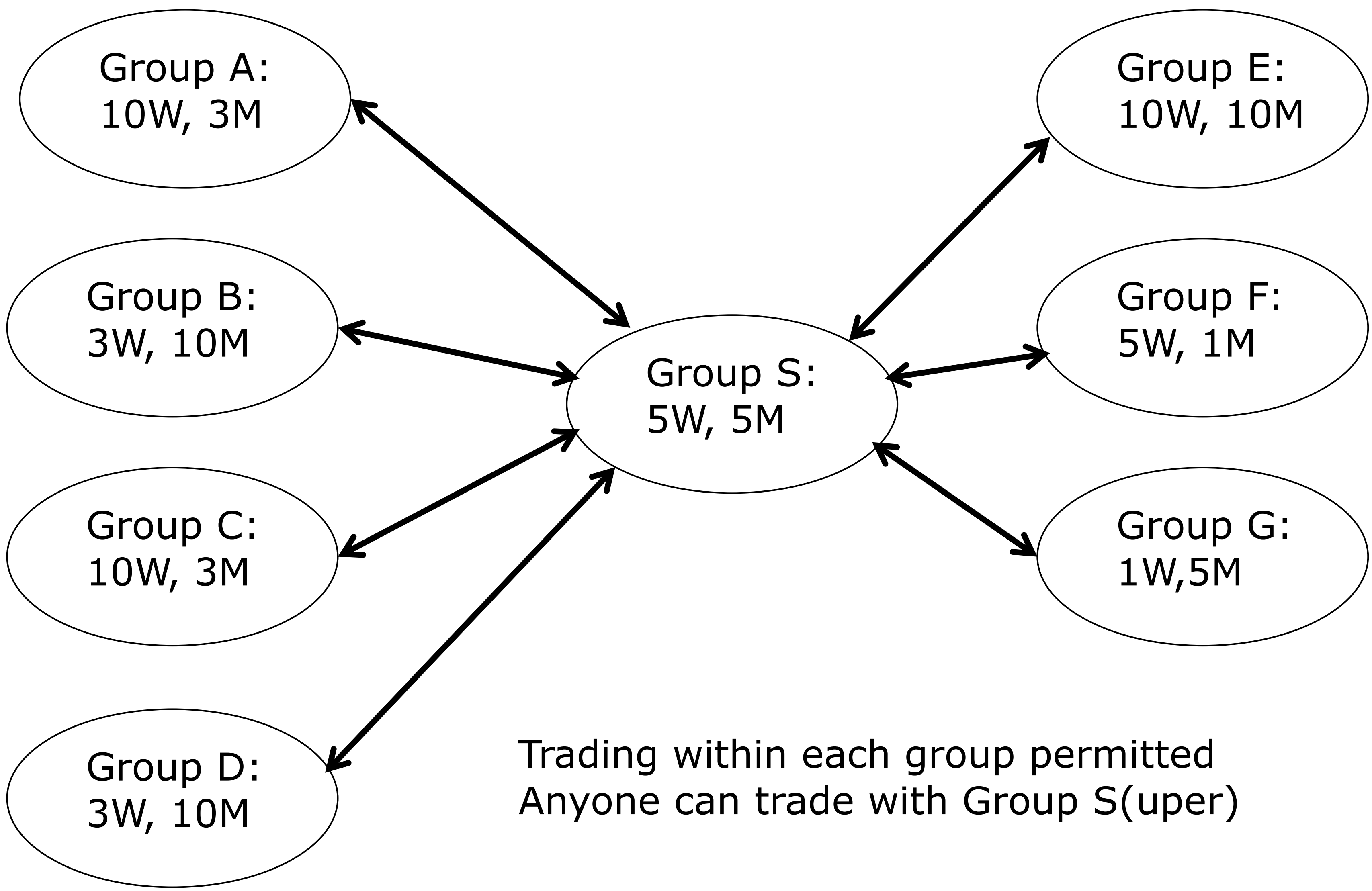
# Examples Revisited




Has no perfect matching




Has a perfect matching


Group A:
10W, 3M
Group B:
3W, 10M
Group C:
10W, 3M
Group D:
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5W, 5M
Group E:
10W, 10M
Group F:
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Group G:
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Trading within each group permitted
Anyone can trade with Group S(uper)

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Group S:
5W, 5M

Group E:
10W, 10M

Group F:
5W, 1M

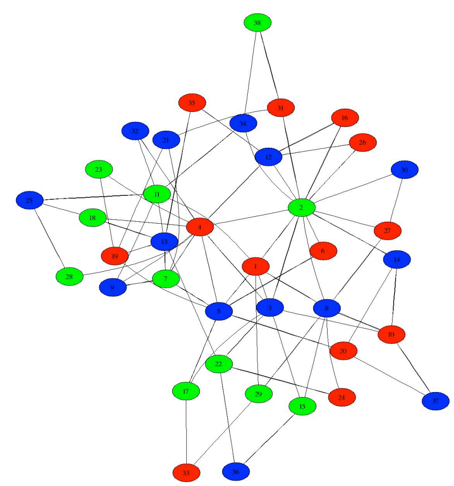
Group G:
1W,5M

What “should” have happened?

- Group E trades internally 1-for-1
- Group A,C,F Ws trade with A,C,F,S Ms
  Exchange rate: 25/12 W for 1 M
- Group B,D,G Ms trade with B,D,G,S Ws
  Exchange rate: 25/12 M for 1 W

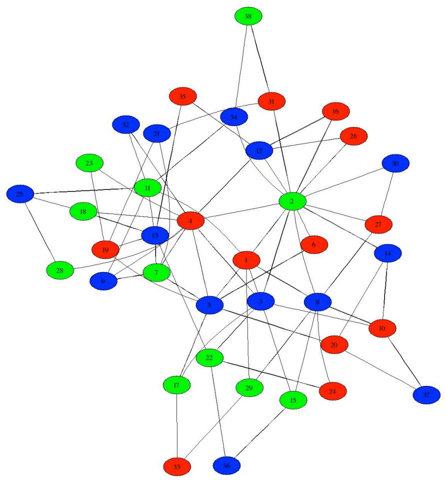
# Inequality in Formation Models

- Bipartite version of Erdös-Renyi: even at low edge density, very likely to have a perfect matching → *no wealth variation* at equilibrium
- Bipartite version of Preferential Attachment: wealth variation will *grow rapidly* with population size
- Erdös-Renyi generates economically "fairer" networks



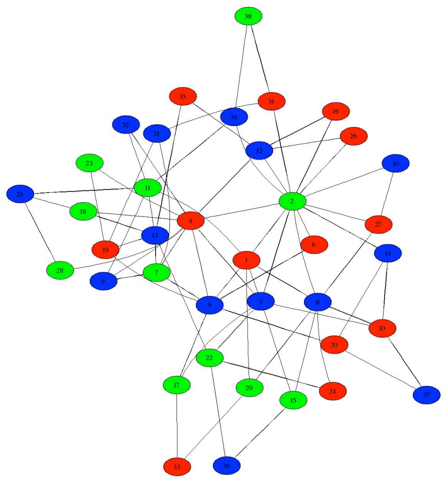
# Summary

- Ratios v(S) completely characterize equilibrium
- Determined entirely by network structure
- More subtle and global than trader degrees
- Next: comparing equilibrium predictions with human behavior



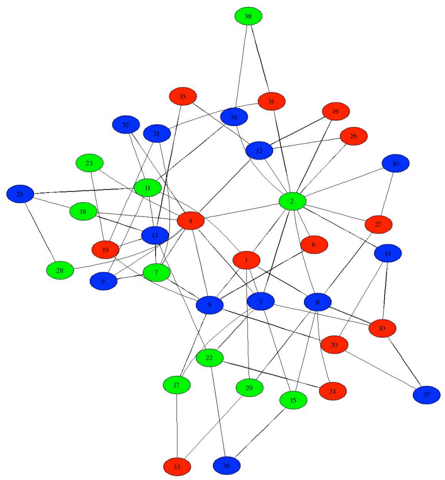
# Trading in Networks:
# III. Behavioral Experiments

## Networked Life
## Prof. Michael Kearns



# Roadmap

- Experimental framework and trading mechanism/interface
- Networks used in the experiments
- Visualization of actual experiments
- Results and comparison to equilibrium theory predictions

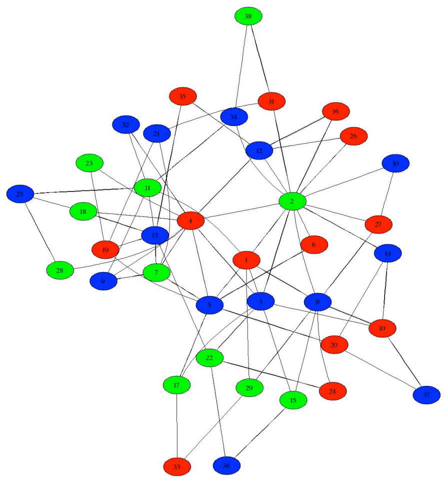


# Equilibrium Theory Review

- Equilibrium prices/wealths entirely determined by network structure
- Largest/smallest wealths determined by largest ratios:

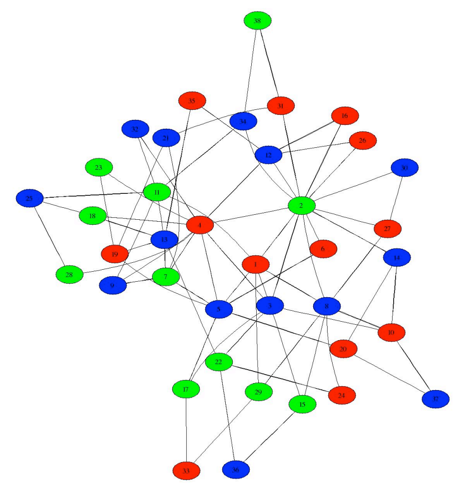
  v(S) = (size of S)/(size of N(S))   N(S) "winners", S "losers"

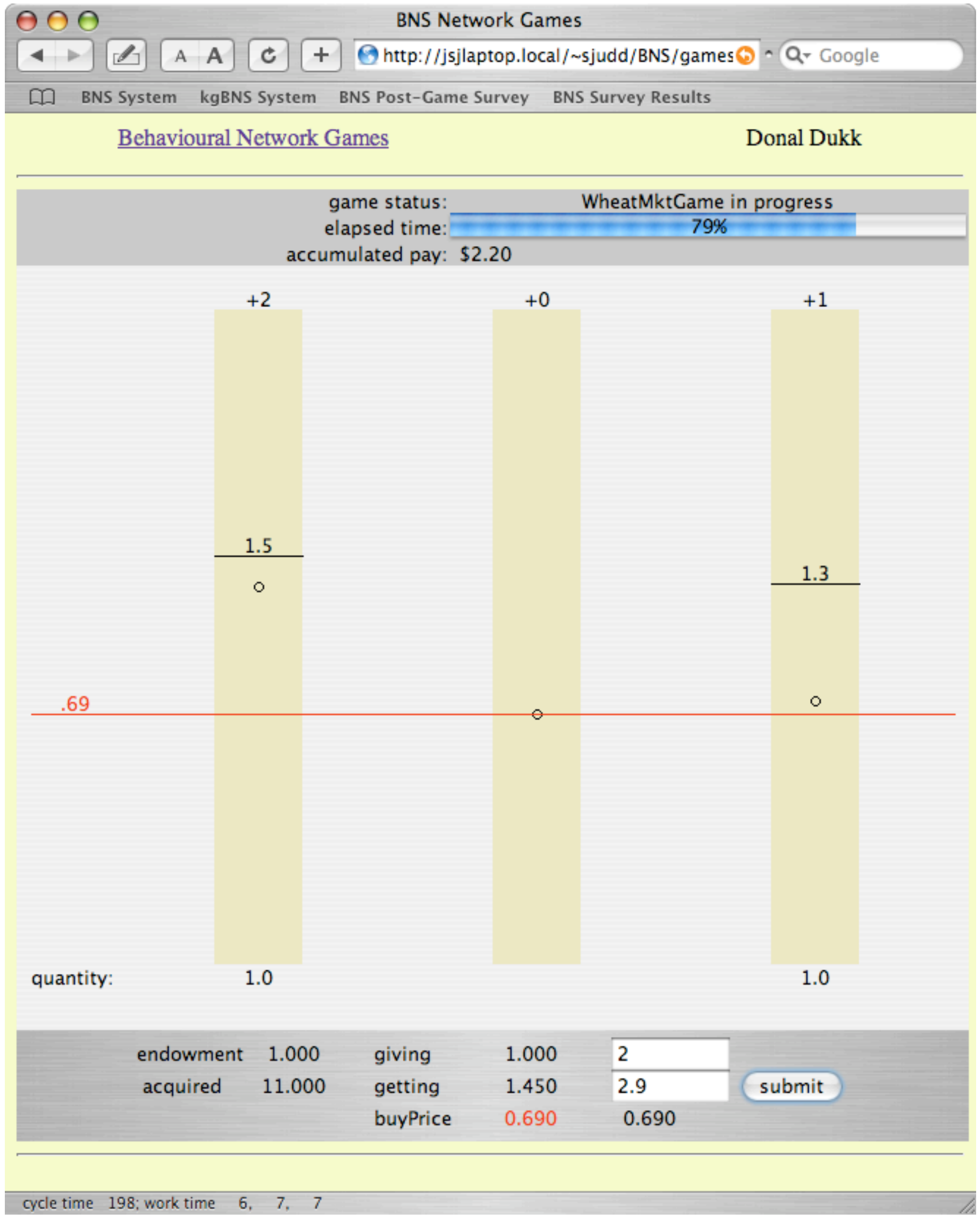
- Network has a perfect matching: all wealths = 1

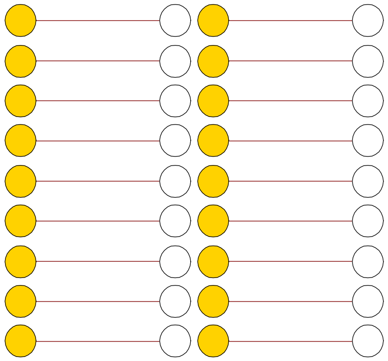


# Experimental Framework

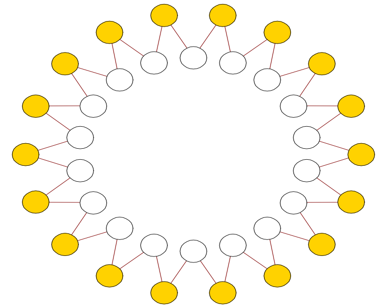
- Same framework as coloring, consensus and biased voting experiments
- 36 simultaneous human subjects in lab of networked workstations
- In each experiment, subjects play our trading model on varying networks
- In equilibrium theory, prices are magically *given* ("invisible hand")
- In experiments, need to provide a mechanism for price *discovery*
- Experiments used simple *limit order* trading with neighbors
  - networked version of standard financial/equity market mechanism
- Each player starts with 10 fully divisible units of Milk or Wheat
  - payments proportional to the amount of the other good obtained



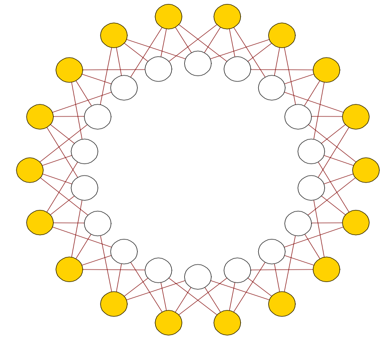

BNS Network Games
http://jsjlaptop.local/~sjudd/BNS/games
Google
BNS System
kgBNS System
BNS Post-Game Survey
BNS Survey Results
Behavioural Network Games
Donal Dukk
game status:
WheatMktGame in progress
elapsed time:
79%
accumulated pay: $2.20
+2
+0
+1
1.5
1.3
.69
quantity:
1.0
1.0
endowment 1.000
acquired 11.000
giving 1.000
getting 1.450
buyPrice 0.690
2
2.9
0.690
submit
cycle time 198; work time 6, 7, 7



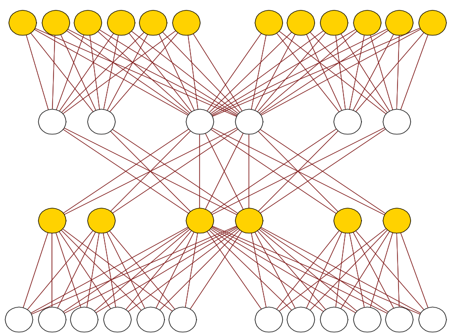
Pairs



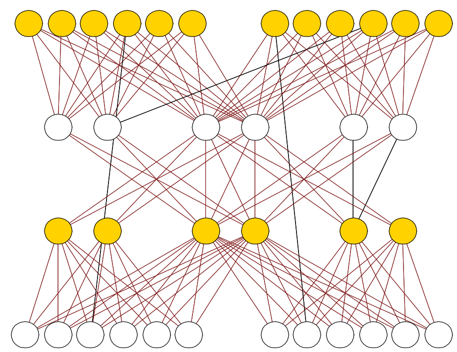
2-Cycle



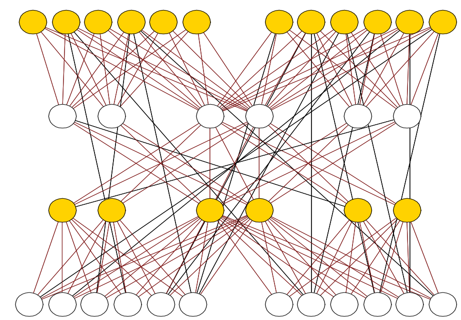
4-Cycle



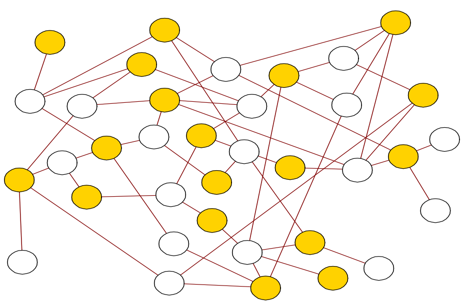
Clan



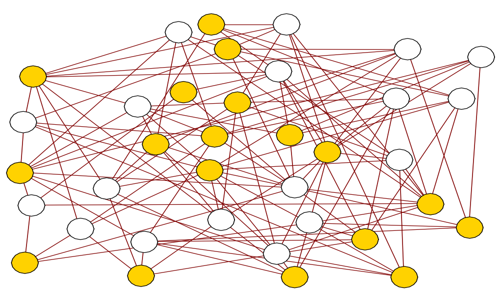
Clan + 5%



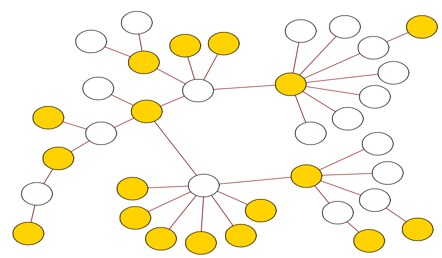
Clan + 10%



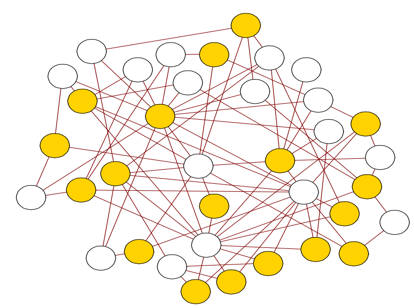
Erdos-Renyi, p=0.2



E-R, p=0.4



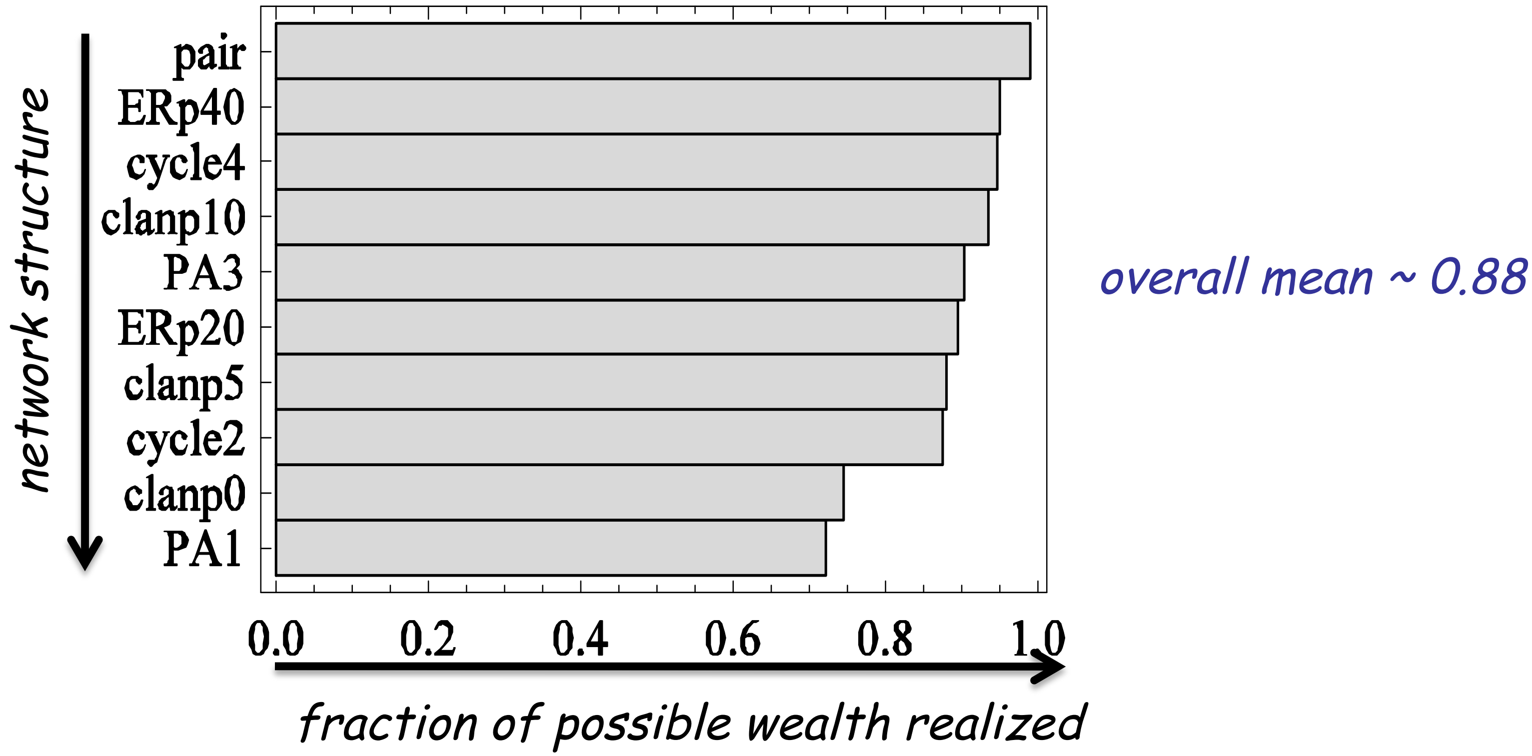
Pref. Att. Tree



Pref. Att. Dense

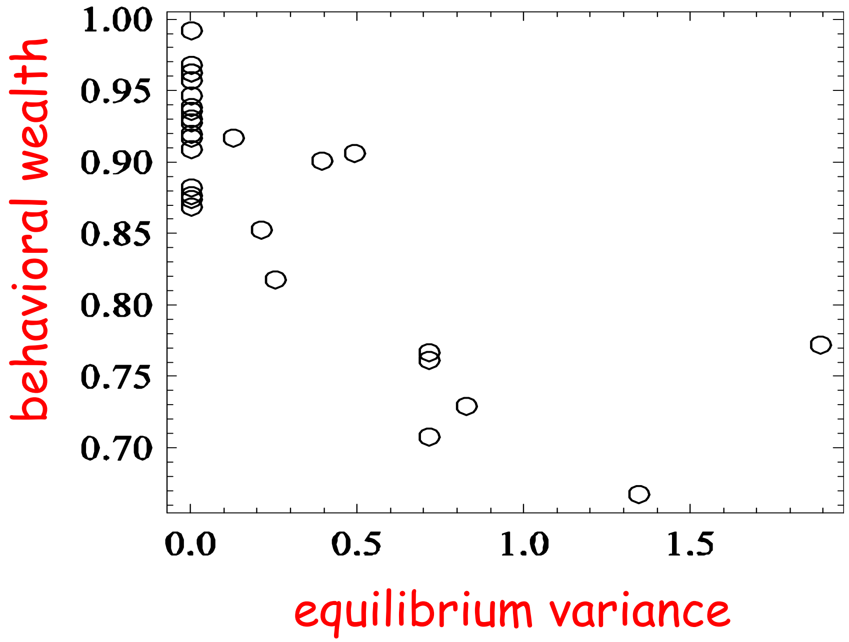
[movies]

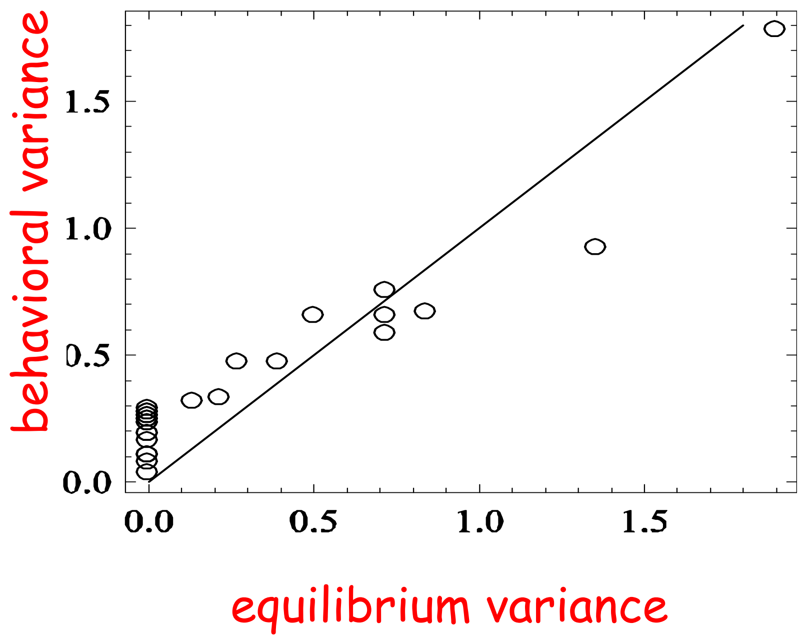
# Collective Performance and Structure




- overall behavioral performance is strong
- structure matters; many (but not all) pairs distinguished

# Equilibrium vs. Behavior



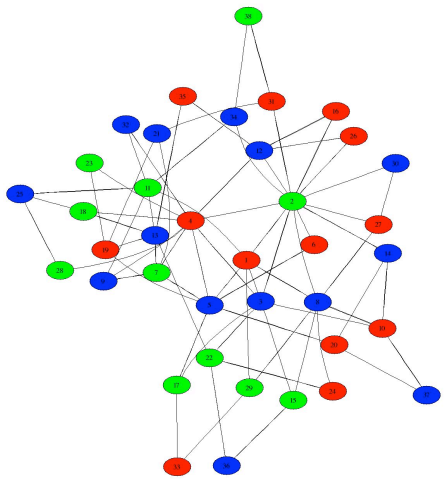

correlation ~ -0.8 ($p < 0.001$)

correlation ~ 0.96 ($p < 0.001$)

- greater equilibrium variation → behavioral performance degrades
- greater equilibrium variation → greater behavioral variation

# Best Model for Behavioral Wealths?

- The equilibrium wealth predictions are better than:
  - degree distribution and other centrality/importance measures
  - uniform distribution
- Best behavioral prediction: 0.75(equilibrium prediction) + 0.25(uniform)
- "Networked inequality aversion" (recall Ultimatum Game)



# Summary

- Trading model most sophisticated "rational dynamics" we've studied
- Has a detailed equilibrium theory based entirely on network structure
- Equilibrium theory matches human behavior pretty well