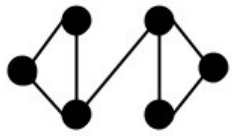
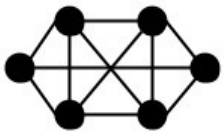
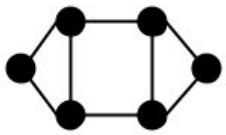
Networked Life (NETS 112) Fall 2013
Professor Michael Kearns
Homework #2

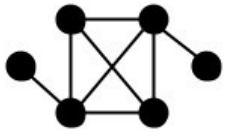
1. Edge Density and Clustering Coefficient

Calculate the edge density and clustering coefficient of the following networks. For each network, say whether you think the network is highly clustered or not, and explain why.

a) 

b) 

c) 

d) 

2. Erdos-Renyi Model

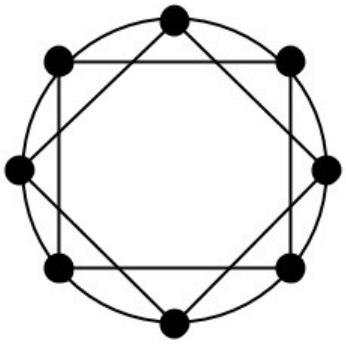
a) What is the clustering coefficient of a network generated in the Erdos-Renyi model? Explain why.

b) Suppose we are generating a network in the Erdos-Renyi model, and we reach a point where there are two large components, each of size N/2. Each component has edge density ½, and there are no edges between the two components. What is the probability that, in the next step, we add an edge connecting the two components? Explain your reasoning.

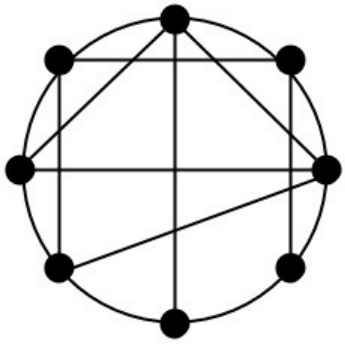
3. Ring-Rewiring Model

Recall the model discussed in lecture in which we start with all vertices arranged on a ring or cycle, and we connect each vertex to all others within two steps. Then, with probability q, we rewire each local vertex to a random vertex.

a) Compute the clustering coefficient and diameter of the following initial network.



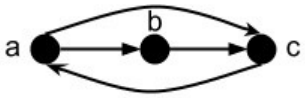
b) Compute the clustering coefficient and diameter of the network after some edges have been rewired.



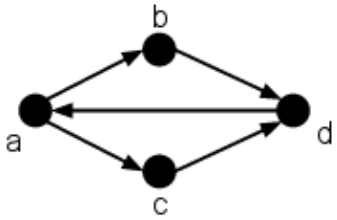
c) As we increase the rewiring parameter q, both clustering coefficient and diameter decrease. But there is still a "sweet spot" at which we can achieve both high clustering coefficient and small diameter simultaneously. Why is this?

4. PageRank Algorithm

Run the PageRank algorithm on each of the following networks. Assuming each vertex initially has rank 1, and perform the updates in alphabetical order. What is the rank of each vertex at equilibrium?

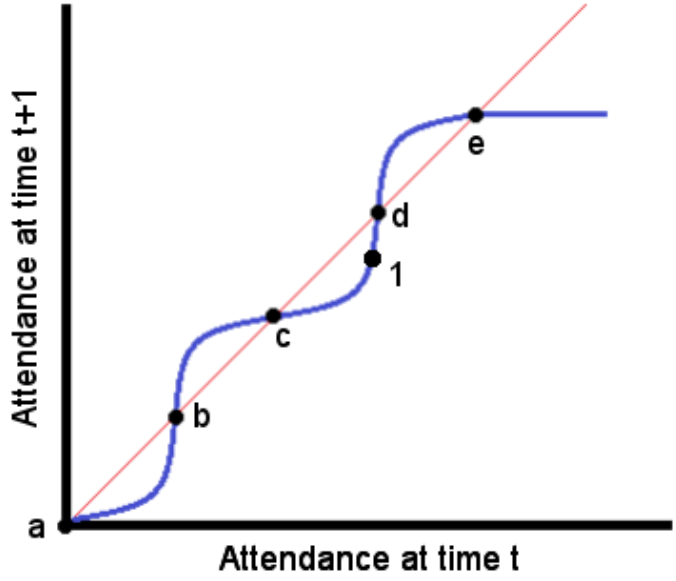
a) 

b)



5. Attendance Dynamics

The graph below plots the number of people who will attend an event at time $t + 1$ as a function of the number of people who attended at time t. The points labeled a … e represent equilibrium states.



a) Why does an equilibrium occur when the blue (curved) line crosses the red (straight) line?

b) If we start at the point labeled 1, which equilibrium will we converge to?

c) What does it mean for an equilibrium to be stable or unstable?

d) For each equilibrium *a* ... *e*, determine whether it is stable or unstable.

6. Payoff Matrices and Equilibrium

a. Circle the pure equilibrium state(s) in the payoff matrix below.

	Action A	Action B
Action A	0, 0	2, 2
Action B	1, 3	1, 2

b. Fill in the following incomplete payoff matrix in such a way that the resulting matrix has no pure equilibria.

	Action A	Action B
Action A	3, 2	
Action B		