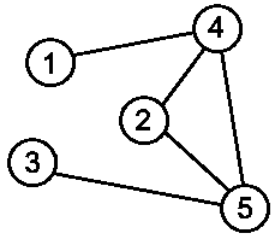
Networked Life (MKSE 112) Fall 2013
Professor Michael Kearns
Homework #1

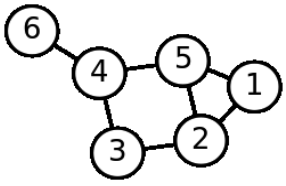
Name:
Penn ID:

Due as hardcopy in class on October 1. Remember that collaboration on homework is not permitted. Don't forget to staple and write your name. Please show all your work on calculation problems for full credit.

1. (5 points) Consider the network shown below. What is the diameter of this network? Recall that we have defined diameter to be the average shortest path distance between pairs of vertices.



2. (5 points) Consider the network shown below. In the altruistic contagion model, if each vertex starts with $1, what is the wealth of each vertex at equilibrium? Recall that, in this model, at each time step, each vertex divides its cash equally among its neighbors. As time goes to infinity, we reach an equilibrium in which the amount of cash each vertex receives is equal to the amount of cash it gives away.



3. (6 points) Answer the following True/False questions. If the answer is False, give a brief one-sentence explanation.

a) (2 points) The largest possible number of edges in a network of N vertices is two times the largest possible number of edges in a network with N/2 vertices.

b) (2 points) Consider a network in which there are two vertices, A and B, that are connected by at least one path. Suppose we remove some edges at random from the network. The distance between vertices A and B may either stay the same or increase, but it cannot decrease.

c) (2 points) Suppose we randomly add some edges to a network. It is possible that the diameter of the new network will be larger than the diameter of the old network.

4. (10 points) Suppose you are starting a new airline company, and you are considering two different designs for your flight map. We can model this map as a network in which each airport is a vertex, and there is an edge between two airports if there is a flight between them.

In the first design, there are a small number of airport "hubs," which have many connections to other airports. The remaining non-hub airports have very few connections. In the second design, there are no hubs, but each airport is connected to several other airports, chosen randomly.

What are the advantages and disadvantages of each design? Consider factors such as the cost of the implementation, the length of a trip between two cities, the possibility of airport closings due to weather conditions, runway congestion, etc.

5. (10 points) Consider the two contagion simulations discussed in class: the “forest fire” simulation and the “viral spread” simulation.

a) (5 points) Compare and contrast the network formation models of the two simulations.

b) (5 points) Compare and contrast the contagion models of the two simulations.

6. (5 points) Describe the navigation or “small world” problem. When solving this problem, what information is available to you, and what is not? Discuss the role of small diameter and “connector” individuals.

7. (5 points) In the Travers and Milgram experiment, as chain length increased, “location” was less frequently cited as an explanation for forwarding the letter, whereas “work” and “education” were more frequently cited. What are some possible reasons for this?

8. (10 points) Consider the network in which each vertex is an actor, and there is an edge between two actors if they have starred in a movie together. An actor's *Bacon number* is the length of the shortest path between the actor and Kevin Bacon. Begin this exercise by going to oracleofbacon.org. This website provides an online tool for calculating Bacon numbers.

a) (2 points) Choose 5 actors, and calculate their Bacon numbers. What is the minimum, maximum, and average?

b) (2 points) Go to http://oracleofbacon.org/center.php. Read the page, and describe in your own words what it means for one actor to be a "better" center than another.

c) (2 points) Go to oracleofbacon.org/onecenter.php. Enter your favorite actor or actress. What is the average __ number? (Replace __ with his or her name).

d) (4 points) From your exploration in this exercise, what information can you infer about this actor network? Consider properties such as diameter, average vertex degree, ease of navigation, etc.