

**Homework 2: Networked Life (MKSE 112), Fall 2012, Prof. Michael Kearns**

**Due as hardcopy at the start of class on Tuesday, December 4. Don't forget to staple and write your name.**

**Problem 1 (20 points)** Clearly draw a *connected*, bipartite network between 5 Milk players and 5 Wheat players in which (a) there is wealth variation at equilibrium, but (b) there are exactly two (and no fewer) edges whose addition eradicates wealth variation at equilibrium. Clearly annotate your diagram with the numerical wealths of all players for part (a), and the wealths with the added edges for part (b).

**Problem 2 (40 points)** Consider the types of systems considered in "Micromotives and Macrobehavior" and discussed in class, where if the fraction of people participating at the current round is  $x$ , then the fraction participating at the next round is  $F(x)$ , for some function  $F(x)$ .

- (a) Suppose  $F(0) = 0$ ,  $F(1) = 1$ , and that  $F(x)$  is increasing in  $x$  (that is, if  $x > x'$ ,  $F(x) > F(x')$ ). Give an example of a real-world activity that might be modeled by such an  $F$ . What, if any, equilibria of the system are implied by only these conditions? Could there be others?
- (b) Suppose  $F(0) = 1$ ,  $F(1) = 0$ , and that  $F(x)$  is decreasing in  $x$  (that is, if  $x > x'$ ,  $F(x) < F(x')$ ). Give an example of a real-world activity that might be modeled by such an  $F$ . What, if any, equilibria of the system are already implied by only these conditions? Could there be others?
- (c) Suppose  $F(0) = 0$ ,  $F(1) = 0$ , and that  $F$  is first increasing in  $x$  and then decreasing (that is, there is some unique value of  $x$  such that  $F$  is increasing below  $x$ , and then decreasing above  $x$ ). Give an example of a real-world activity that might be modeled by such an  $F$ . What, if any, equilibria of the system are already implied by only these conditions? Could there be others?
- (d) Suppose  $F(0) = 1$ ,  $F(1) = 1$ , and  $F$  is first decreasing in  $x$  and then increasing. Give an example of a real-world activity that might be modeled by such an  $F$ , or explain why it is difficult to imagine such an activity. What, if any, equilibria of the system are already implied by only these conditions? Could there be others?

**Problem 3 (30 points)** Consider the game given by the matrix below, where X, Y and Z represent numerical payoffs, the row player's payoffs are shown in red, and the column player's payoffs are blue.

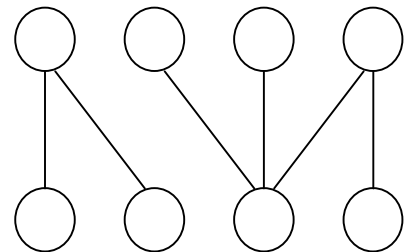
- (a) Give numerical values for X, Y and Z such that the game has a pure-strategy Nash equilibrium.
- (b) Give numerical values for X, Y and Z such that the game does NOT have a pure-strategy Nash equilibrium.
- (c) Briefly describe why examples for (a) and (b) never need to consider negative values for X, Y or Z.

	Column 1	Column 2
Row 1	X, Y	X, Z
Row 2	0, Z	Z, Y

**Problem 4 (10 points)** For the networks below, write the equilibrium wealths in the space beside each vertex, assuming all vertices are initially endowed with one unit of milk or wheat and have preferences only for the other good.

(a)

Wealth: \_\_\_\_\_

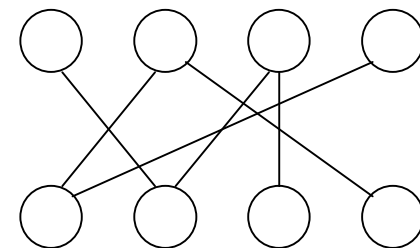


Milks

Wheats

(b)

Wealth: \_\_\_\_\_



Milks

Wheats

Wealth: \_\_\_\_\_