Homework 1
Networked Life (NETS 112)
Fall 2019
Prof Michael Kearns

Posted September 29, 2016. Due in hard-copy format at the start of lecture on Tuesday, October 15. Please don’t forget to write your name and staple the pages together.

Collaboration of any kind is NOT permitted on the homework.

Your Name:
**Problem 1.** For each of the following networks, compute the worst-case diameter, the average-case diameter, the edge density (number of edges divided by the maximum possible), and plot the degree distribution. Show your work where appropriate.

(a)

(b)
Problem 2. A *cycle* in a network is a sequence of vertices $v_1$, $v_2$, $v_3$,...,$v_n$, $v_1$ such that each vertex is distinct (other than the first and last), and each adjacent pair in the sequence are neighbors (i.e. $v_1$ and $v_2$ are neighbors, $v_2$ and $v_3$ are neighbors, etc. and $v_n$ and $v_1$ are neighbors). Since we start at $v_1$ and end at $v_1$, and all vertices in between are distinct, we’ve taken a tour of the vertices in the sequence. Note that other edges may be present as well (e.g. $v_1$ and $v_7$ might be neighbors). As an example, in the network for problem 1(c), the sequence A, B, F, C, G, E, A is a cycle of length 6 edges.

We can also consider cycles in the directed network of the Web. For instance, the following sequence of URLs:

- https://www.cis.upenn.edu/~mkearns/
- https://www.cis.upenn.edu/~aaroth/
- https://www.cse.huji.ac.il/~katrina/
- https://www.cse.huji.ac.il/~katrina/research.html
- http://www.cs.cmu.edu/~avrim/
- http://hunch.net/~jl/
- http://hunch.net/~jl/projects/projects.html
- https://www.cis.upenn.edu/~mkearns/

is a cycle of length 8: each of the pages above has a link to the next one in the sequence, and we end up back where we started. Note that the hard part of finding a long cycle is not simply finding a long list of links; it’s then being able to find your way “home”.

Using any methods you like, find the longest cycle you can in the Web, and list the corresponding sequence of URLs. Carefully describe your methodology. We will spot-check for correctness of your cycles (i.e. that each page links to the next). Rewards of some kind will be given for the longest cycle found, and for the most creative/interesting.
**Problem 3.** Consider the following network, which consists of multiple connected components:

(a) Consider the process in which a vertex $v$ is chosen at random to be infected, and the infection then spreads deterministically to kill all vertices in the connected component of $v$. What is the expected or average number of vertices killed for this network? Show your work.

(b) Suppose you are allowed to “immunize” exactly one vertex that can no longer be infected, and cannot infect others. Which vertex would you choose to make the average number of vertices killed as small as possible? What would the new value for this average be? Show your work.

(c) Suppose you are forced to add an edge between vertices in different connected components. Which edge would you choose to make the average number of vertices killed as small as possible? What would the new value for this average be? Show your work.
Problem 4. For each of the following parts, carefully draw an undirected network with the specified properties.

(a) Draw a network with three connected components of sizes 7, 4 and 3 vertices, in which the total number of edges is 12 and the worst-case diameter (longest shortest-path distance) of the largest component is 5.

(b) Draw a network with three connected components of sizes 5, 4 and 4 vertices, in which the total number of edges is 19 and the worst-case diameter of the largest component is 1.

(c) Draw a network with three connected components of sizes 12, 5 and 3 vertices, in which the total number of edges is 17 and the worst-case diameter of the largest component is 7.
**Problem 5.** In this problem you are asked to carefully define, create and visualize a network of your own choosing, ideally reflecting a personal interest. You should begin by carefully specifying what the vertices are in your network, and stating exactly what constitutes an edge between two vertices. As long as you’re precise, your network can be about any topic you like; the vertices don’t have to correspond to people at all. As one example, the vertices in your network could represent your favorite musical artists (people or bands), and the edges could represent pairs of artists who have collaborated on a song, for some precise definition of collaboration.

Carefully define your vertices and edge definition, making sure your network has at least 15 vertices and 20 edges. Specify any research or data sources you use to identify the edges in your network. Draw a clear visualization of your network, showing all vertices and edges. Compute the worst-case diameter, edge density and degree distribution for your network.