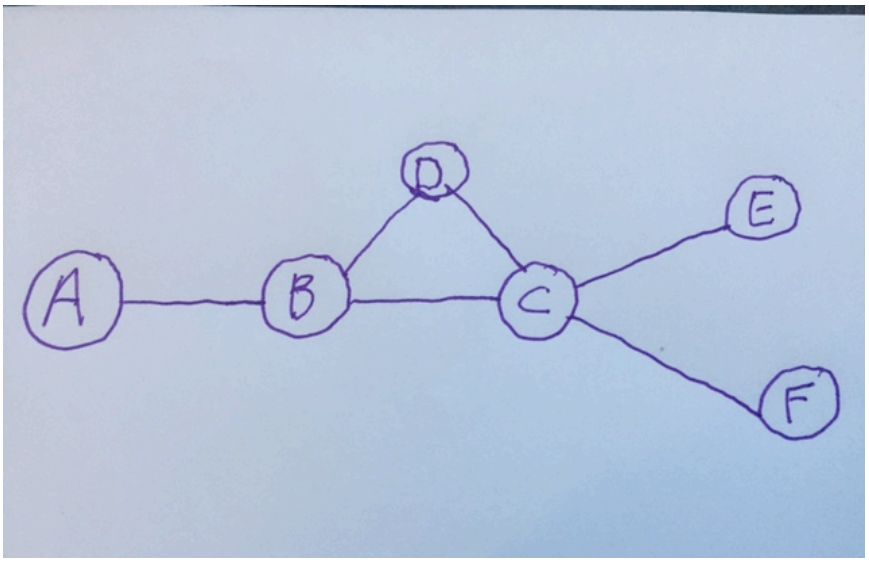
**Homework 1**
**Networked Life (NETS 112)**
**Fall 2017**
**Prof Michael Kearns**

***Posted October 1, 2017. Due in hard-copy format at the start of lecture on Tuesday, October 17. Please don't forget to write your name and staple the pages together.***

***Collaboration of any kind is NOT permitted on the homework.***

**Your Name:**

1. Consider the following network:



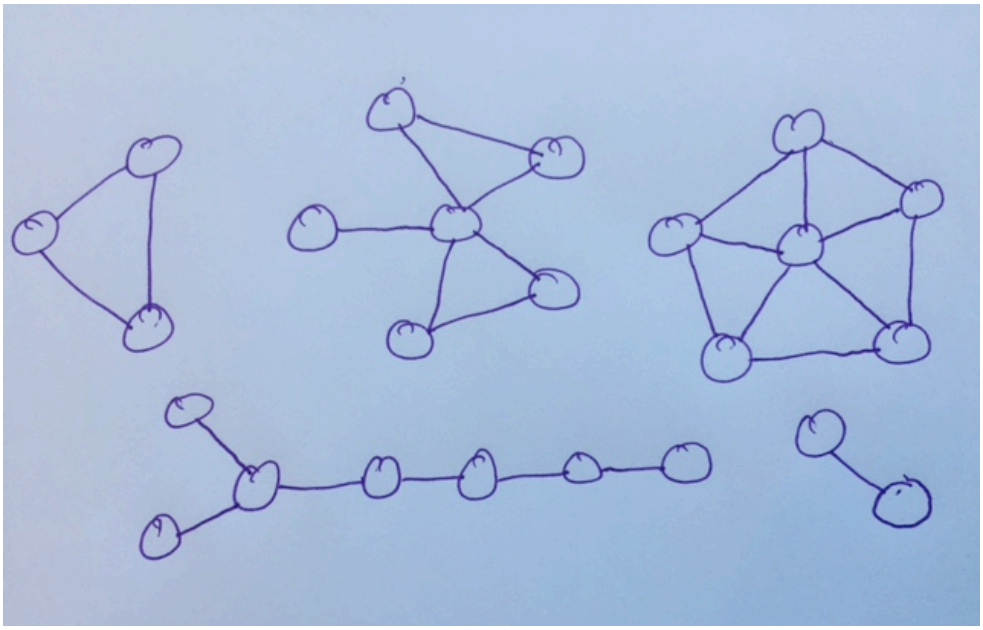

(a) Compute the average-case diameter of the network. Show your work.

(b) Recall the "economic altruism" model in which each vertex starts with $1, and at each step divides its current wealth evenly amongst its neighbors. Compute the equilibrium wealth for each vertex. Show your work.

(c) Consider modifying the network by deleting some edges, and adding some new edges. Clearly indicate the modification with the *fewest* deletions and additions that causes all vertices to have the same equilibrium wealth, and indicate what that wealth is.

(d) Draw a connected network with the fewest vertices you can in which the number of distinct or different equilibrium values is exactly 4. Hint: it can be done with N = 6; I'm not sure if smaller N is possible or not.

2. Consider the following network, which consists of multiple connected components:



(a) Consider the contagion process considered in class, in which a vertex v is chosen at random to be infected, and the infection then spreads deterministically to kill all vertices in the connected component of v. What is the expected or average number of vertices killed for this network? Show your work.

(b) Suppose you are allowed to “immunize” exactly one vertex that can no longer be infected. Which vertex would you choose to make the average number of vertices killed as small as possible? What would the new value for this average be? Show your work.

(c) Suppose you are forced to add an edge between vertices in different connected components. Which edge would you choose to make the average number of vertices killed as small as possible? What would the new value for this average be? Show your work.

3. Carefully consider the assigned reading “Can Cascades be Predicted?”.

(a) In 5 brief bullet points, succinctly describe the main questions the authors are interested in.

(b) In 5 brief bullet points, succinctly describe their problem formulation and methodology.

(c) In 5 brief bullet points, succinctly describe the main findings of the study.

(d) Briefly discuss some of the limitations or drawbacks of the methodology and study that were discussed in lecture.

(e) Discuss what, if any, conclusions can be drawn from the study regarding the possibility of designing or selecting images that can go viral strictly on the basis of their content.

4. The *degree sequence* of a network is a list of the degrees of all the vertices. For example, the degree sequence of the network in Problem 1 is <1,3,4,2,1,1>. Note that the order of the degrees is unimportant; i.e. we could have given it as <1,1,1,2,3,4>.

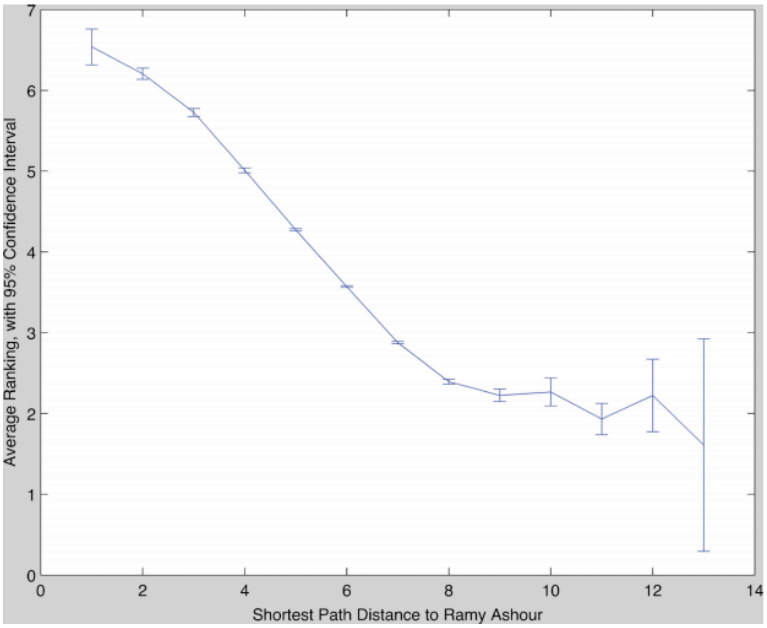
(a) Draw a connected network with N=6 vertices in which the degree sequence is <1,2,2,2,2,5>, or explain why one does not exist.

(b) Draw a connected network with N=7 vertices in which the degree sequence is <1,1,2,2,3,3,4>, or explain why one does not exist.

(c) Draw a connected network with N=7 vertices in which the degree sequence is <1,1,2,2,3,3,5>, or explain why one does not exist.

(d) Think of N as being large, and consider connected networks in which the all of the degrees are 4, i.e. the degree sequence is <4,4,4,…,4>. Clearly describe the networks meeting this condition that have the lowest and highest diameters you can design.

5. Consider the figure below, which is from the assigned article “The Small-World Network of Squash”.

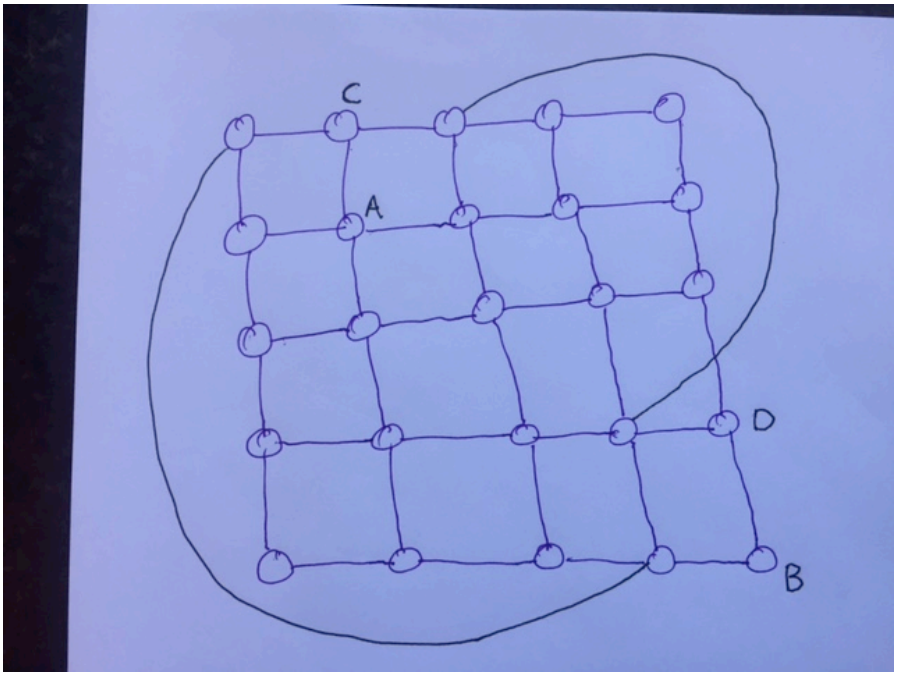



(a) Carefully and clearly describe exactly how this plot was generated, and what point it is making. Use precise terminology and definitions from class.

(b) Why are the vertical bars wider at the left and the right?

(c) Suppose that we used the same methodology to generate the figure, but used a mediocre player as the focal vertex instead of Ramy Ashour. Discuss how you think the figure would now look, and justify or explain your answer. Both “mathematical” (e.g. in which you make assumptions about the distribution of ratings) or “sociological” (e.g. in which you discuss the possible forces governing the arrangement of matches) answers are acceptable; just be clear about your assumptions.

6. Consider the assigned article “Navigation in a Small World” in the context of the network below, which is an underlying 5 by 5 grid augmented by two “long distance” edges. Using the algorithm for navigation examined in the article, for each source-destination pair below, write the length of the *shortest* and *longest* possible paths found by the algorithm.




(a) From A to B Shortest: Longest:

(b) From B to A Shortest: Longest:

(c) From C to B Shortest: Longest:

(d) From B to C Shortest: Longest:

(e) From A to D Shortest: Longest:

(f) From D to A Shortest: Longest:

(g) From C to D Shortest: Longest:

(h) From D to C Shortest: Longest: