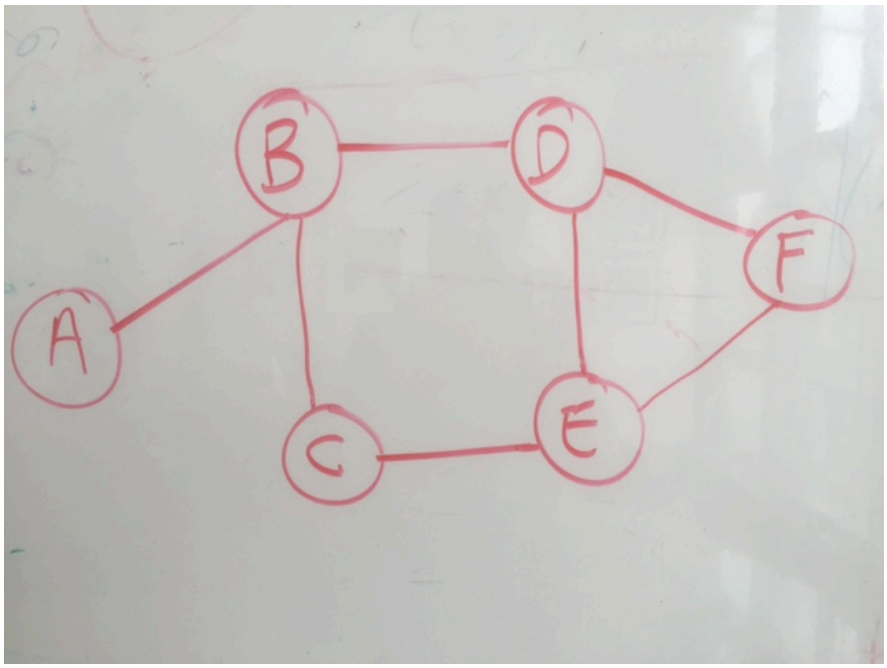
Homework 1
Networked Life (NETS 112)
Fall 2016
Prof Michael Kearns

Posted September 20, 2016. Due in hard-copy format at the start of lecture on Thursday, September 29. Please don't forget to write your name and staple the pages together.

Collaboration of any kind is NOT permitted on the homework.

Your Name:

1. Consider the following network:



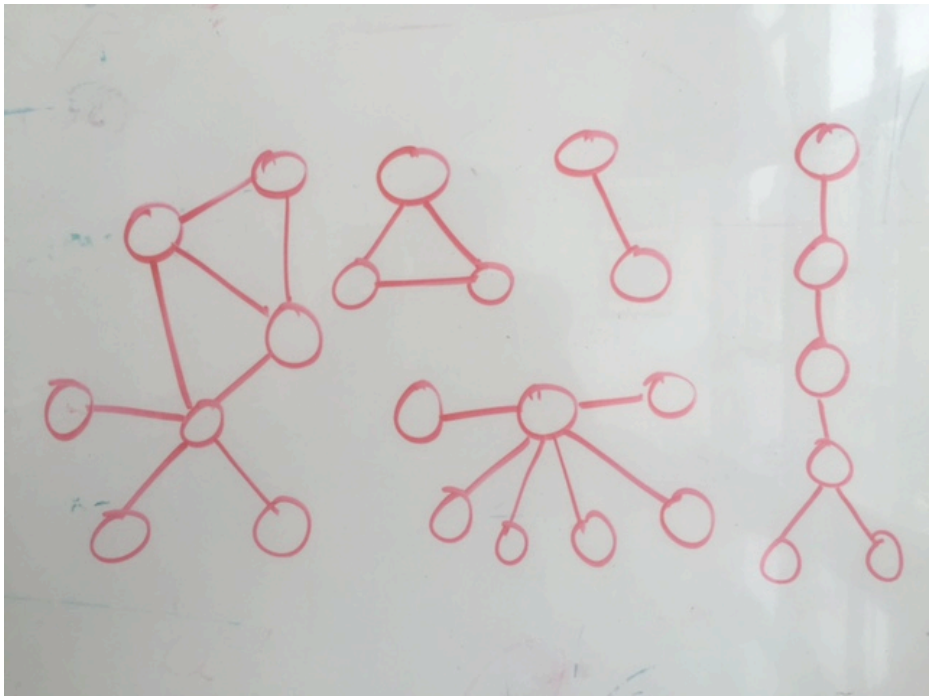
(a) Compute the diameter of the network. Show your work.

(b) Recall the “economic altruism” model in which each vertex starts with $1, and at each step divides its current wealth evenly amongst its neighbors. Compute the equilibrium wealth for each vertex. Show your work.

(c) Draw exactly two new edges whose addition to the network would cause all vertices to have the same equilibrium wealth.

(d) Draw a network with the fewest vertices you can in which the number of distinct or different equilibrium values is exactly 4. Hint: it can be done with N = 6; I’m not sure if smaller N is possible or not.

2. Consider the following network, which consists of multiple connected components:



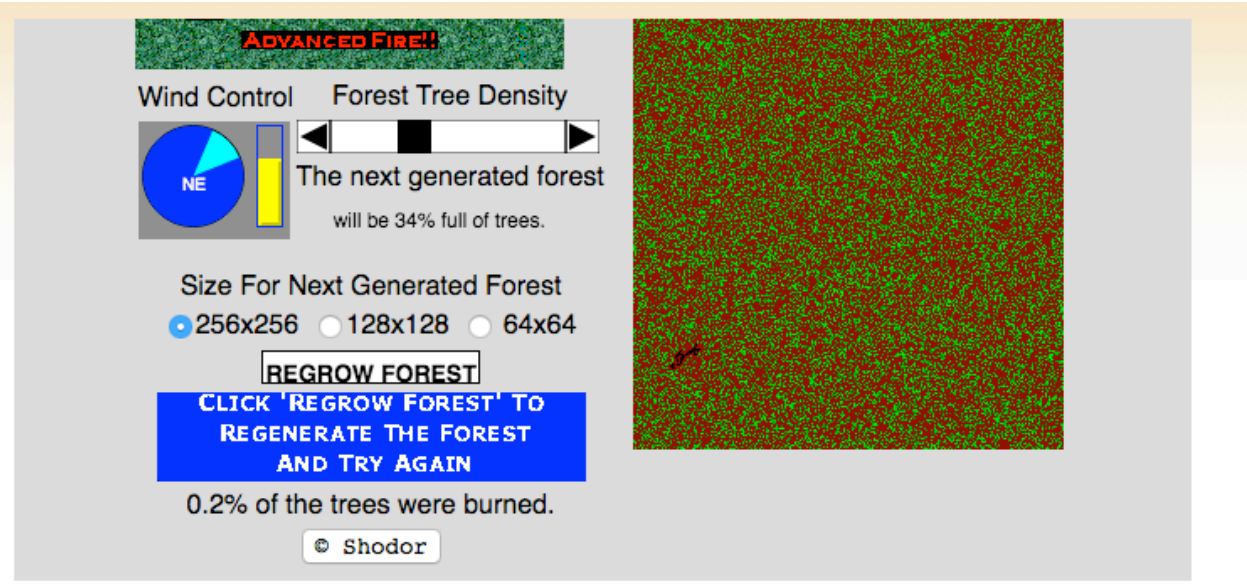
(a) Consider the process in which a vertex v is chosen at random to be infected, and the infection then spreads deterministically to kill all vertices in the connected component of v. What is the expected or average number of vertices killed for this network? Show your work.

(b) Suppose you are allowed to “immunize” exactly one vertex that can no longer be infected. Which vertex would you choose to make the average number of vertices killed as small as possible? What would the new value for this average be? Show your work.

(c) Suppose you are forced to add an edge between vertices in different connected components. Which edge would you choose to make the average number of vertices killed as small as possible? What would the new value for this average be? Show your work.

3. As a fun introduction to our study of navigation in networks, visit the site http://thewikigame.com/, and select “least clicks” as the game mode at the bottom. Play this game at least 10 times (but feel free to do more, it’s kind of addictive). Note that your experience may vary with how many competitors you have. Describe any strategies or heuristics you adopted in as much detail as you can. What was the average number of hops during the trials in which you were a winner (found a path)? How did it compare to the shortest paths found by your competitors? What is the pair of topics for which the path found was the shortest? What is the pair for which it was the longest?

4. Visit http://www.shodor.org/interactivate/activities/ABetterFire/, which is a forest fire simulator similar to the one discussed in class, but a bit fancier. Here is a screenshot:



Set the direction of the wind to be NE, as shown above. There are three values you can select for the wind speed (shown above as a yellow bar, which is the medium setting), and you can also select the forest density by either sliding the black bar, or using the black arrows (which give you finer-grained control). For each of the three settings of the wind speed, experiment with the forest density value and start your fire near the lower left corner (the direction from which the wind is blowing). For each wind speed, find the forest density at which at least roughly half is burned, to the greatest numerical accuracy you can. For each wind speed, how gradual or sudden is the transition from a small amount burned to a large amount --- i.e. is there a “tipping point” or threshold, or not? How might you model the effects of wind in terms of the network structure?

5. Consider the three articles you were asked to read in the “Contagion in Networks” section of the course schedule. Briefly compare, contrast and synthesize these articles (do not simply recite or summarize their content), touching on their motivation, methodology, results, and their limitations and weaknesses.