

**Some Analysis of Coloring Experiments  
and  
Intro to Competitive Contagion Assignment**

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**Networked Life**

**NETS 112**

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# Coloring Assignment: Experimental Design

- 30 graphs total
- 10 each from the following generative models
  - Erdos-Renyi
  - Small Worlds (multi-hop cycle with rewirings)
  - Preferential Attachment
- Controlled to keep number of edges and vertices constant
- Also designed graphs to elicit different running times for heuristics

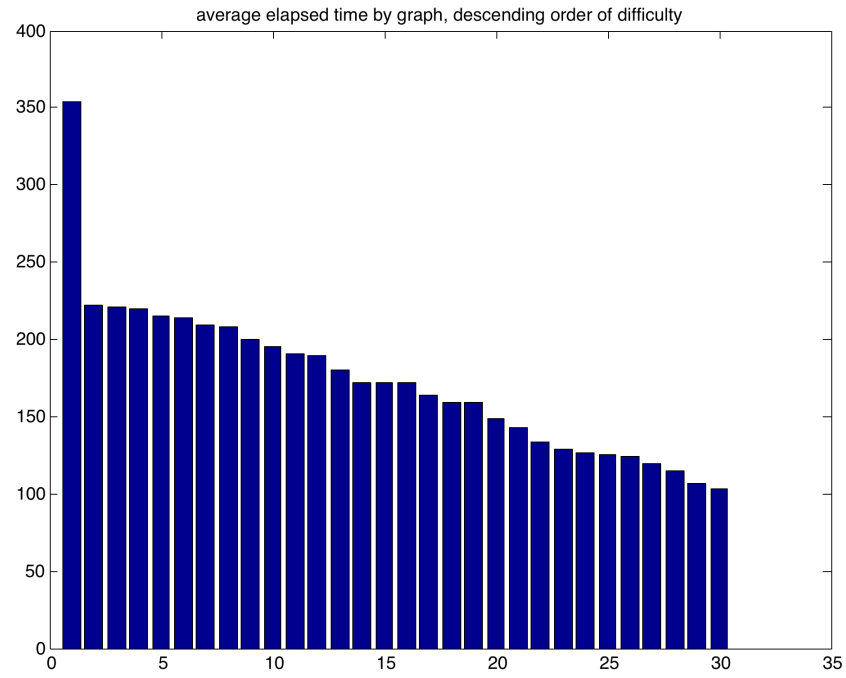
# Are Some Families Harder Than Others?

## 1.1.2 Comparing graph families

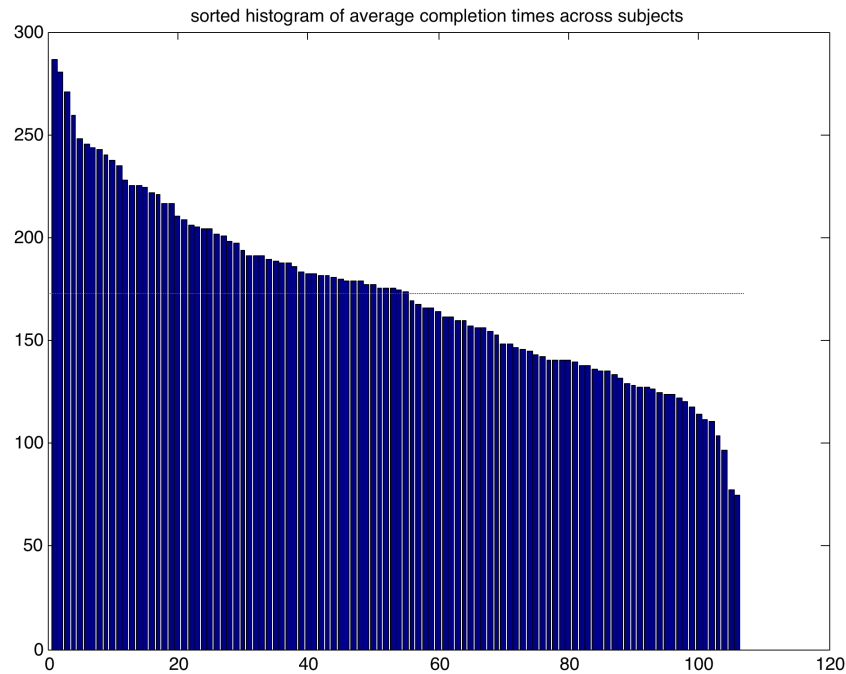
	Ave Elapsed	Ave # Attempts	Ave # Conflicts
Random	163.85	1.18	28.07
Pref Att	157.37	1.1520	33.54
Sm World	196.56	1.3697	46.94

Ordering Small World > Erdos-Renyi > Preferential Attachment holds with each pairwise comparison passing  $P < 0.05$  significance.

# Are Some Graphs Harder Than Others?



# Are Some Players Better Than Others?



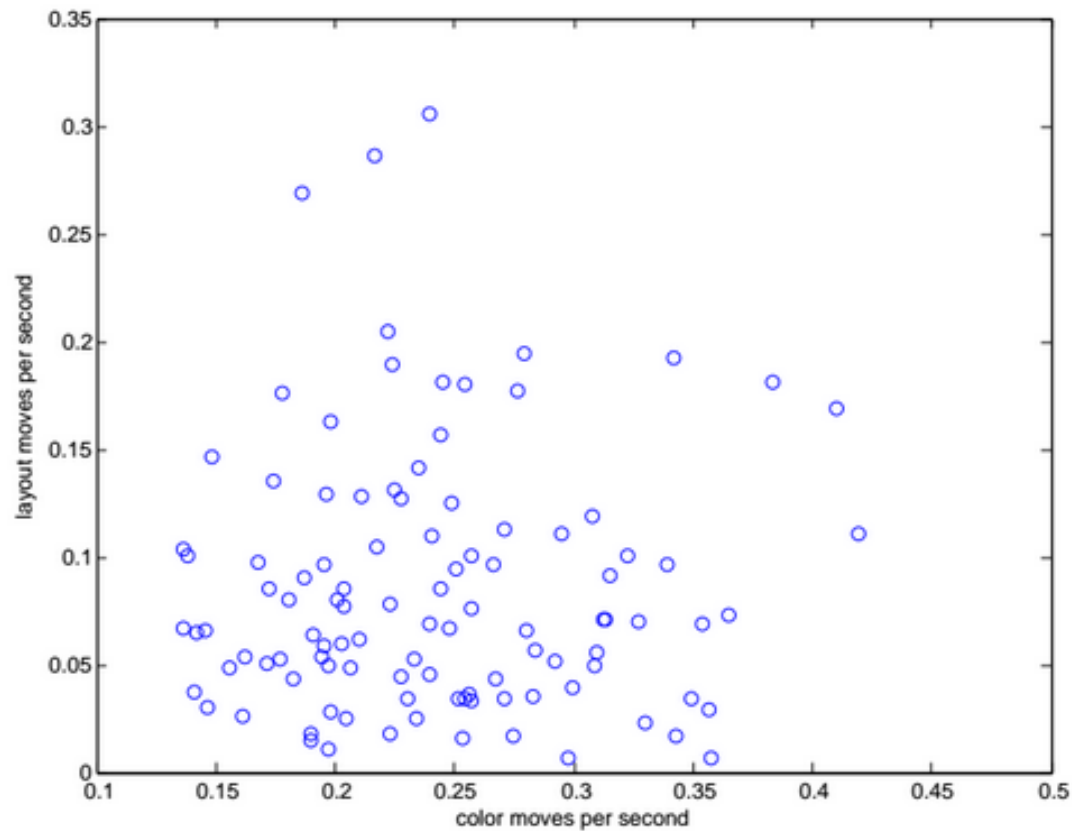
# What Correlates With Population Solution Time?

Methodology: Create vector of 30 population average solution times;  
Correlate with properties of graphs or other population properties.

## 1.1.1 Correlations with elapsed time

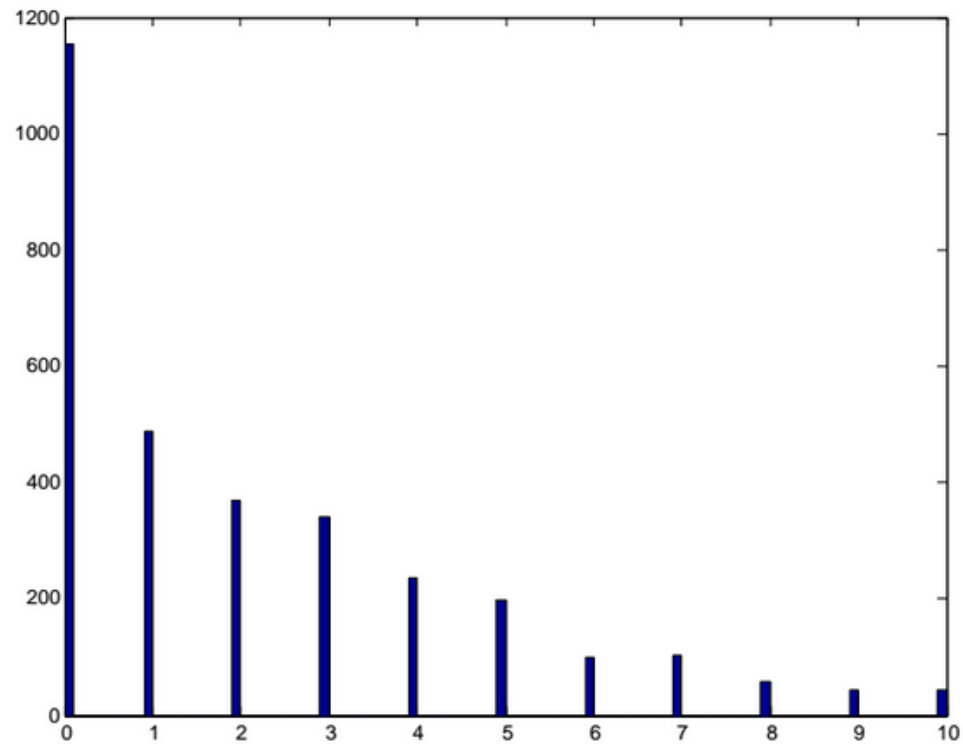
Property	Coefficient	P-value
Max-Degree	-0.3950	0.03
Crossing #	-0.0875	0.6457
Backtrack	0.0233	0.9028
Annealing	0.67	0.0001
Optimal	0.108	0.5680
Color Changes	0.9338	0
Display Changes	0.9327	0
# Attempts	0.8506	0
# Conflicts	0.8929	0

# How Do People Play?



Faster is Better: At the subject level, correlation between average solution time and average color changes/second = -0.53

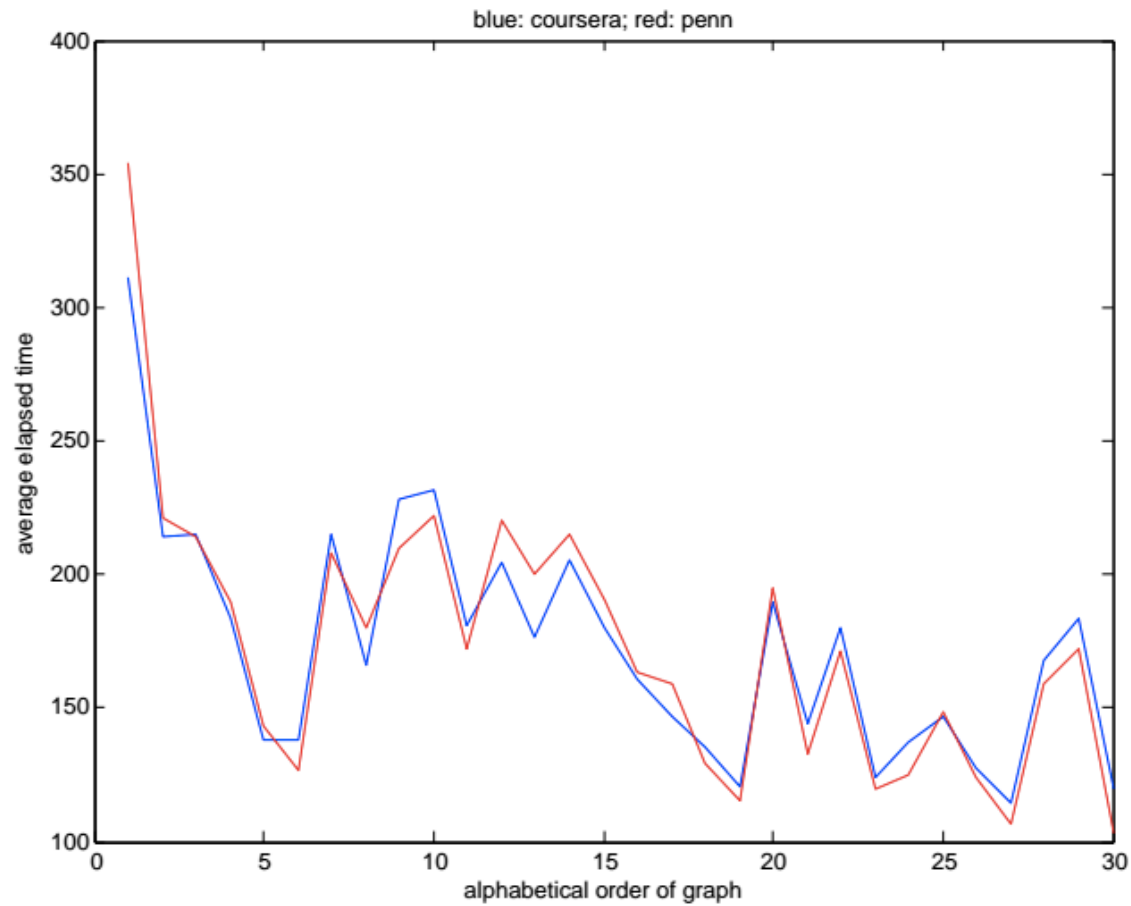
# How Do People Play?



Histogram of max degree – degree of first vertex colored



# Penn vs. Coursera: Average Solution Times



**Next Assignment:  
Experiments in Competitive Contagion**

# Scenario

- You are the head of marketing for the Red Widget Company
- You are tasked with creating the “viral spread” of Red Widgets on Facebook
- **Widgets are very compelling**: once someone learns about them via their friends, they simply must have one
- Your budget permits you to give away two Red Widgets to seed their spread
- Unfortunately, your counterpart at Blue Widget Co also has two seeds
- **Red and Blue Widgets are identical except for color**, and have extreme network/platform effects: you want to adopt the color your friends have
- For a given network, your goal is to win more market share than your Blue opponent(s)

# Detailed Dynamics

- Red and Blue each pick two seeds (duplicates chosen randomly)
- At the first step, all neighbors of the seeds will adopt/buy a widget
- At the next step, all their neighbors will buy a widget
- In general, if step  $T$  is the first step at which some neighbor of  $v$  has adopted a widget, then  $v$  will adopt on step  $T+1$
- To decide which **color** widget to adopt,  $v$  looks at the number of Red and Blue widgets in their neighborhood, and adopts **majority color** (ties broken randomly)
- Any vertex in the connected component of any seed will eventually adopt
- Two sources of randomization: duplicate seeds, ties in neighborhoods

# Discussion

- This is a (complex) game between Red and Blue
- Pure strategies: all choose  $(N,2)$  choices of 2 seeds
- Mixed strategies: all distributions over seed pairs
- Payoffs: number of adoptions won
- We will play a **population opponent** variant of this game
- Let  $\text{pay}(s_1, s_2)$  denote the (expected) payoff to Red when Red chooses seed set  $s_1$  and Blue chooses seed set  $s_2$
- Let  $\text{pay}(s_1, P)$  denote the (expected) payoff to Red when Red chooses seed set  $s_1$  and Blue chooses a seed set **randomly** according to distribution  $P$
- Then payoff to Red is  $\text{pay}(s_1, P)$ , where  $P$  is the empirical distribution of seed choices of all your classmates/opponents
- In general, there is no right/best choice for  $s_1$ : depends on  $P$ !
- Let's go to the app

# Questions Worth Pondering

- What does it mean for the population distribution  $P$  to be an equilibrium?
- If  $P$  is an equilibrium what can we say about different players' payoffs?
- If  $P$  is an equilibrium and  $G$  is connected, what can we say about payoffs?
- What if  $G$  is not connected?

# How We Will Compute Scores

- Let  $P$  be the population distribution of seed choices on graph  $G$
- For every seed set  $s$  that appears with non-zero probability in  $P$ , we will compute its *expected payoff with respect to  $P$* :
  - average of  $\text{pay}(s,s')$  over many trials and many draws of  $s'$  from  $P$
  - enough draws/trials to distinguish/rank expected payoffs accurately
- We will then rank the  $s$  that appear in  $P$  by their expected payoffs
- If you played  $s$  on  $G$ , you will receive a number of points equal to the *number of other players* you *strictly beat* in expected payoff
- Example: Suppose  $s_1$ ,  $s_2$  and  $s_3$  appear in  $P$ , and have expected payoffs and population counts as follows:
  - $s_1$ : payoff 0.57, count 11;  $s_2$ : payoff 0.48, count 71;  $s_3$ : payoff 0.31, count 18
  - if you play  $s_1$ , your score is  $71+18=89$ ; if  $s_2$ , your score is 18; if  $s_3$ , your score is 0
- If everyone plays the same thing, nobody receives any points
- You must submit seeds for *all* graphs in order to receive any credit
- Your overall score/grade for the assignment is the sum of your scores over all graphs, which will then be curved
- In general, there is no right/best choice for seeds: depends on  $P$ !

## More Details

- You can (and should) change seeds as often as you like
- Important: Since  $P$  will change/evolve during the assignment, you should revisit your seed choices in response
- Deadline for assignment: 11:59PM on Monday November
- URL for app: <http://upenn-nwlife-contagion.herokuapp.com/>
- Active at noon today