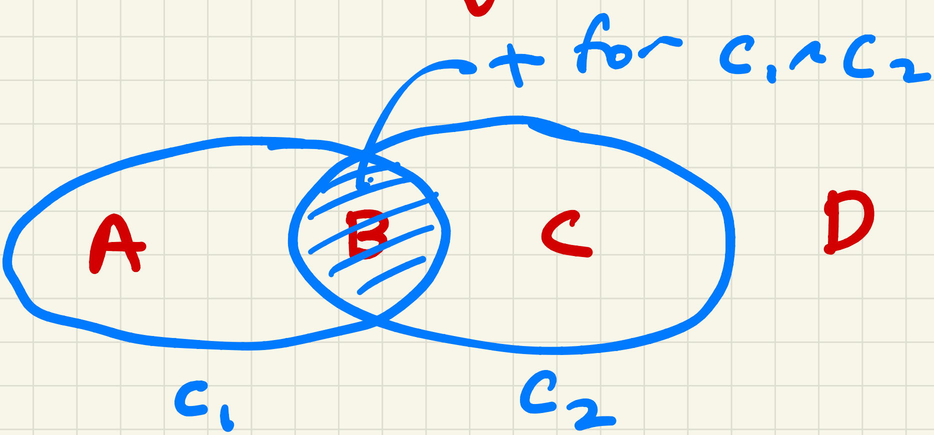
Musings on Problem Set 1

Problem 1

- C_1 PAC by $\mathcal{H}_1$, + only, L_1
- C_2 PAC by $\mathcal{H}_2$, + only, L_2

Show $C_1 \cap C_2$ PAC by $\mathcal{H}_1 \cap \mathcal{H}_2$, + only.



- P over A, B, C, D
- L_1 only "looks" at B, but may call all of A +

$\Rightarrow \varepsilon(h_1) \gg \varepsilon$.

Define P_1:

$$P_1[A] = 0$$
$$P_1[B] = P[B]$$
$$P_1[C] = P[C]/Z$$
$$P_1[D] = P[D]/Z$$

$$Z = 1 - P_1[A]$$

Then P_1 is indistinguishable from P by L_1, & P_1 is a "legal" distribution for

$c_1 \in \mathcal{C}$ so $\varepsilon_1(h_1) \leq \varepsilon$

wrt P_1

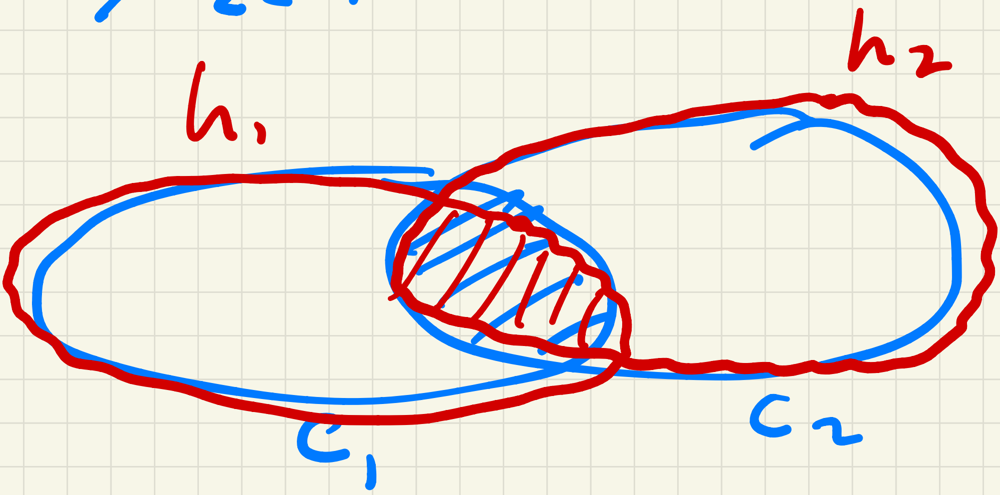
Similar for P_2, c_2, h_2

$\Rightarrow \varepsilon_2(h_2) \leq \varepsilon$

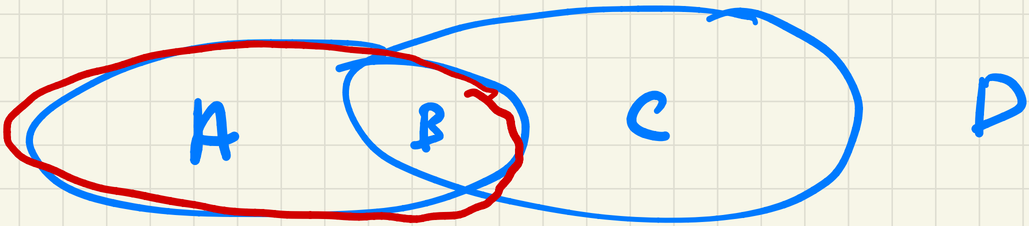
Then show

$\varepsilon_1(h_1), \varepsilon_2(h_2) \leq \varepsilon$

$\Rightarrow \varepsilon(h_1 \wedge h_2) \leq \varepsilon.$



Problem 2



- Can still run L_1 on $P_1 \to h_1$
- Now force error of h_1 on $B, C, D \ll 1/m_2$

Simulation of L_2:

- draw $\langle x, y\rangle \sim P$
- $\langle x, +\rangle \Rightarrow$ give to L_2
- $\underbrace{\langle x, -\rangle \;\&\; h_1(x) = + \Rightarrow L_2}_{\text{all } m_2 \in A \text{ w.h.p.}}$

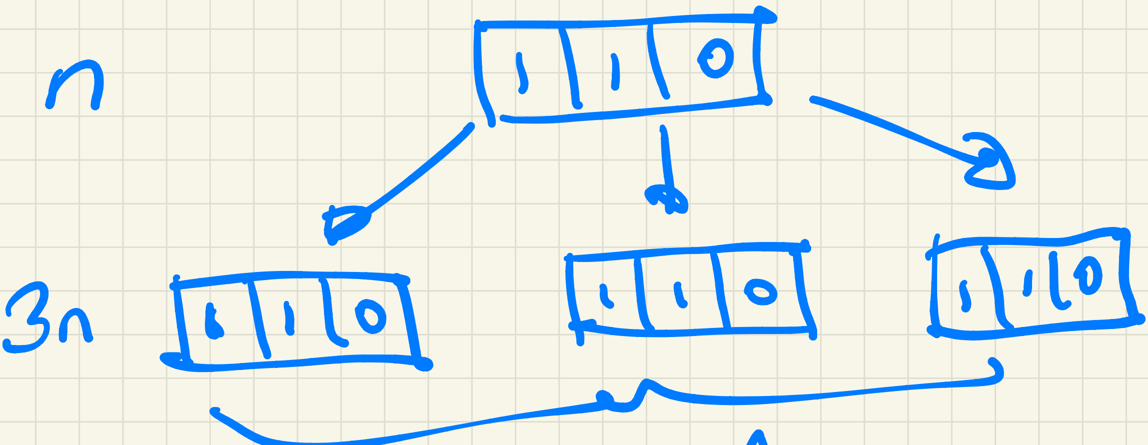
Problem 5

$$x_1 x_2 \neg x_3 \vee \neg x_1 x_2 x_3 \vee \neg x_2 x_3$$

$$=$$

$$x_1 x_2 \neg x_3 \vee \neg y_1 y_2 y_3 \vee \neg z_2 z_3$$

where $y_i = x_i$, $z_i = x_i$



run L for read once,

convert h back to x_i

Problem 8

- Let L have running time $= a\left(\frac{1}{\varepsilon}\right)^b$ for constants $a, b \geq 1$

includes dependence on n, size(c)

- Let P be uniform on S
- Run L with $\varepsilon < 1/m \Rightarrow$ size(h) could be $am^b \geq m$ X

Let's force size(h) $\sim \sqrt{m}$

Choose ε s.t.

$$a\left(\frac{1}{\varepsilon}\right)^b = \sqrt{m}$$

$$\left(\frac{1}{\varepsilon}\right)^b = \frac{m^{1/2}}{a}$$

$$\frac{1}{\varepsilon} = \frac{m^{1/2b}}{a_2}$$

$$\varepsilon = \frac{a_2}{m^{1/2b}} \to L \to h$$

Then $\text{size}(h) \leq \sqrt{m}$ and

$$\#\text{mistakes} \leq \varepsilon m$$

$$= \frac{a_2}{m^{1/2b}} \cdot m = \underline{a_2 m^{1-1/2b}}$$

sublinear!

So final hypothesis:

$$\underbrace{h, \langle x_1, y_1 \rangle, \ldots \langle x_\ell, y_\ell \rangle}_{\text{mistakes of } h \text{ on } S}$$

total size:

$$\sqrt{m} + a_2 m^{1-1/2b}$$

Better: balance terms

$$\text{size}(h) = m^c$$

Set

$$a(1/\varepsilon)^b = m^c$$

$$(1/\varepsilon)^b = \frac{m^c}{a}$$

$$1/\varepsilon = \frac{m^{c/b}}{a_2}, \quad \varepsilon = \frac{a_2}{m^{c/b}}$$

$$\#\text{errors} = \frac{a_2}{m^{c/b}} \cdot m$$

$$= a_2 m^{1-c/b}$$

$$\text{set} = m^c, \text{ so } c = 1 - c/b$$

$$c(1 + 1/b) = 1$$

$$c = \frac{1}{1 + 1/b}$$

$$\therefore \text{size}(h) \sim m^{\frac{1}{1+1/b}}$$