Algorithms for VWAP and Limit Order Trading

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Technological Revolutions in Financial Markets

- Competition
  - amongst exchanges
  - rise of the ECNs; NASDAQ vs. NYSE
- Automation
  - exchanges
  - technical analysis/indicators
  - algorithmic trading
- Transparency
  - real-time revelation of low-level transactional data
  - market microstructure

Outline

- formal models for market microstructure
- competitive algorithms for canonical execution problems
- provide a price for VWAP trading

Market Microstructure

- Consider a typical exchange for some security
- Order books: buy/sell side
  - sorted by price; top prices are the bid and ask
- Market order:
  - give volume, leave price to "the market"
  - matched with opposing book
- Limit order:
  - specify price and volume
  - placed in the buy or sell book
- Market orders guaranteed transaction but not price; limit orders guaranteed price but not transaction
- last price / ticket price

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MSFT

GET STOCK

MSFT go

Symbol Search

LAST MATCH		TODAY'S ACTIVITY	
Price	24.0700	Orders	52,983
Time	14:57:07.72	Volume	10,243,212

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
500	24.0620	500	24.0690
6,000	24.0610	500	24.0690
5,000	24.0600	500	24.0700
100	24.0600	200	24.0800
1,100	24.0550	1,981	24.0900
100	24.0500	412	24.0900
5,000	24.0500	3,000	24.0980
200	24.0500	500	24.1000
3,294	24.0500	100	24.1200
1,000	24.0500	2,800	24.1400
3,000	24.0430	5,000	24.1400
100	24.0400	1,000	24.1400
5,503	24.0400	5,000	24.1500
2,100	24.0300	400	24.1600
2,800	24.0300	1,000	24.1700
(412 more)		(694 more)	

As of 14:57:16.178

Commercial and Academic Interest in Market Microstructure

- Real-time microstructure revelation enables:
  - optimized execution
  - new automated trading strategies?
    - order books express "market sentiment"
- Early microstructure research:
  - equilibria of limit order games (Parlour et al.)
  - power laws relative to bid/ask (Bouchaurd et al.)
  - dynamics of price evolution (Farmer et al.)
- What about the algorithmic issues?

One way trading (OWT)

- The common objective in online analysis
- Sequence of prices:
  - $p1, p_2, \ldots, p_t$
  - $p_{max} = MAX_i \{p_i\}$; $p_{min} = MIN_i \{p_i\}$; $\mathbf{R = p_{max}/p_{min}}$
- Q: Compete with the maximum price p_{max}?
  - "Yes", assuming infinite liquidity [EFKT]
  - O(log R) competitive

The VWAP

- Given a sequence of price-volume trades:
  - (p_1,v_1) , (p_2,v_2) , … , (p_T, v_T)
- Volume Weighted Average Price (VWAP)
  - $VWAP=\Sigma p_t v_t/\Sigma v_t$
- Objective: sell (or buy) tracking VWAP
- A much more modest goal
  - a "trading benchmark"?
  - Why is it important?
  - Can we achieve it?

Typical Trading scenario

- Large mutual fund owns 3% of a company
- Likes to sell 1% of the shares
  - over a month
  - likes to get a "fair price"
- Option 1: Simply sell all the shares
  - huge market impact!

Typical Trading scenario (more)

- Option 2: Sell it to a brokerage
  - What should be the price
  - The future VWAP over the next month

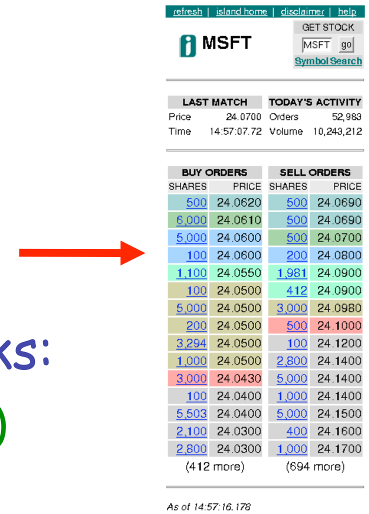
  [minus some commission cost]
- Brokerage: Needs to sell the shares at the VWAP (more or less)
  - brokerage takes on risk

VWAP Issues

- Psychological Factors:
  - increased supply
  - market impact
  - less of an issue for the 'brokerage'
- Mechanics:
  - liquidity is the key
- Algorithmic Challenge:
  - get close to the VWAP?
  - what about psychology?

An Online Microstructure Model

- Market places a sequence of price-volume limit orders:
  - M = (p_1,v_1),(p_2,v_2),…,(p_T,v_T)   (+ order types)
  - possibly adversarial
  - ignore market orders!
- Algorithm is allowed to interleave its own limit orders:
  - A = (q_1,w_1),(q_2,w_2),…,(q_T,w_T)
- Merged sequence determines executions and order books:
  - merge(M,A) = (p_1,v_1), (q_1,w_1),…, (p_T,v_T), (q_T,w_T)
  - Now have complex, high-dimensional state



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VWAP Results

- Goal: Sell K shares at VWAP.
- How to measure "time"?
  - measure time by amount of volume traded
  - assume no order larger than β shares
- Theorem: After βK shares traded,

$$\text{AvgRevenue}(S,A) \geq \text{VWAP}(S,A) - (2p_{max}/\sqrt{K})$$

- Worst case commission cost of $2p_{max}/\sqrt{K}$
  - relatively mild assumptions
  - don't address 'psychology'
- If time horizon is fixed, "guess" volume

VWAP Algorithm

- Divide time into equal (executed) volume intervals I_1, I_2,...
- Let VWAP_j be the VWAP in volume interval I_j
- consider price levels $(1-\varepsilon)^k$

Algorithm:

After I_j, place sell limit order for 1 share at the price $(1-\epsilon)^k$ nearest VWAP_j

- Note if all orders executed, we are within $(1-\varepsilon)$ of overall VWAP
  - since each limit order is $(1-\varepsilon)$ close to VWAP_j

The Proof

Algorithm:

After I_j, place sell limit order for 1 share at ~ $(1-\epsilon)^k$ nearest VWAP_j

Proof:

- say after interval I_j, algo. places order at level $(1-\epsilon)^m$
- Key Idea: after interval j, if price ever rises above the price $(1-\epsilon)^m$, then our limit order is executed
- Hence, at end of trading, can't "strand" more than one order at any given price level
- This implies:

$$\mathrm{AvgRevenue}(S,A) \geq (1-\epsilon)\,\mathrm{VWAP}(S,A) - (p_{max}/\epsilon K)$$

- optimize ε!

Implications:

- note that algorithm may not sell any shares?
- Algorithm exploits the power of limit orders!

One Way Trading & Order Books

- Goal: sell K shares at highest prices
  - compete with optimal "offline" algorithm
- Assumptions:
  - The price is in: $[p_{min}, p_{max}]$
  - define $R = p_{max}/p_{min}$
- Theorem: Algo A has performance that is within a multiplicative factor of 2log(R)log(K) of "optimal"
  - worst-case market impact of large trades

  proof:
  - order prices p_1 > p_2 >... are exec/buy prices
  - want to obtain Kp_1, but cant
  - try to "guess" and obtain max{kp_k}