(In)Stability Properties of Limit Order Dynamics

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LAST MATCH		TODAY'S ACTIVITY	
Price	23.7790	Orders	1,630
Time	9:01:55.614	Volume	44,839

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
1,000	23.7600	100	23.7800
3,087	23.7500	800	23.7990
200	23.7500	500	23.8000
100	23.7400	1,720	23.8070
1,720	23.7280	900	23.8190
2,000	23.7200	200	23.8500
1,000	23.7000	1,000	23.8500
100	23.7000	1,000	23.8500
100	23.7000	1,000	23.8600
800	23.6970	200	24.0000
500	23.6500	500	24.0000
3,000	23.6500	1,000	24.0300
4,300	23.6500	200	24.0300
2,000	23.6500	1,100	24.0400
200	23.6200	500	24.0500
(195 more)		(219 more)	

“Backtesting” of Trading Strategies

- Common microstructure backtesting process:
  - assume access to historical limit order data
  - reconstruct complete order books at each point in time
  - insert hypothetical limit orders into the stream
  - simulate forward the execution of the hypothetical orders
- Faithfully simulates the mechanical aspects of market impact
- What about the reactive or “psychological” aspects?
- Formalize as a question about dynamical stability:
  - Make various assumptions about how future orders do or do not react to the past
  - *Can tiny perturbations of the limit order sequence cause dramatic future change?*
  - Butterfly Effects and Chaos

Two Models of Market Impact

- Both models deal with arbitrary, fixed sequences
- Absolute model:
  - market given by a sequence of "absolute" limit order prices (one share each)
  - e.g. M = (p_1,buy),(p_2,buy),(p_3,sell),…
  - order books constructed from sequence M
  - "mechanical" impact only
  - motivation:
    - traders with "inherent" valuations
    - traders with slow time scales, long investment horizons, poor microstructure access
- Relative model:
  - market given by a sequence of limit order prices relative to current bid & ask
  - e.g. M' = (d_1,buy),(d_2,buy),(d_3,sell),…
  - construct order books & actual prices in concert with each other
    - e.g. limit price p_2 = current bid + d_2; limit price p_3 = current ask + d_3; etc.
  - crude form of "psychological" or "reactive" impact
  - motivation:
    - traders "looking for a bargain"; trading off time for price
    - "penny-jumping", optimized execution
    - high-frequency traders with low latency and full microstructure access
- *How do these models differ?*

Stability

- Consider sequences in the two models:
  - absolute: M = (p_1,type_1),(p_2,type_2),…
  - relative: M' = (d_1,type_1),(d_2,type_2),…
- Now consider a small, arbitrary modification to each
  - e.g. deleting or adding a single order
    - (p_i,type_i) from M, (d_i,type_i) from M'
    - think of this as "our" action
- How much can such a change alter basic properties of the sequence?
  - stability = small change not amplified with time
  - instability = small change greatly amplified
- Absolute model: Every "reasonable" property stable!
  - volume executed, VWAP, closing price,…
  - note: must still be careful; some bounds depend on spread of M
  - generalizes to larger modifications, other types
- Relative model: Most properties highly unstable!
  - can find sequences (with bounded spread) such that single deletion causes arbitrarily large changes in volume executed, VWAP, closing price,…

Absolute Model Stability

- <B,S> = original buy and sell books (at some point in simulation)
- <B',S'> = modified buy and sell books (at the same point)
- Introduce "meta-states" with small "edit distance" between simulations
- E.g. meta-state where B = B' and S U {s'} = S' U {s} for some s != s'
- Main technical lemma establishes:

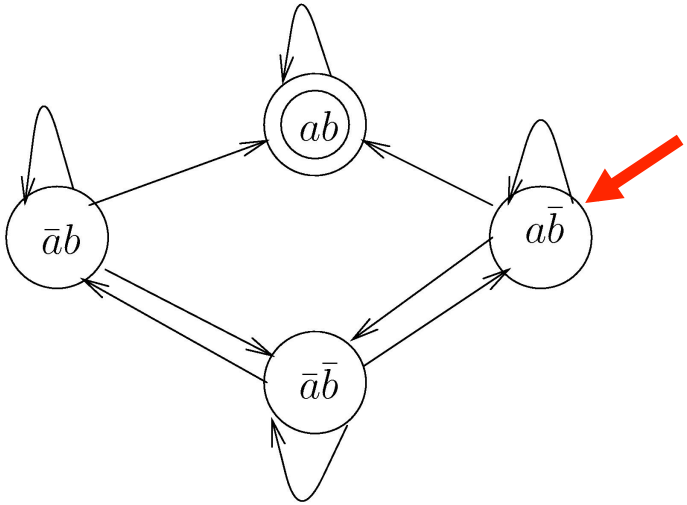

Figure 1: Diagram representing the set S of stable states and the possible movements transitions in it after the change.

Some Sobering Philosophy

- The “usual” backtesting concern:
  - Past strategy performance may not be indicative of hypothetical future performance
    - changes in underlying market conditions
    - overfitting the historical data
- An even worse concern:
  - Past strategy performance may not be indicative of hypothetical past performance!
    - well beyond measurable trading costs, mechanical market impact, etc.
- Standard backtesting methodologies implicitly assume an absolute model
  - May be fine on longer timescales, but potentially dangerous at microstructure level
  - Alternatives: only use actual past trades or live trading

Simulations

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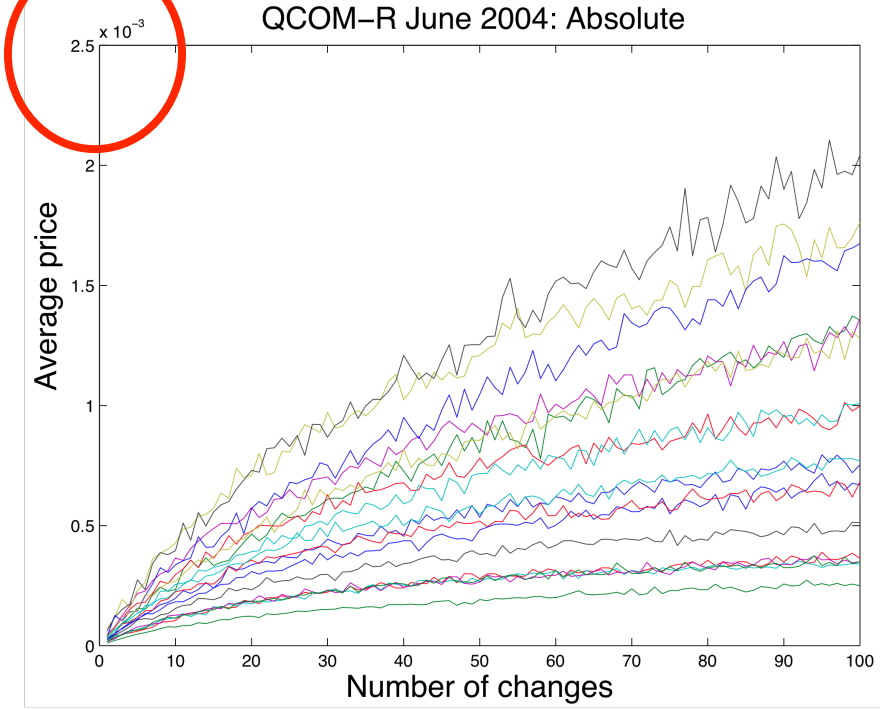
QCOM−R June 2004: Absolute
x 10^−3
Average price
Number of changes
0
0.5
1
1.5
2
2.5
0
10
20
30
40
50
60
70
80
90
100

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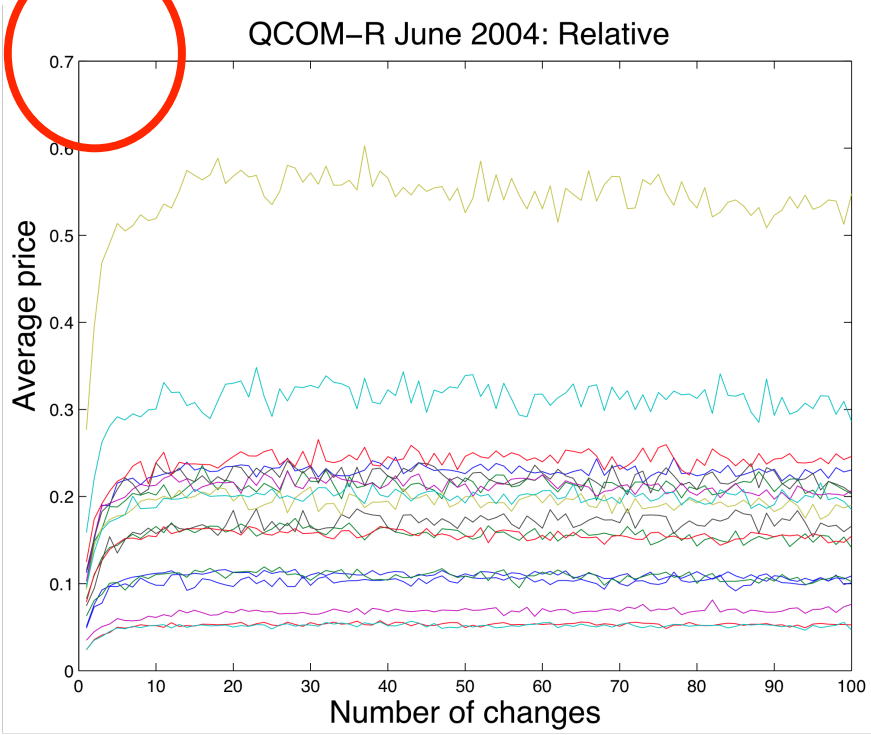
QCOM−R June 2004: Relative
Average price
Number of changes
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
10
20
30
40
50
60
70
80
90
100

A Mixture Model

fraction α of absolute traders, 1-α of relative traders, single order deletion