Learning Minimal Abstractions

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Percy Liang
UC Berkeley

Omer Tripp
Tel-Aviv Univ.

Mayur Naik
Intel Labs Berkeley

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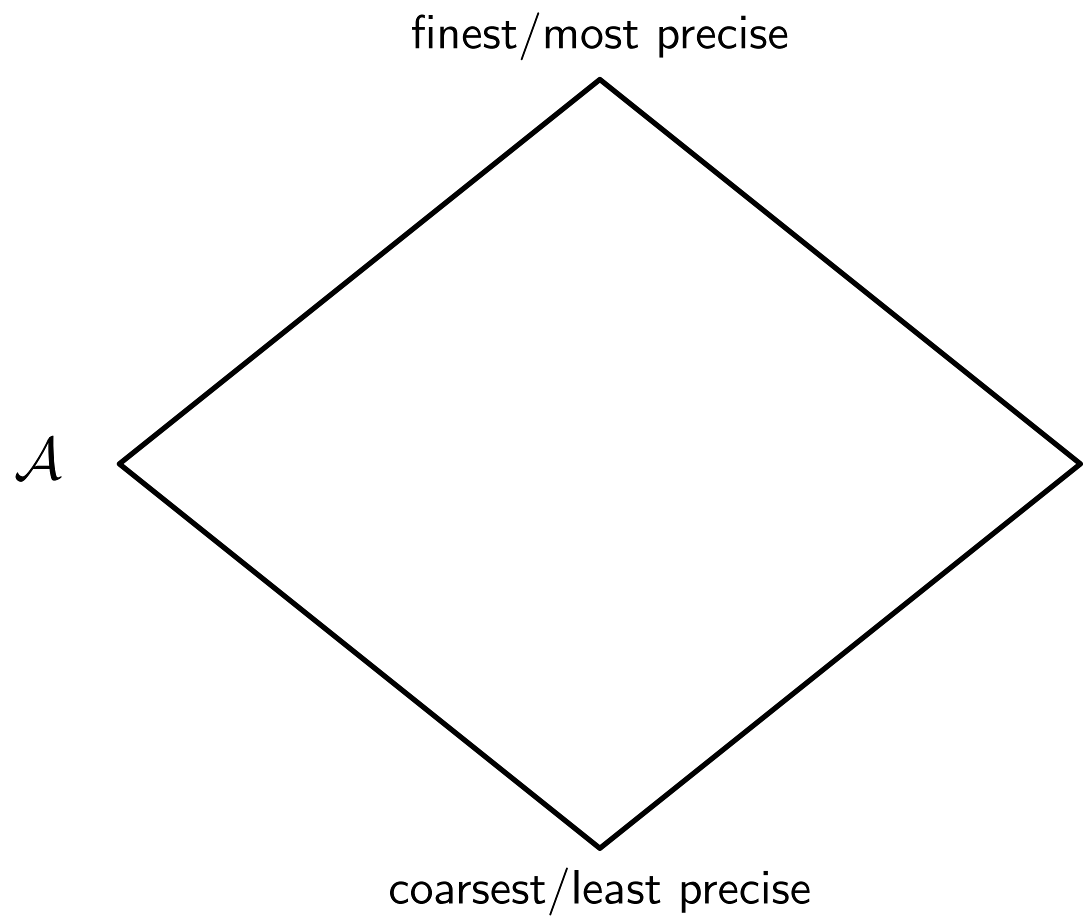

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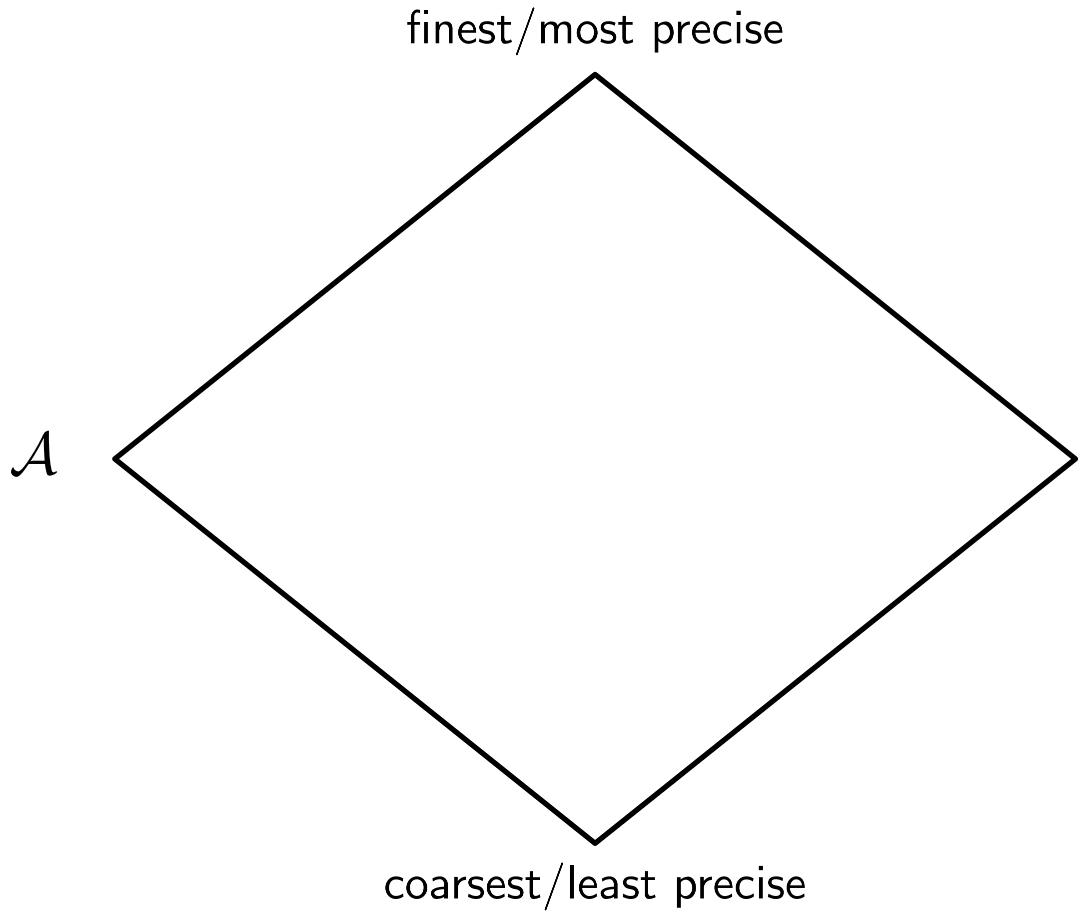
Given a family of abstractions $\mathcal{A}$ and a client query q...



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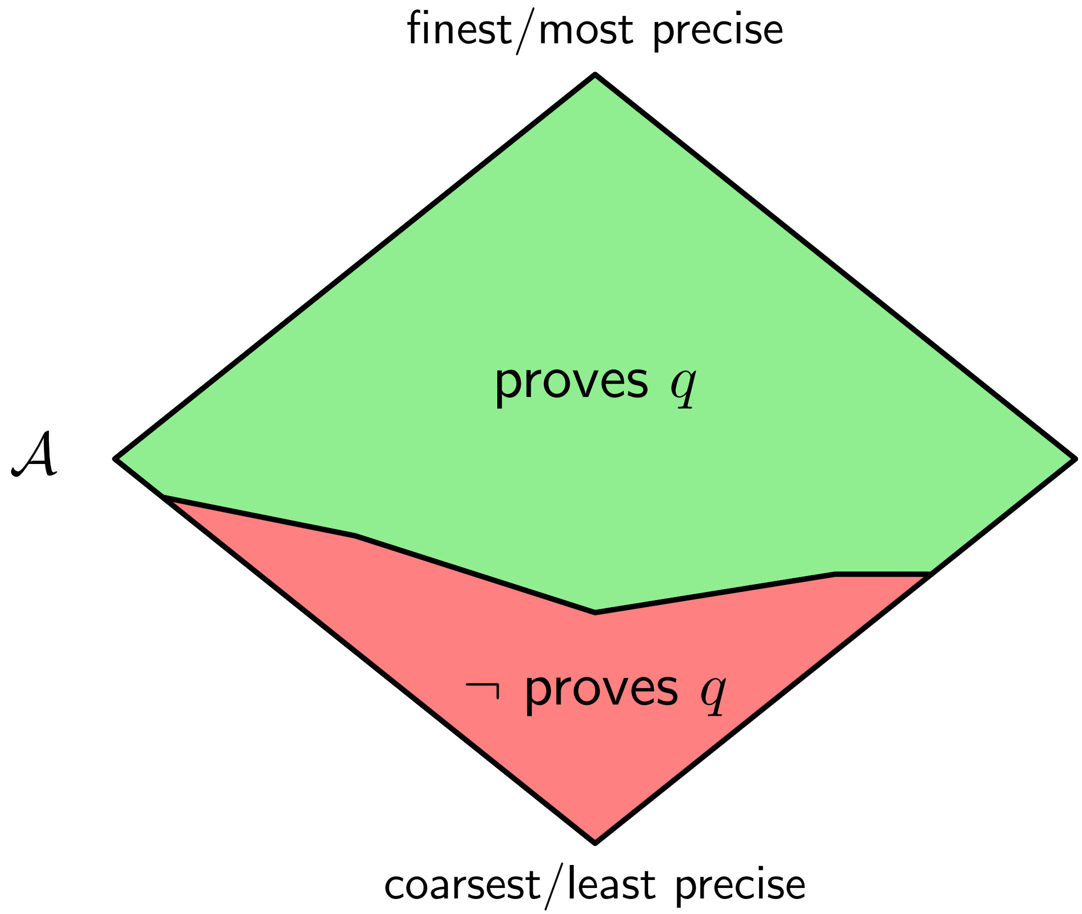

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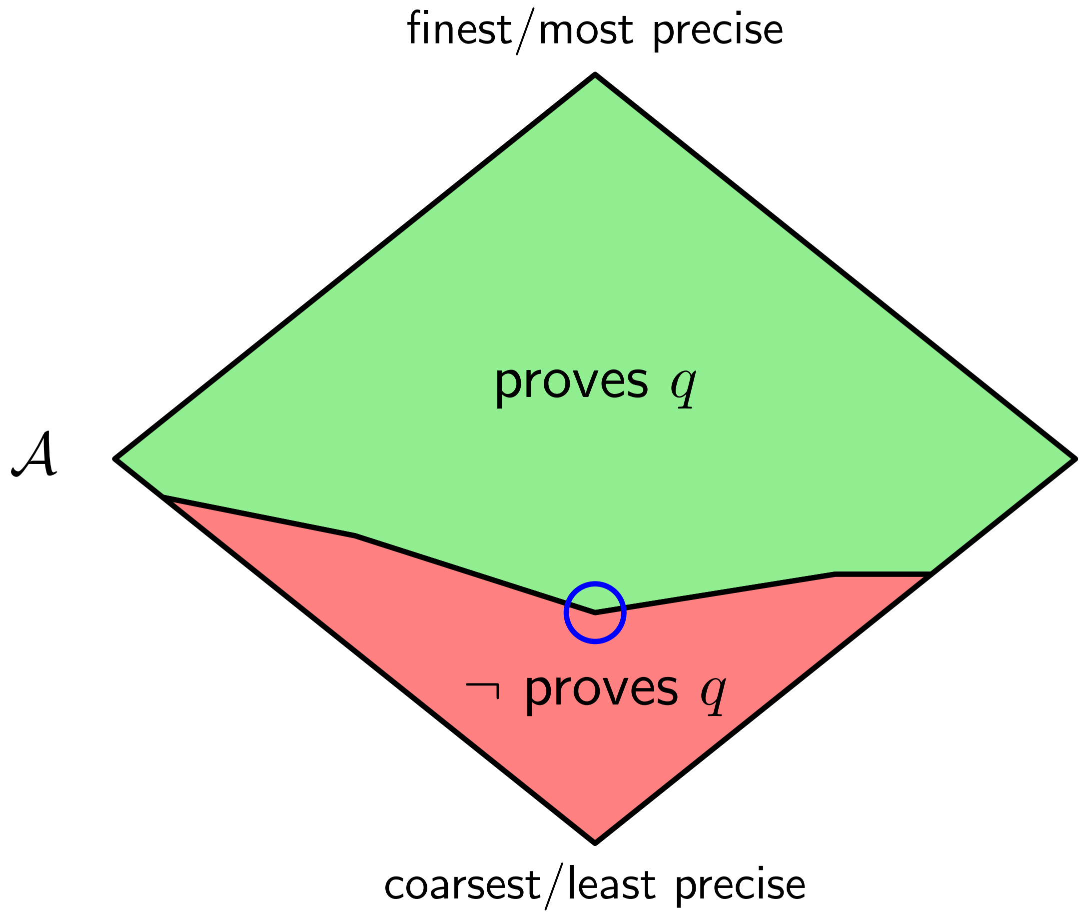

What is the coarsest abstraction $\mathbf{a} \in \mathcal{A}$ that proves the query q?

Motivating application: race detection

```
    getnew() {           // Thread 1       // Thread 2
h1:   z1 = new C         x = getnew()      y = getnew()
h2:   z2 = new C         x.f = ...         y.f = ...
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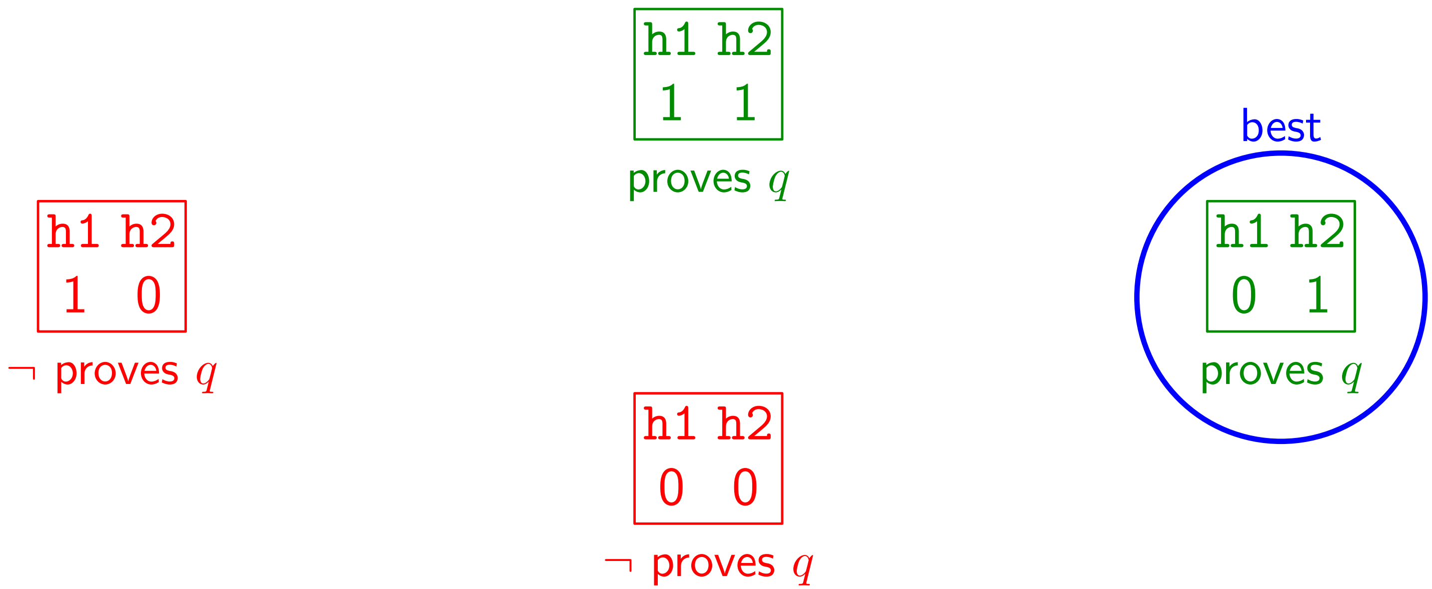

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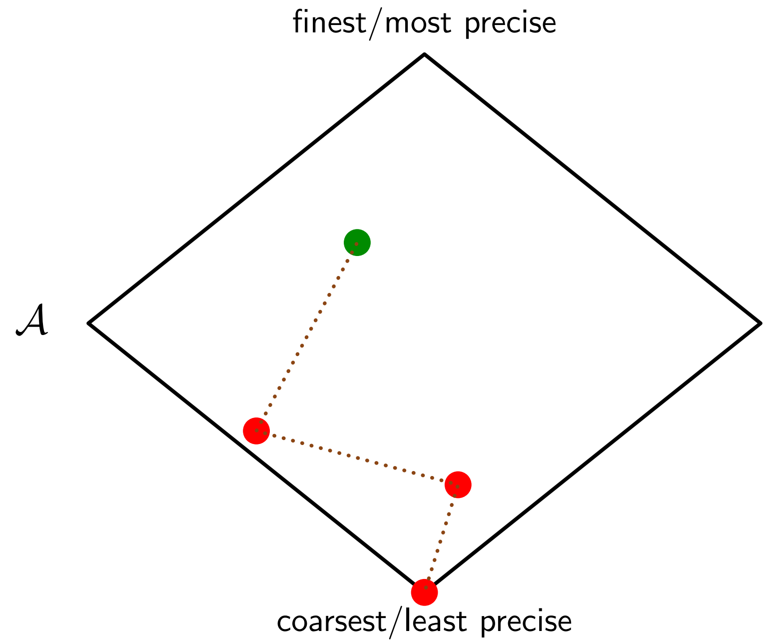

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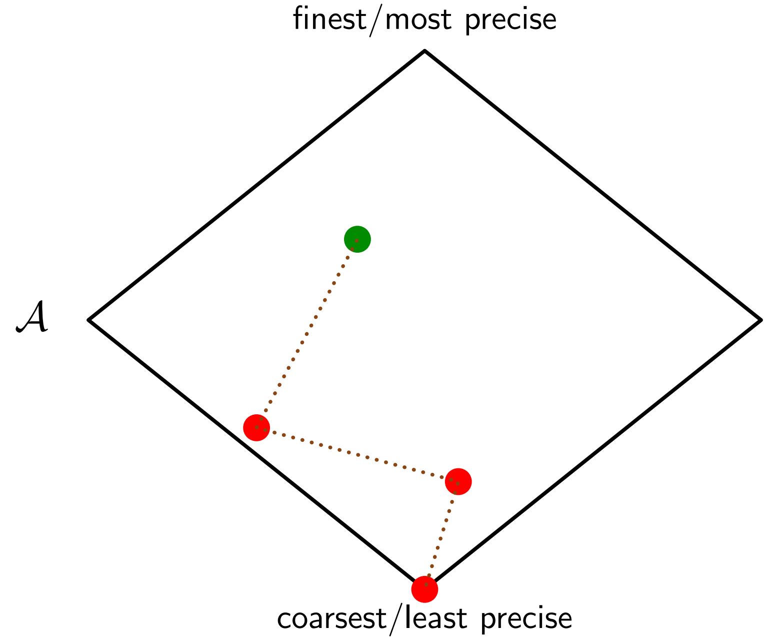

Our problem (scientific question):

Given that we've proved a query,
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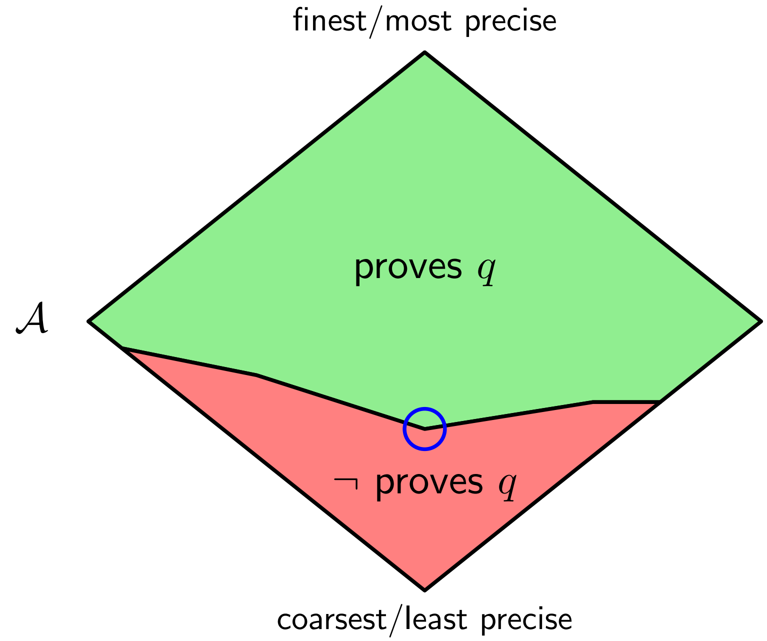

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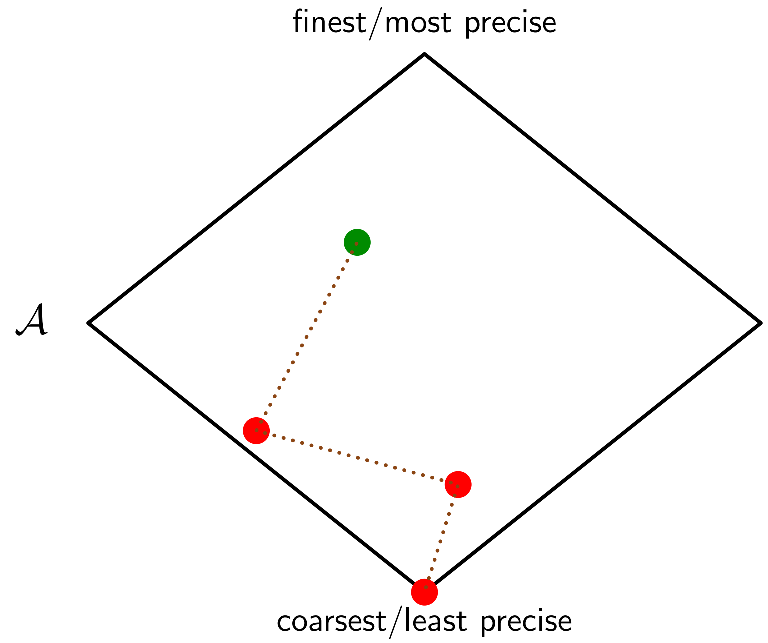

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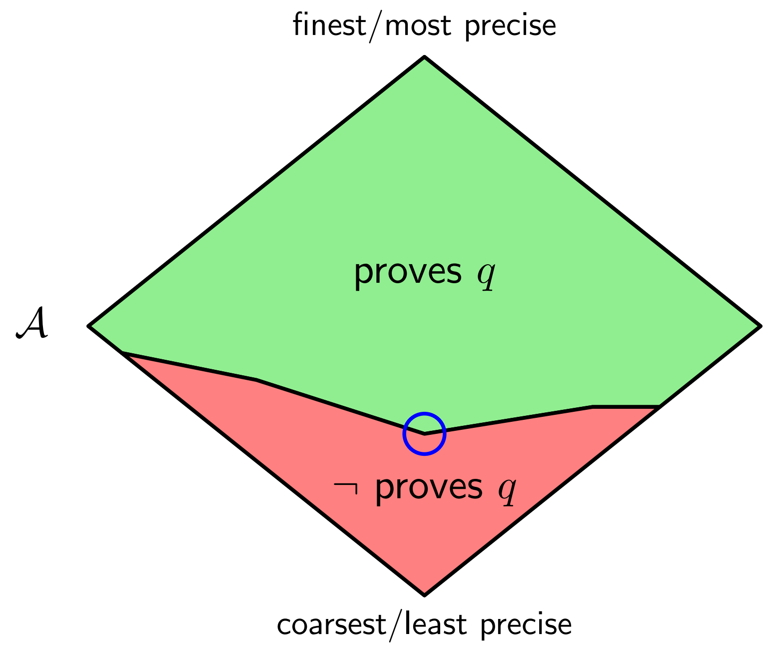
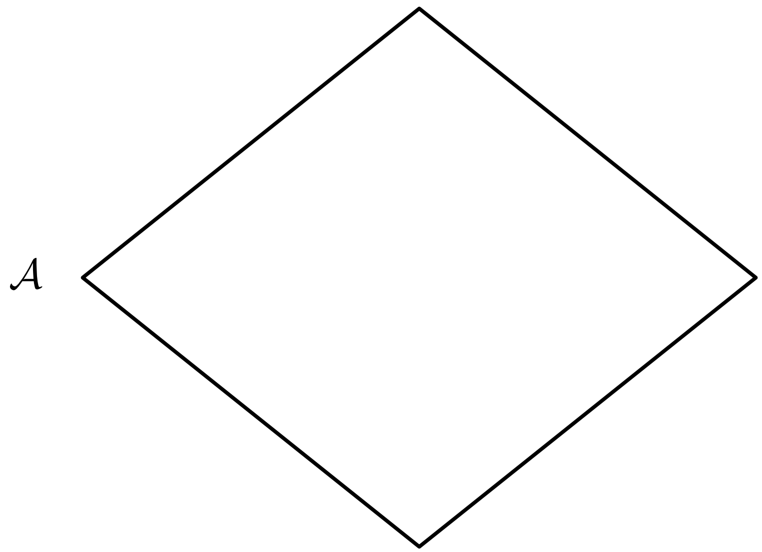
Sufficient/necessary conditions: what aspects of program to model?



Binary representation

Abstraction $\mathbf{a} \in \mathcal{A}$ is a binary vector (subset of components):

$\mathbf{a} = 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$ (most precise)



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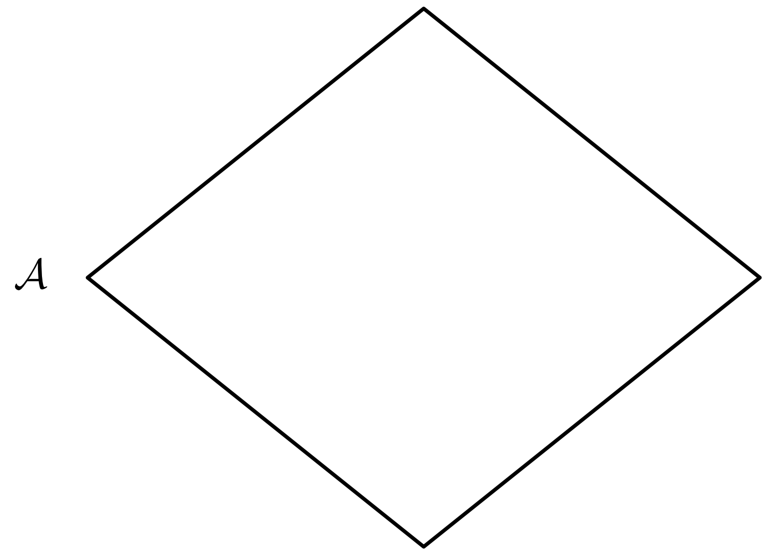

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Examples:

k-limited [Milanova et al. 2002]: treat site context-sensitively?

Predicate abstraction [Ball et al. 2001]: include predicate?

Shape analysis [Sagiv et al. 2002]: treat as abstraction predicate?

Finding a minimal abstraction

Given a static analysis $\mathbf{F}$:

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(i) $\mathbf{F}(\mathbf{a}) = 0$ (proves the query)

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Approach: machine learning algorithms that exploit randomization

Key theme: sparsity

Sparsity hypothesis:
Only a small fraction of components of $\mathbf{a}$ need to be refined

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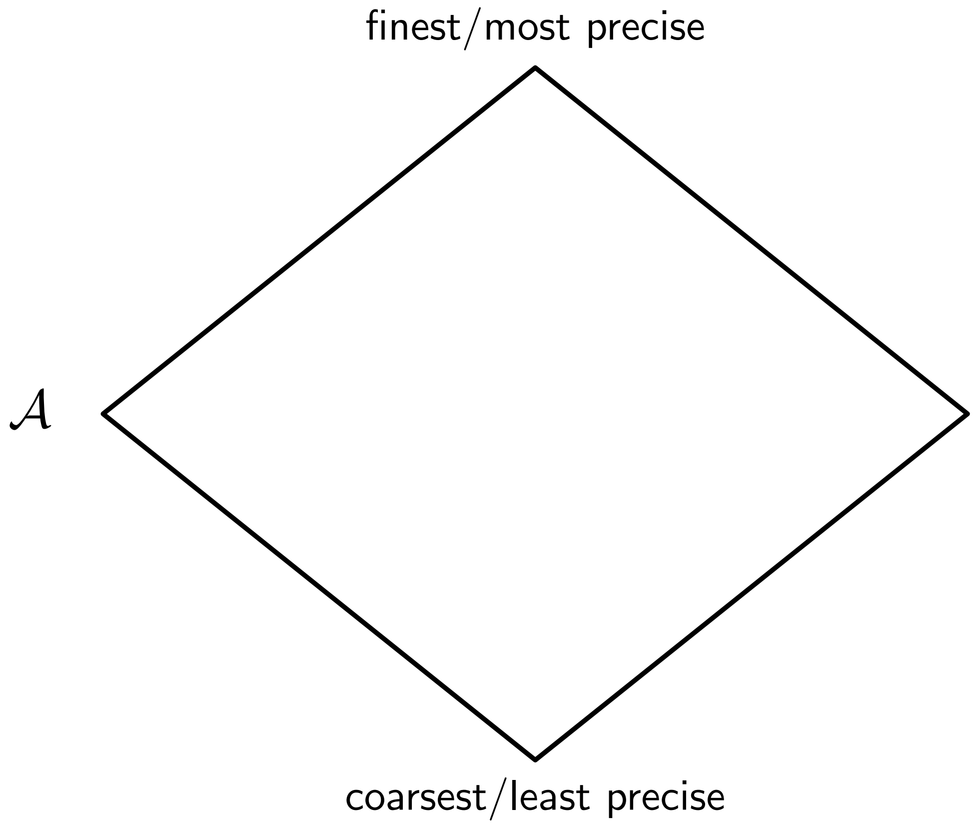

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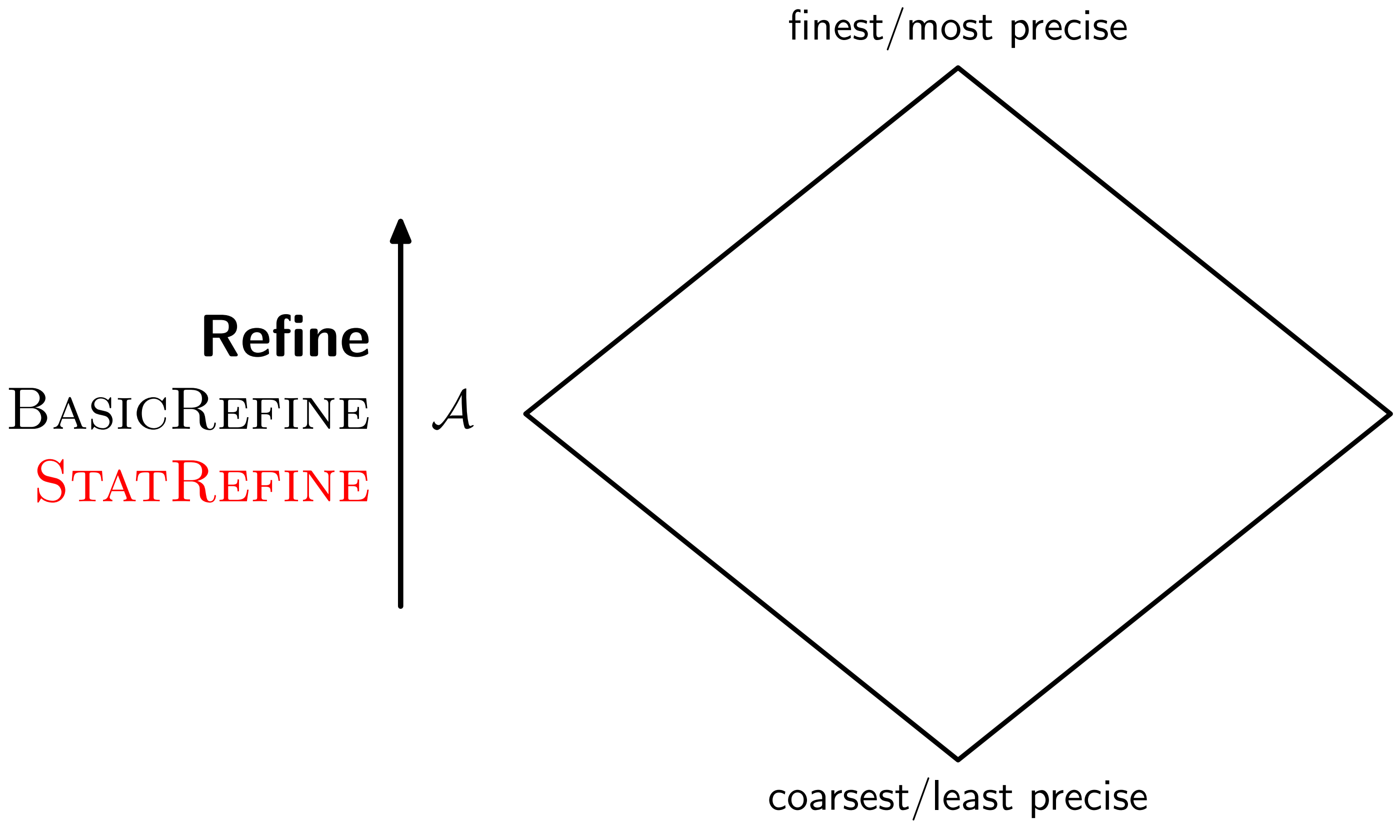

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BasicRefine
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coarsest/least precise

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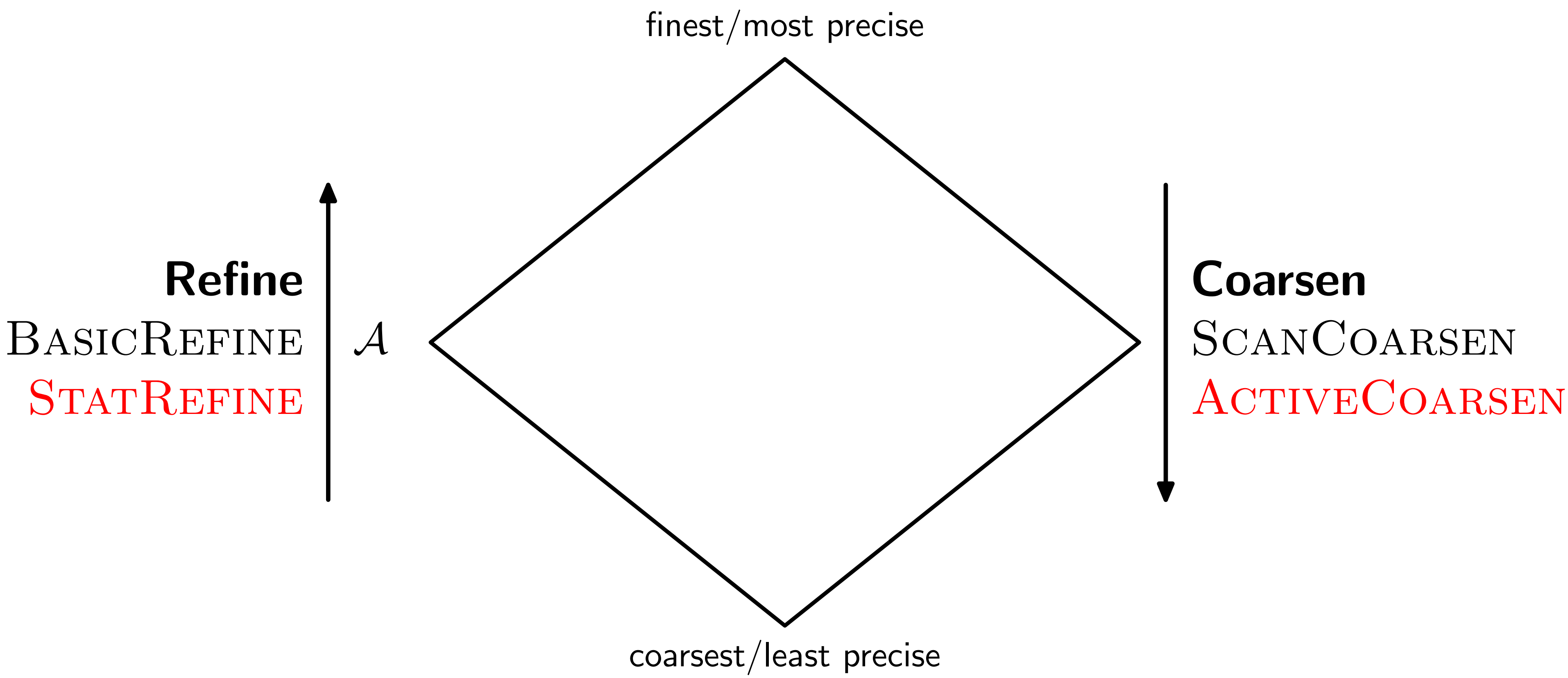
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Does not solve the minimal abstraction problem (it refines too much)

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Problem: takes $O(\#$ components) time (can be $> 10,000 \Rightarrow > 30$ days)

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1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	0
1	1	0	1	1	1	0	0	1	0	0	1	1	1	1	1	1	1	1	1	$\Rightarrow$	1
1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	0
1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	0
0	1	1	1	1	0	0	1	1	0	1	1	0	1	1	1	1	1	1	0	$\Rightarrow$	0
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1	1	1	0	1	1	1	1	0	1	0	1	1	1	0	1	1	0	1	1	$\Rightarrow$	1
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1	0	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1	1	0	1	$\Rightarrow$	1
0	0	1	0	0	1	1	1	0	1	1	1	1	1	1	1	0	0	1	1	$\Rightarrow$	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	$\Rightarrow$	0
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1	1	1	1	1	1	0	0	1	1	1	1	0	1	0	1	0	1	1	1	$\Rightarrow$	0
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	$\Rightarrow$	0
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1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⇒	0
0	1	1	1	1	0	0	1	1	0	1	1	0	1	1	1	1	1	1	0	⇒	0
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0	1	0	1	1	1	1	1	1	0	1	0	1	1	1	0	1	1	1	1	⇒	0
1	1	1	1	1	1	0	0	1	1	1	1	0	1	0	1	0	1	1	1	⇒	0
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	⇒	0
1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	1	1	1	0	1	⇒	0
1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	⇒	0
8	9	7	**11**	9	6	7	10	**11**	7	**11**	9	7	10	10	10	9	8	9	9		

StatRefine

Theorem:

$s = \#$ components in the largest minimal abstraction
$d = \#$ components in any minimal abstraction
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Proof sketch: large deviation bounds $+$ optimization over α

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1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 $\Rightarrow$ 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 1 1 1 1 1 1 1 1 1 1 1 $\Rightarrow$ 1
1 1 1 1 1 1 0 1 1 1 0 1 0 1 1 1 1 0 1 0 1 1 1 0 1 1 1 1 0 0 $\Rightarrow$ 0

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$ 1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	⇒	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			⇒	0
0	1	0	1	1	1		1	1	0		1		1	1	1	1		0		0	1	1		0	1	1	1			⇒	0

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1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$ 0

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1			0		1	1	1	0					1	1			1	1	1			$\Rightarrow$	0

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$	0
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$ 1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$ 0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$ 1

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$	0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$	1
	1		1	1	0		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$	0

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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$ 1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$ 0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$ 1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$ 1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$ 0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$ 1
	1		1	1			1	1					1	1	1						1	1			1	1	1			$\Rightarrow$ 0
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$ 1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$ 0
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1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$ 0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$ 0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$ 1
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	1		1	1			1	1					1	1	1						1	1					1			$\Rightarrow$ 0
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			$\Rightarrow$	0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$	1
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	1		1	1			1	1					1	1	1						1	1					1			$\Rightarrow$	0
	1		1	1			0	1					0	1	0						1	1					1			$\Rightarrow$	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	$\Rightarrow$	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			$\Rightarrow$	0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			$\Rightarrow$	0
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	1		0	0	1		0	0					0	1	1						0	1			1	1	0			$\Rightarrow$	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	⇒	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	⇒	1
1	1	1	1	1	1		1	1	1		1		1	1	1	1		1		1	1	1		1	1	1	1			⇒	0
	1		1	1	1		1	1			1		1	1	1	1					1	1			1	1	1			⇒	0
	1		1	1	1		1	1					1	1	1						1	1			1	1	1			⇒	0
	1		0	0	1		0	0					0	1	1						0	1			1	1	0			⇒	1
	1		1	1			1	1					1	1	1						1	1			1	1	1			⇒	0
	1		1	1			1	1					1	1	1						1	1					1			⇒	0
	1		1	1			0	1					0	1	0						1	1					1			⇒	1
	1		1	1			1	1					1	1	1						1	1								⇒	0
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ACTIVECOARSEN analysis

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$|\mathbb{J}| =$ total $\#$ components

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Significance: s is small,
only logarithmic dependence on total $\#$ components

Summary of algorithms

Algorithm	Minimal	Correct	# calls to $\mathbf{F}$
BasicRefine	no	yes	$O(1)$
ScanCoarsen	yes	yes	$O(\lvert\mathbb{J}\rvert)$
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Extensions:

- Adapatively refinement probability α
- Sharing computation across multiple queries

Experimental setup

Application: static race detector of [Naik et al. 2006]

Pointer analysis using k-object-sensitivity or k-CFA with heap cloning

Combination of call graph, may alias, thread escape, may happen in parallel

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Benchmark statistics (determines # components in abstraction):

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Number of races:

	`hedc`	`weblech`	`lusearch`
0-CFA	21,335	27,941	37,632
1-CFA	17,837	8,208	31,866
diff. (queries)	**3,498**	**19,733**	**5,766**
1-OBJ	17,137	8,063	31,428
2-OBJ	16,124	5,523	20,929
diff. (queries)	**1,013**	**2,540**	**10,499**

Experimental results (all queries)

Setting: find **one** abstraction to prove **all** queries

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k-object-sensitivity:

	total # components	BasicRefine	ActiveCoarsen (minimal)
`hedc`	1,580	906 (57%)	37 (**2.3%**)
`weblech`	2,584	1,768 (68%)	48 (**1.9%**)
`lusearch`	2,873	2,085 (73%)	56 (**1.9%**)

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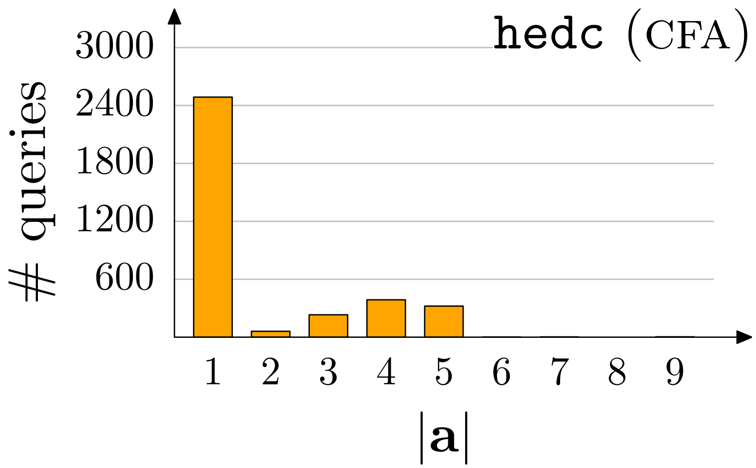

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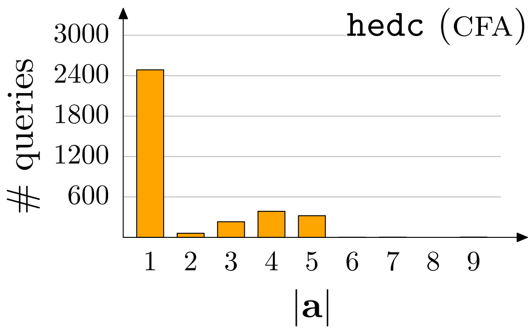

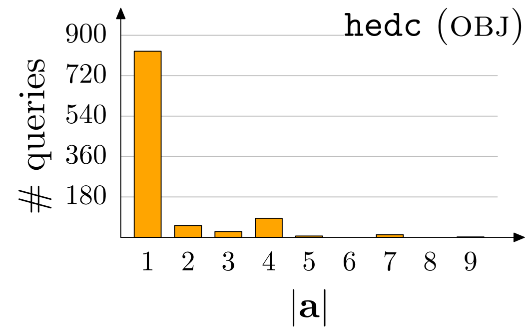

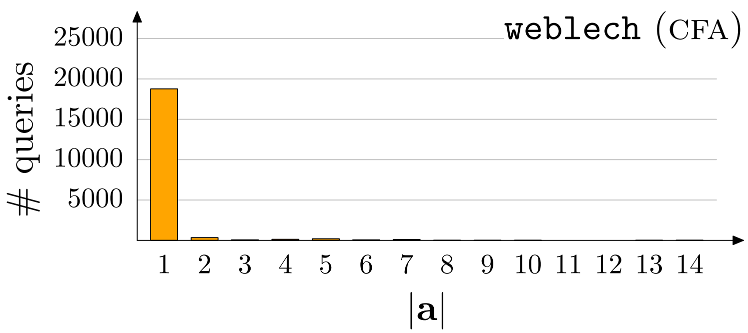

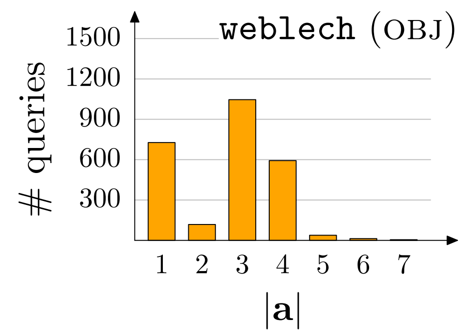

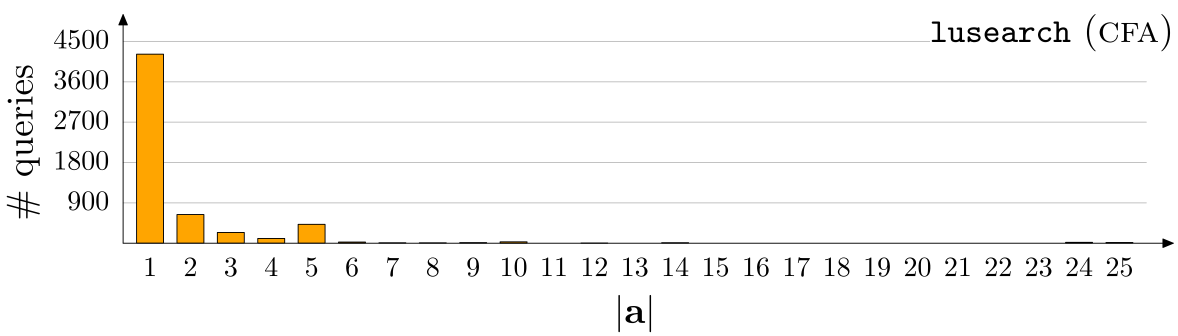

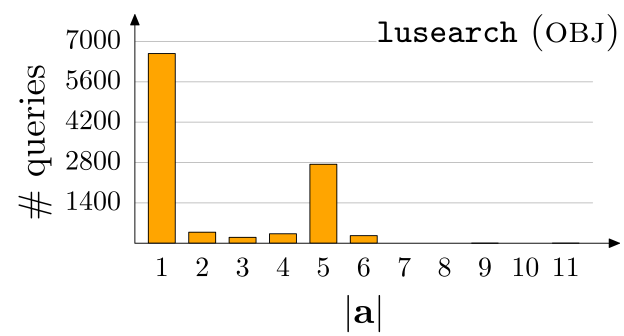

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- Motivating problem: to scale static analyses, need cheap abstractions
- Scientific question: what's the minimal abstraction to prove a query?
- **Sparsity**: very few components are needed
  - Theoretical result: leads to efficient machine learning algorithms
  - Empirical result: leads to cheap abstractions
- Future work: tackle motivating problem with information gathered from minimal abstractions