

# On Incremental Core-Guided MaxSAT Solving

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X. Si<sup>1</sup>, X. Zhang<sup>1</sup>, V. Manquinho<sup>2</sup>, M. Janota<sup>3</sup>, A. Ignatiev<sup>4,5</sup>, and M. Naik<sup>1</sup>

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<sup>1</sup> Georgia Institute of Technology, USA

<sup>2</sup> INESC-ID, IST, University of Lisbon, Portugal

<sup>3</sup> Microsoft Research, Cambridge, UK

<sup>4</sup> LaSIGE, FC, University of Lisbon, Portugal

<sup>5</sup> ISDCT SB RAS, Irkutsk, Russia

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$|F_i|$  — up to  $10^8$

(e.g. Markov Logic Networks<sup>1</sup>)

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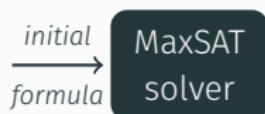
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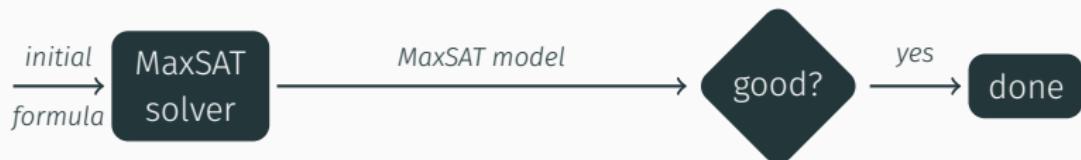
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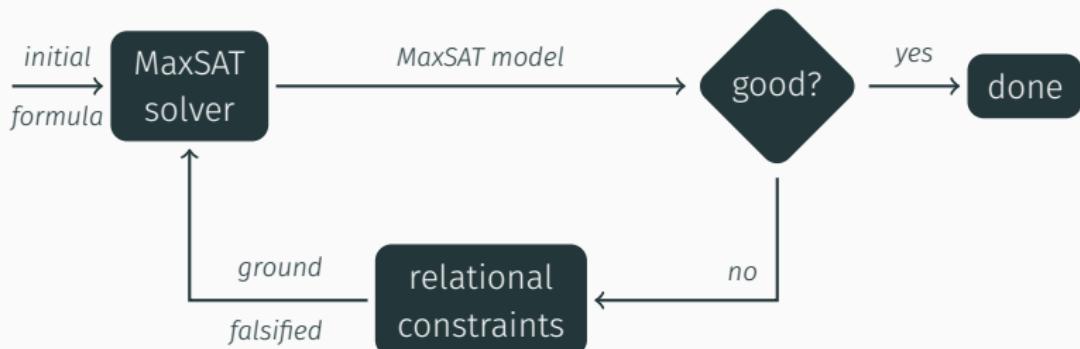
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## Fu&Malik algorithm for MaxSAT

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$$\begin{array}{lll} F_{hard} & = & (\neg x \vee \neg y) \quad (\neg x \vee \neg z) \quad (\neg y \vee \neg z) \\ & & r_1 + r_2 \leq 1 \\ F_{soft} & = & (\cancel{10}, \cancel{x}) \quad (\cancel{20}, \cancel{y}) \quad (40, z) \\ & & (10, x \vee r_1) \quad (10, y \vee r_2) \\ & & (10, y) \end{array}$$

$$cost = 10$$

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Poor quality cores

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$$2 \quad F_{hard} = \quad (b)$$



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( $split\_lim_c \leq k \quad \forall c \in F_{soft}$ )

## Experimental results

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# Experimental evaluation

- Applications:
  1. abstraction refinement

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  - running Linux

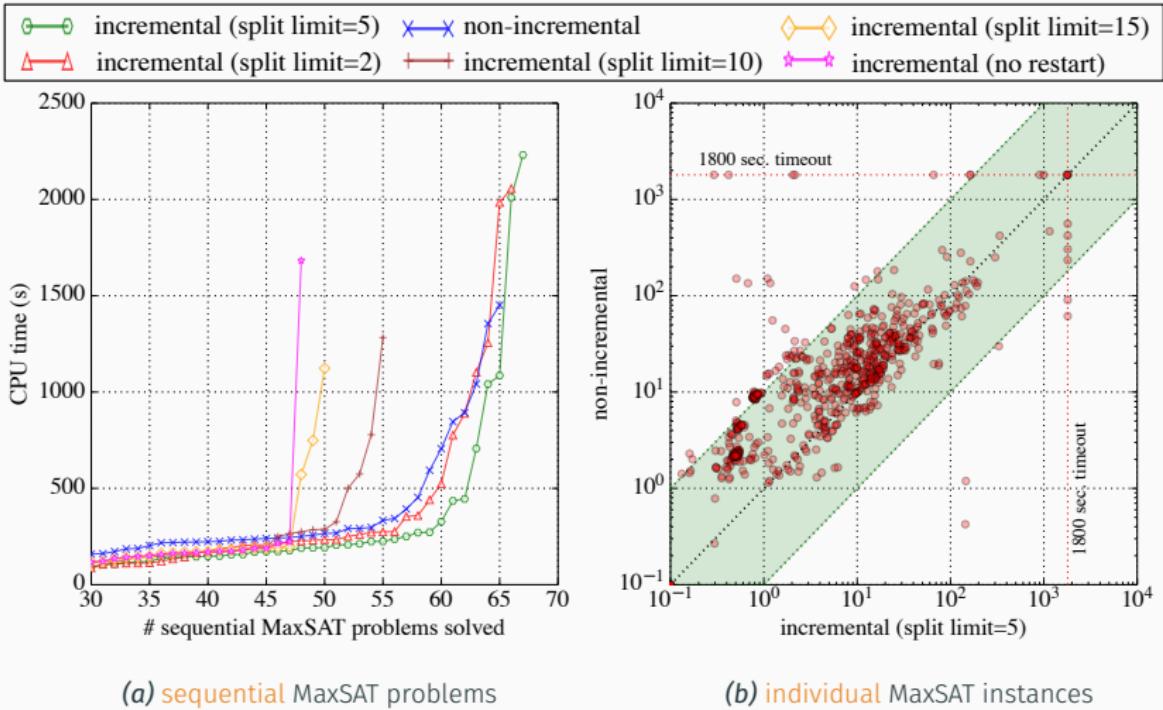
# Experimental evaluation

- Applications:
  1. abstraction refinement +
  2. user-guided analysis +
  3. statistical relational inference
    - = 74 sequential MaxSAT problems
    - = 669 individual MaxSAT instances (avg.  $10^7$  clauses)
- new approach in Open-WBO — state of the art
  1. non-incremental
  2. incremental-without-restarts
  3. incremental (clause split 2, 5, 10, 15)
- Machine configuration:
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  - 30m timeout

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  - 32GB memout

# Experimental results



(a) sequential MaxSAT problems

(b) individual MaxSAT instances

## Speedup over non-incremental approach

Split limit 5 vs. non-incremental:

- average speedup —  $1.8\times$
- best speedup —  $296\times$  !

## Summary and future work

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- new **incremental** approach to sequential MaxSAT:
  - incremental MaxSAT calls +
  - incremental SAT calls inside MaxSAT +
  - adaptive restarts
- better restart strategies
- state-of-the-art MaxSAT algorithms
- not only **add** but also **delete** clauses

Questions?