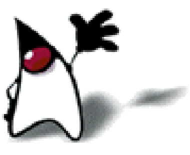
Hashing



Preview

- A hash function is a function that:
  - When applied to an Object, returns a number
  - When applied to *equal* Objects, returns the *same* number for each
  - When applied to *unequal* Objects, is *very unlikely* to return the same number for each
- Hash functions turn out to be very important for searching, that is, looking things up fast
- This is their story....

Searching

- Consider the problem of searching an array for a given value
  - If the array is not sorted, the search requires O(n) time
    - If the value isn’t there, we need to search all n elements
    - If the value is there, we search n/2 elements on average
  - If the array is sorted, we can do a binary search
    - A binary search requires O(log n) time
    - About equally fast whether the element is found or not
  - It doesn’t seem like we could do much better
    - How about an O(1), that is, constant time search?
    - We can do it *if* the array is organized in a particular way

Hashing

- Suppose we were to come up with a "magic function" that, given a value to search for, would tell us exactly where in the array to look
  - If it's in that location, it's in the array
  - If it's not in that location, it's not in the array
- This function would have no other purpose
- If we look at the function's inputs and outputs, they probably won't "make sense"
- This function is called a hash function because it "makes hash" of its inputs

Example (ideal) hash function

- Suppose our hash function gave us the following values:

```
hashCode("apple") = 5
hashCode("watermelon") = 3
hashCode("grapes") = 8
hashCode("cantaloupe") = 7
hashCode("kiwi") = 0
hashCode("strawberry") = 9
hashCode("mango") = 6
hashCode("banana") = 2
```

0	kiwi
1	
2	banana
3	watermelon
4	
5	apple
6	mango
7	cantaloupe
8	grapes
9	strawberry

Sets and tables

- Sometimes we just want a set of things—objects are either in it, or they are not in it
- Sometimes we want a map—a way of looking up one thing based on the value of another
  - We use a *key* to find a place in the map
  - The associated *value* is the information we are trying to look up
- Hashing works the same for both sets and maps
  - Most of our examples will be sets

	key	*value*
. . .		
141		
142	robin	robin info
143	sparrow	sparrow info
144	hawk	hawk info
145	seagull	seagull info
146		
147	bluejay	bluejay info
148	owl	owl info

Finding the hash function

- How can we come up with this magic function?
- In general, we cannot--there is no such magic function ☹
  - In a few specific cases, where all the possible values are known in advance, it has been possible to compute a perfect hash function
- What is the next best thing?
  - A perfect hash function would tell us exactly where to look
  - In general, the best we can do is a function that tells us where to *start* looking!

Example imperfect hash function

- Suppose our hash function gave us the following values:
  - hash("apple") = 5
    hash("watermelon") = 3
    hash("grapes") = 8
    hash("cantaloupe") = 7
    hash("kiwi") = 0
    hash("strawberry") = 9
    hash("mango") = 6
    hash("banana") = 2
    hash("honeydew") = 6

0	kiwi
1	
2	banana
3	watermelon
4	
5	apple
6	mango
7	cantaloupe
8	grapes
9	strawberry

• Now what?

Collisions

- When two values hash to the same array location, this is called a collision
- Collisions are normally treated as “first come, first served”—the first value that hashes to the location gets it
- We have to find something to do with the second and subsequent values that hash to this same location

Handling collisions

- What can we do when two different values attempt to occupy the same place in an array?
  - **Solution #1:** Search from there for an empty location
    - Can stop searching when we find the value *or* an empty location
    - Search must be end-around
  - **Solution #2:** Use a second hash function
    - ...and a third, and a fourth, and a fifth, ...
  - **Solution #3:** Use the array location as the header of a linked list of values that hash to this location
- All these solutions work, provided:
  - We use the same technique to *add* things to the array as we use to *search* for things in the array

Insertion, I

- Suppose you want to add seagull to this hash table
- Also suppose:
  - hashCode(seagull) = 143
  - table[143] is not empty
  - table[143] != seagull
  - table[144] is not empty
  - table[144] != seagull
  - table[145] is empty
- Therefore, put seagull at location 145

. . .	
141	
142	robin
143	sparrow
144	hawk
145	seagull
146	
147	bluejay
148	owl
. . .	

Searching, I

- Suppose you want to look up seagull in this hash table
- Also suppose:
  - hashCode(seagull) = 143
  - table[143] is not empty
  - table[143] != seagull
  - table[144] is not empty
  - table[144] != seagull
  - table[145] is not empty
  - table[145] == seagull !
- We found seagull at location 145

. . .	
141	
142	robin
143	sparrow
144	hawk
145	seagull
146	
147	bluejay
148	owl
. . .	

Searching, II

- Suppose you want to look up `cow` in this hash table
- Also suppose:
  - `hashCode(cow) = 144`
  - `table[144]` is not empty
  - `table[144] != cow`
  - `table[145]` is not empty
  - `table[145] != cow`
  - `table[146]` is empty
- If `cow` were in the table, we should have found it by now
- Therefore, it isn't here

. . .	
141	
142	robin
143	sparrow
144	hawk
145	seagull
146	
147	bluejay
148	owl
. . .	

Insertion, II

- Suppose you want to add `hawk` to this hash table
- Also suppose
  - `hashCode(hawk) = 143`
  - `table[143]` is not empty
  - `table[143] != hawk`
  - `table[144]` is not empty
  - `table[144] == hawk`
- `hawk` is already in the table, so do nothing

. . .	
141	
142	robin
143	sparrow
144	hawk
145	seagull
146	
147	bluejay
148	owl
. . .	

Insertion, III

- Suppose:
  - You want to add cardinal to this hash table
  - hashCode(cardinal) = 147
  - The last location is 148
  - 147 and 148 are occupied
- Solution:
  - Treat the table as circular; after 148 comes 0
  - Hence, cardinal goes in location 0 (or 1, or 2, or ...)

. . .	
141	
142	robin
143	sparrow
144	hawk
145	seagull
146	
147	bluejay
148	owl

Clustering

- One problem with the above technique is the tendency to form "clusters"
- A cluster is a group of items not containing any open slots
- The bigger a cluster gets, the more likely it is that new values will hash into the cluster, and make it ever bigger
- Clusters cause efficiency to degrade
- Here is a *non*-solution: instead of stepping one ahead, step n locations ahead
  - The clusters are still there, they're just harder to see
  - Unless n and the table size are mutually prime, some table locations are never checked

Efficiency

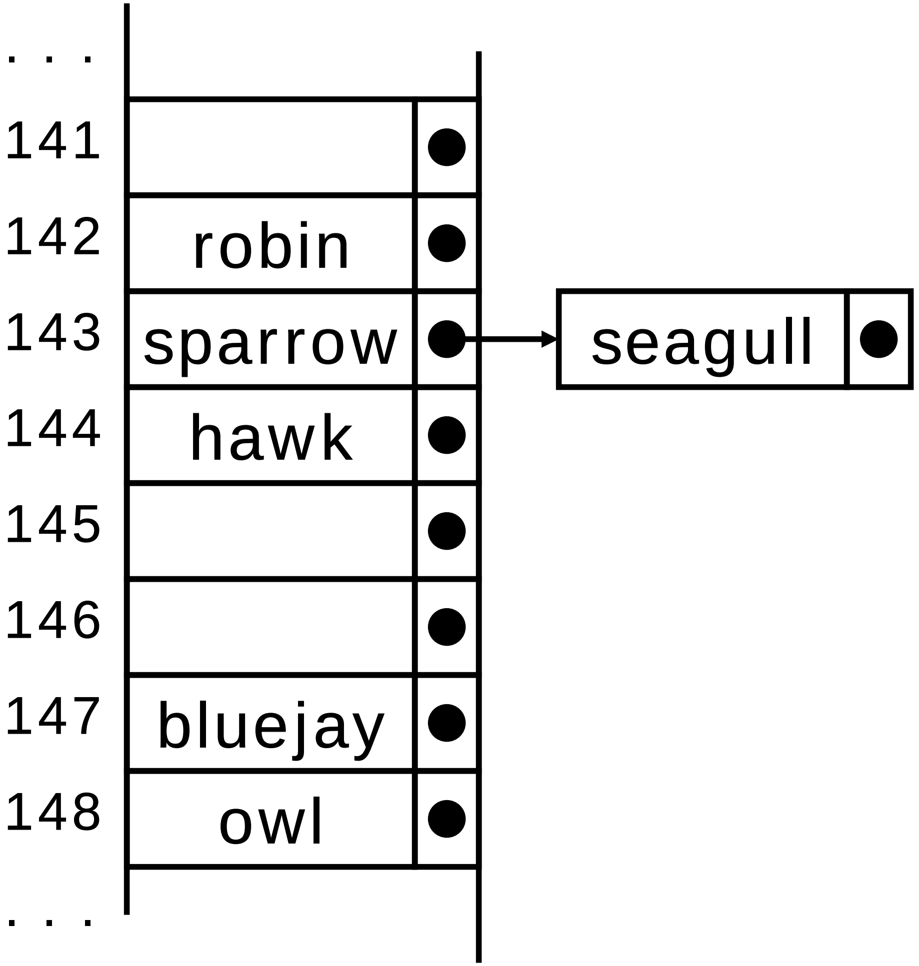
- Hash tables are actually surprisingly efficient
- Until the table is about 70% full, the number of probes (places looked at in the table) is typically only 2 or 3
- Sophisticated mathematical analysis is required to *prove* that the expected cost of inserting into a hash table, or looking something up in the hash table, is O(1)
- Even if the table is nearly full (leading to occasional long searches), efficiency is usually still quite high

Solution #2: Rehashing

- In the event of a collision, another approach is to rehash: compute another hash function
  - Since we may need to rehash many times, we need an easily computable sequence of functions
- Simple example: in the case of hashing Strings, we might take the previous hash code and add the length of the String to it
  - Probably better if the length of the string was not a component in computing the original hash function
- Possibly better yet: add the length of the String plus the number of probes made so far
  - Problem: are we sure we will look at every location in the array?
- Rehashing is a fairly uncommon approach, and we won't pursue it any further here

Solution #3: Bucket hashing

- The previous solutions used open hashing: all entries went into a “flat” (unstructured) array
- Another solution is to make each array location the header of a linked list of values that hash to that location



The hashCode function

- `public int hashCode()` is defined in `Object`
- Like `equals`, the default implementation of `hashCode` just uses the address of the object—probably not what you want for your own objects
- You can override `hashCode` for your own objects
- As you might expect, `String` overrides `hashCode` with a version appropriate for strings
- Note that the supplied `hashCode` method can return *any* possible `int` value (including negative numbers)
  - You have to adjust the returned `int` value to the size of your hash table

Why do you care?

- Java provides HashSet, Hashtable, and HashMap for your use
- These classes are very fast and very easy to use
- They work great, without any additional effort, for Strings
- But...
- They will ***not work*** for your own objects ***unless either:***
  - You are satisfied with the inherited equals method (no object is equal to any other, separately created object)
- ***Or:***
  - You have defined equals for your objects *and*
  - You have *also* defined a hashCode method that is *consistent with* your equals method (that is, equal objects have equal hash codes)

Writing your own hashCode()

- A `hashCode()` method *must:*
  - Return a value that is (or can be converted to) a legal array index
  - Always return the same value for the same input
    - It can’t use random numbers, or the time of day
  - Return the same value for *equal* inputs
    - Must be consistent with your `equals` method
- It does *not* need to guarantee different values for different inputs
- A *good* `hashCode()` method *should:*
  - Make it *unlikely* that different objects have the same hash code
  - Be efficient to compute
  - Give a uniform distribution of values
  - *Not* assign similar numbers to similar input values

Mutable keys

- Suppose you have a class `Customer` with fields `firstName` and `lastName`
- How would this work for a hash code?
```
@Override
public int hashCode() {
    return hashCode(firstName + lastName);
}
```
- Does it matter if other customers might have exactly the same name?
- Does it matter if "Mary Smith" gets married and changes her name to "Mary Brown"?

Other considerations

- The hash table might fill up; we need to be prepared for that
  - Not a problem for a bucket hash, of course
- You cannot easily delete items from an open hash table
  - This would create empty slots that might prevent you from finding items that hash before the slot but end up after it
  - Again, not a problem for a bucket hash
- Generally speaking, hash tables work best when the table size is a prime number

Hash tables in Java

- Java provides classes Hashtable, HashMap, and HashSet (and many other, more specialized ones)
- Hashtable and HashMap are maps: they associate *keys* with *values*
- Hashtable is synchronized; that is, it can be accessed safely from multiple threads
    - Hashtable uses an open hash, and has a rehash method, to increase the size of the table
- HashMap is newer, faster, and usually better, but it is not synchronized
    - HashMap uses a bucket hash, and has a remove method
- HashSet is just a set, not a collection, and is not synchronized

Hash table operations

- HashSet, Hashtable and HashMap are in java.util
- All have no-argument constructors, as well as constructors that take an integer table size
- The maps have methods:
  - public Object put(Object key, Object value)
    - (Returns the previous value for this key, or null)
  - public Object get(Object key)
  - public void clear()
  - public Set keySet()
    - Dynamically reflects changes in the hash table
  - ...and many others

Bottom line

- You do ***not*** have to write a `hashCode()` method if:
  - You never use a built-in class that depends on it, or
  - You put only `String`s in hash sets, and use only `String`s as *keys* in hash maps (values don't matter), or
  - You are happy with `equals` meaning `==`, and don't override it
- You ***do*** have to write a `hashCode()` method if:
  - You use a built-in hashing class for your own objects, and you override `equals` for those objects
- Finally, if you ever override `hashCode`, you ***must*** also override `equals`

The End