

UPPAAL tutorial

- What's inside UPPAAL
- The UPPAAL input languages

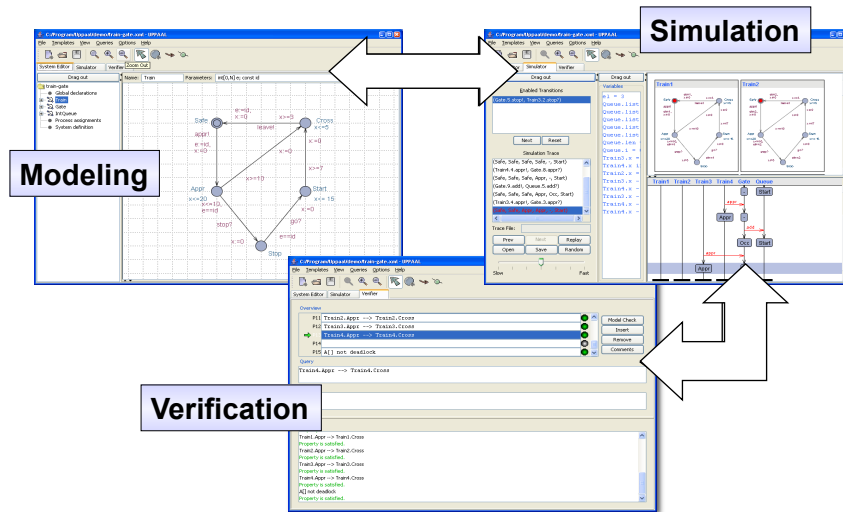
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UPPAAL tool

- Developed jointly by Uppsala & Aalborg University
- >>28,000 downloads since 1999

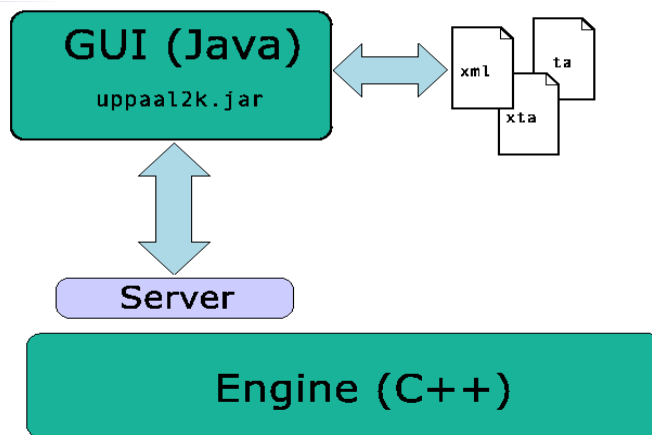
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UPPAAL Tool



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Architecture of UPPAAL



Linux, Windows, Solaris, MacOS

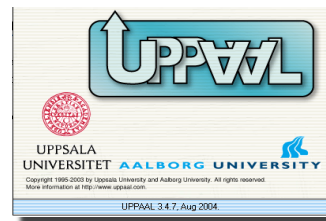
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What's inside UPPAAL

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OUTLINE

- Data Structures
 - DBM's (Difference Bounds Matrices)
 - Canonical and Minimal Constraints
- Algorithms
 - Reachability analysis
 - Liveness checking
- Verification Options

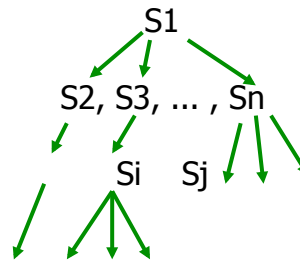


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All Operations on Zones

(needed for verification)

- Transformation
 - Conjunction
 - Post condition (delay)
 - Reset
- Consistency Checking
 - Inclusion
 - Emptiness



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Zones = Conjunctive constraints

- A zone Z is a conjunctive formula:
 $g_1 \ \& \ g_2 \ \& \ \dots \ \& \ g_n$
 where g_i may be $x_i \sim b_i$ or $x_i - x_j \sim b_{ij}$
- Use a zero-clock x_0 (constant 0), we have
 $\{x_i - x_j \sim b_{ij} \mid \sim \text{ is } < \text{ or } \leq, i, j \leq n\}$
- This can be represented as a MATRIX, DBM
 (Difference Bound Matrices)

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Datastructures for Zones in UPPAAL

- **Difference Bounded Matrices**

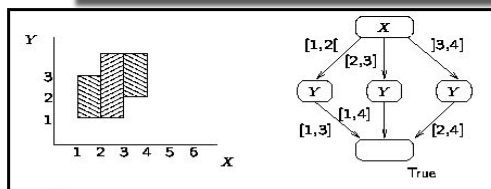
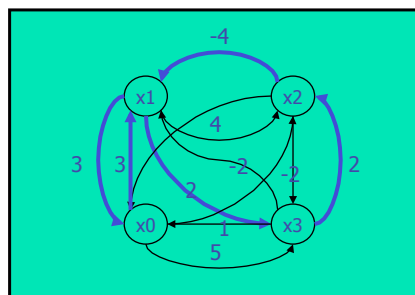
[Bellman58, Dill89]

- **Minimal Constraint Form**

[RTSS97]

- **Clock Difference Diagrams**

[CAV99]



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Canonical Datastructures for Zones

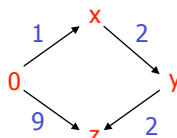
Difference Bounded Matrices Bellman 1958, Dill 1989

Inclusion

Z1

$x \leq 1$
 $y - x \leq 2$
 $z - y \leq 2$
 $z \leq 9$

Graph

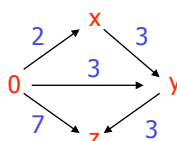


$? \subseteq ?$

Z2

$x \leq 2$
 $y - x \leq 3$
 $y \leq 3$
 $z - y \leq 3$
 $z \leq 7$

Graph



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Canonical Datastructures for Zones

Difference Bounded Matrices

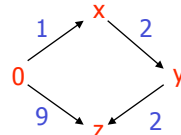
Bellman 1958, Dill 1989

Inclusion

Z1

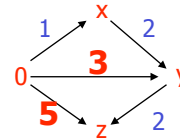
$x \leq 1$
 $y - x \leq 2$
 $z - y \leq 2$
 $z \leq 9$

Graph



$? \subseteq ?$

Shortest
Path
Closure

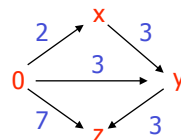


$Z1 \subseteq Z2 !$

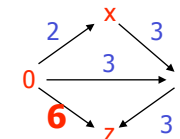
Z2

$x \leq 2$
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 $z \leq 7$

Graph



Shortest
Path
Closure



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Canonical Datastructures for Zones

Difference Bounded Matrices

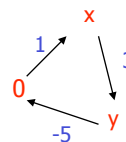
Bellman 1958, Dill 1989

Emptiness

Z

$x \leq 1$
 $y \geq 5$
 $y - x \leq 3$

Graph

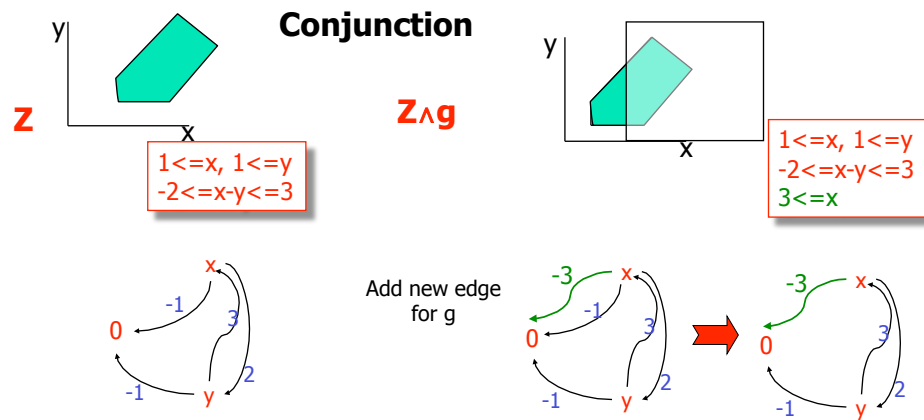


**Negative Cycle
 iff
 empty solution set**

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Canonical Datastructures for Zones

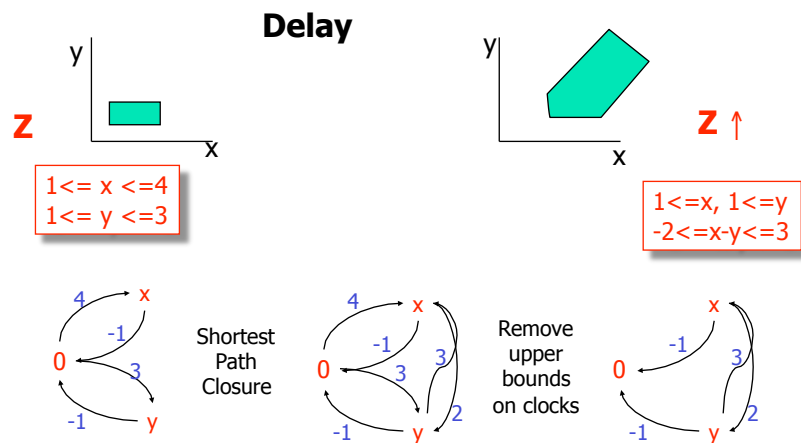
Difference Bounded Matrices



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Canonical Datastructures for Zones

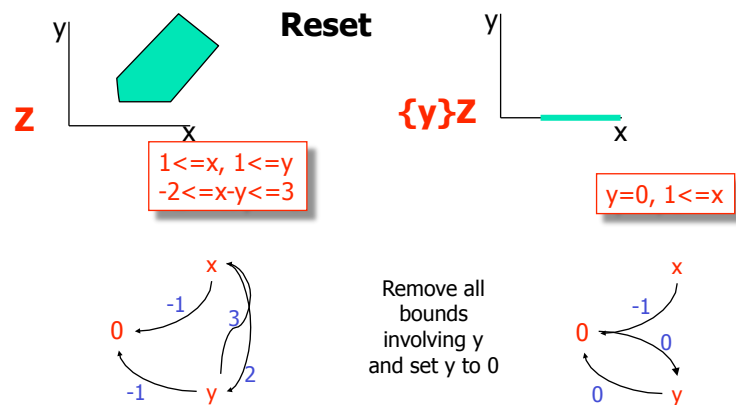
Difference Bounded Matrices



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Canonical Datastructures for Zones

Difference Bounded Matrices



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COMPLEXITY

- Computing the shortest path closure, the canonical form of a zone: $O(n^3)$ [Dijkstra's alg.]
- Run-time complexity, mostly in $O(n)$ (when we keep all zones in canonical form)

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Datastructures for Zones in UPPAAL

- **Difference Bounded Matrices**

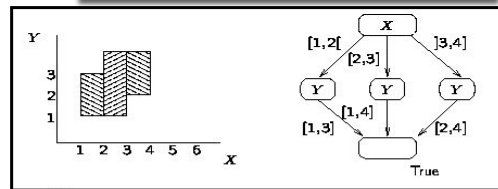
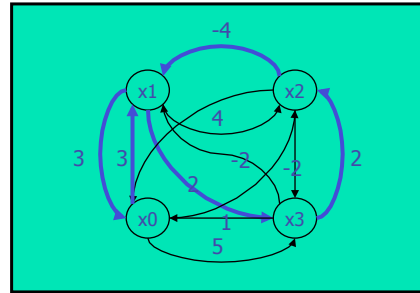
[Bellman58, Dill89]

- **Minimal Constraint Form**

[RTSS97]

- **Clock Difference Diagrams**

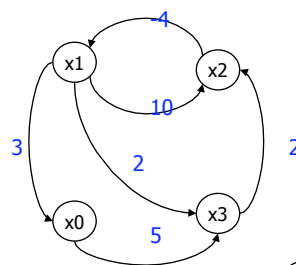
[CAV99]



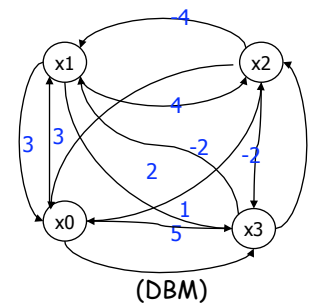
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Minimal Graph

$x_1 - x_2 \leq -4$
 $x_2 - x_1 \leq 10$
 $x_3 - x_1 \leq 2$
 $x_2 - x_3 \leq 2$
 $x_0 - x_1 \leq 3$
 $x_3 - x_0 \leq 5$

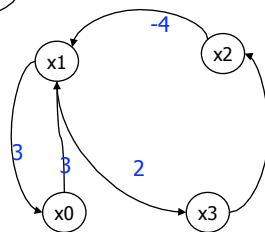


**Shortest
Path
Closure**
 $O(n^3)$



(DBM)

**Shortest
Path
Reduction**
 $O(n^3)$

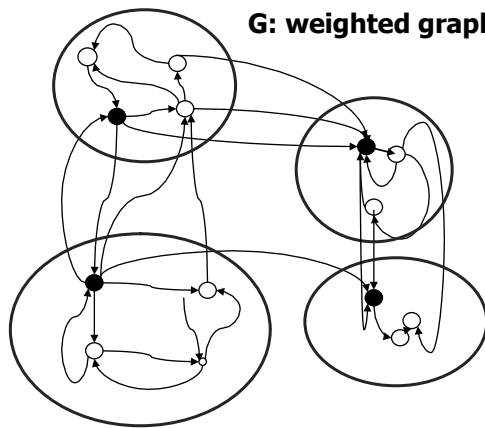


Space worst $O(n^2)$
practice $O(n)$

(Minimal graph, a.k.a.
compact data structure)

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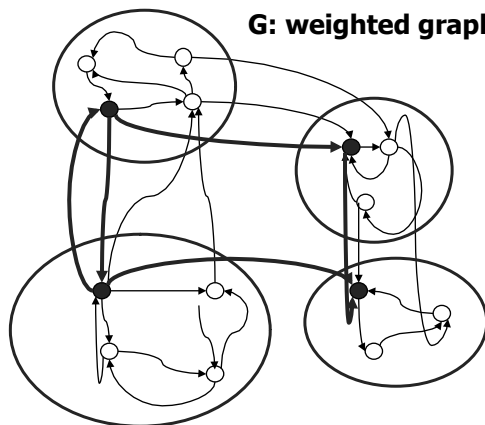
Graph Reduction Algorithm



1. Equivalence classes based on 0-cycles.

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Graph Reduction Algorithm

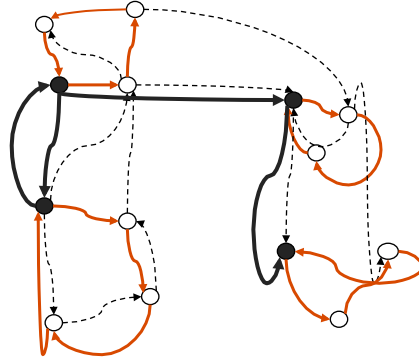


1. Equivalence classes based on 0-cycles.
2. Graph based on representatives.
Safe to remove redundant edges

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Graph Reduction Algorithm

G: weighted graph

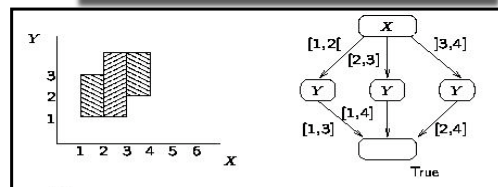
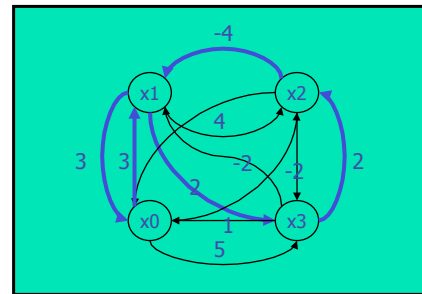


1. Equivalence classes based on 0-cycles.
2. Graph based on representatives.
Safe to remove redundant edges
3. **Shortest Path Reduction**
= One cycle pr. class
+ Removal of redundant edges between classes

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Datastructures for Zones in UPPAAL

- **Difference Bounded Matrices**
[Bellman58, Dill89]
- **Minimal Constraint Form**
[RTSS97]
- **Clock Difference Diagrams**
[CAV99]

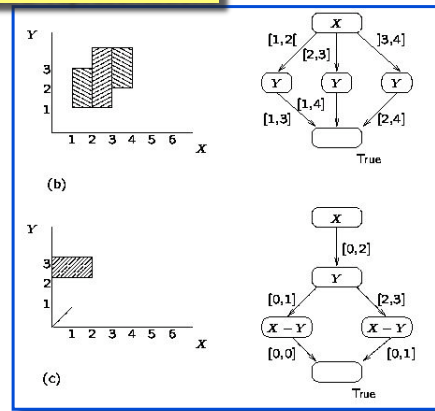


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Other Symbolic Datastructures

- NDD's Maler et. al.
- CDD's UPPAAL/CAV99
- DDD's Møller, Lichtenberg
- Polyhedra HyTech
-

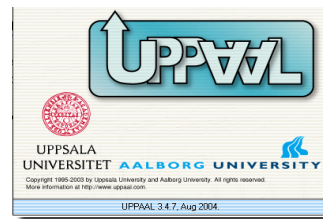
CDD-representations



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Inside the UPPAAL tool

- Data Structures
 - DBM's (Difference Bounds Matrices)
 - Canonical and Minimal Constraints
- ➔ ■ Algorithms
 - Reachability analysis
 - Liveness checking
- Verification Options



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Timed CTL in UPPAAL

EF p | AG p | EG p | AF p | p -> q

P ::= A.l | g_c | g_d | not p | p or p | p and p | p imply p

*Process
Location
(a location in
automaton A)*

*Clock
constraint*

*predicate
over data variables*

p leads to q
denotes
AG (p imply AF q)

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Timed CTL in UPPAAL

EF p | AG p | EG p | AF p | p -> q

P ::= A.l | g_c | g_d | not p | p or p | p and p | p imply p

*Process
Location
(a location in
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over data variables*

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denotes
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SAFETY PROPERTIES

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SAFETY Properties

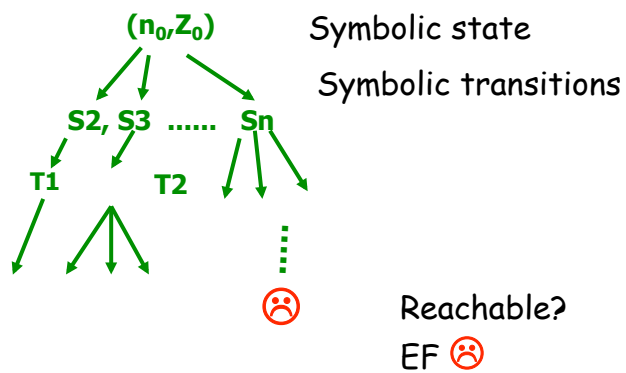
F ::= EF P | AG P

Reachability

Invariant = $\neg EF \neg P$
Thus, **AG P** is also checked by reachability analysis!

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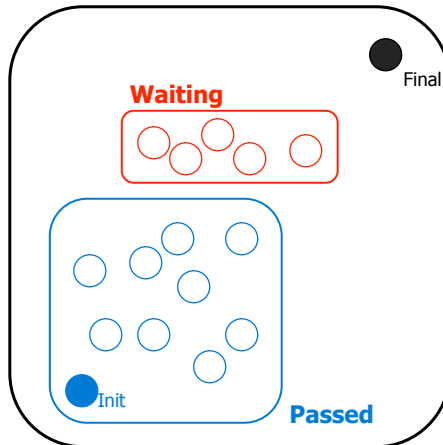
We have a search problem



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Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

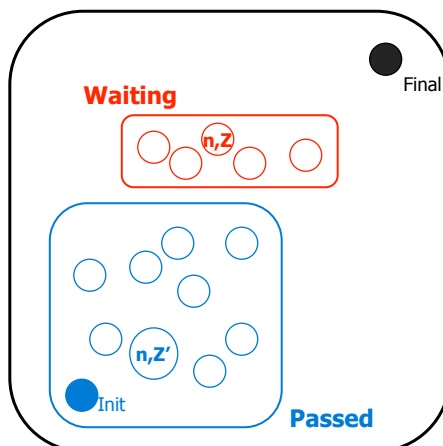
- pick (n, Z) in **Waiting**
- **if** for some $Z' \models Z$
 (n, Z') in **Passed** **then STOP**
- **else** /explore/ add
 $\{ (m, U) : (n, Z) \Rightarrow (m, U) \}$
to **Waiting**;
Add (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

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Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
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REPEAT

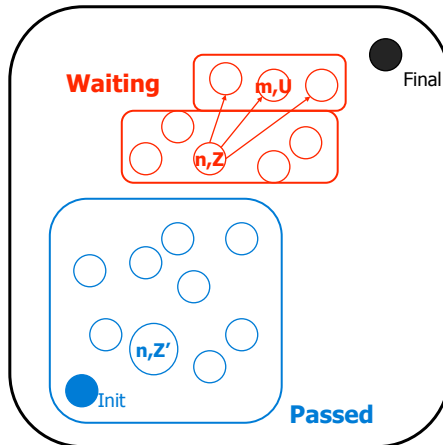
- pick (n, Z) in **Waiting**
- **if** for some $Z' \models Z$
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- **else** (explore) add
 $\{ (m, U) : (n, Z) \Rightarrow (m, U) \}$
to **Waiting**;
Add (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

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Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
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REPEAT

- pick (n,Z) in **Waiting**

- if for some $Z' \models Z$

(n,Z') in **Passed** then **STOP**

- else /explore/ add
 $\{ (m,U) : (n,Z) \Rightarrow (m,U) \}$
to **Waiting**;

Add (n,Z) to **Passed**

UNTIL **Waiting** = \emptyset

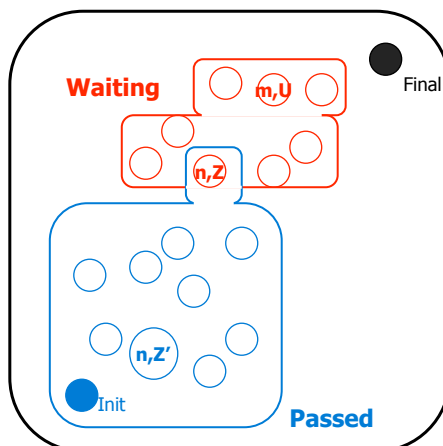
or

Final is in **Waiting**

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Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n0,Z0)\}$

REPEAT

- pick (n,Z) in **Waiting**

- if for some $Z' \models Z$

(n,Z') in **Passed** then **STOP**

- else /explore/ add
 $\{ (m,U) : (n,Z) \Rightarrow (m,U) \}$
to **Waiting**;

Add (n,Z) to **Passed**

UNTIL **Waiting** = \emptyset

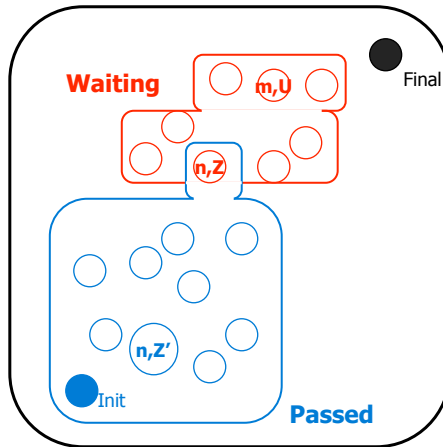
or

Final is in **Waiting**

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Forward Reachability

Init -> **Final** ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

- pick (n, Z) in **Waiting**
- if for some $Z' \preceq Z$
 (n, Z') in **Passed** then **STOP**
- else /explore/ add
 $\{ (m, U) : (n, Z) \Rightarrow (m, U) \}$
to **Waiting**;
Add (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset
or
Final is in **Waiting**

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Further question

Can we find the path with **shortest delay**, leading to **P** ?
(i.e. a state satisfying **P**)

OBSERVATION:

Many scheduling problems can be phrased naturally as
reachability problems for timed automata.

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Verification vs. Optimization

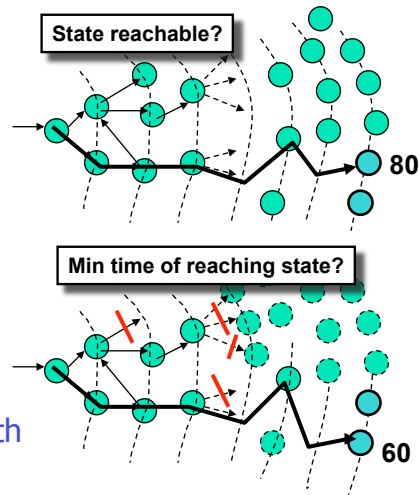
- **Verification Algorithms:**

- Checks a logical property of the entire state-space of a model.
- Efficient Blind search.

- **Optimization Algorithms:**

- Finds (near) optimal solutions.
- Uses techniques to avoid non-optimal parts of the state-space (e.g. Branch and Bound).

- **Goal:** solve opt. problems with verification.



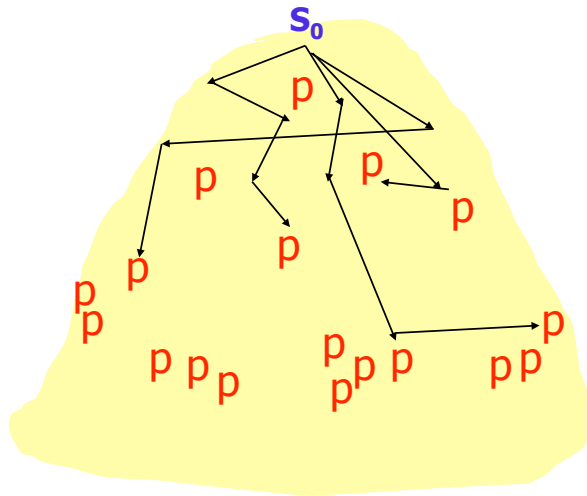
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OPTIMAL REACHABILITY

The maximal and minimal delay problem

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Find the trace leading to P with **min** delay

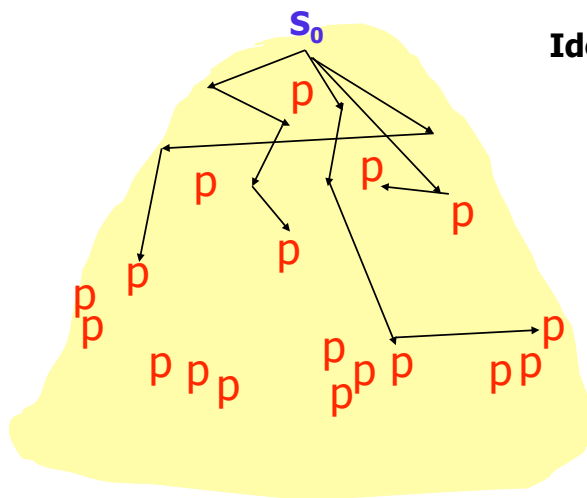


There may be a lot of pathes leading to P

Which one with the shortest delay?

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Find the trace leading to P with **min** delay

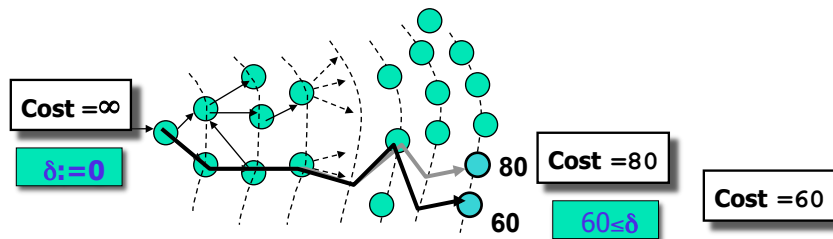


Idea: delay as "**Cost**" to reach a state, thus **cost** increases with time at rate 1

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An Simple Algorithm for minimal-cost reachability

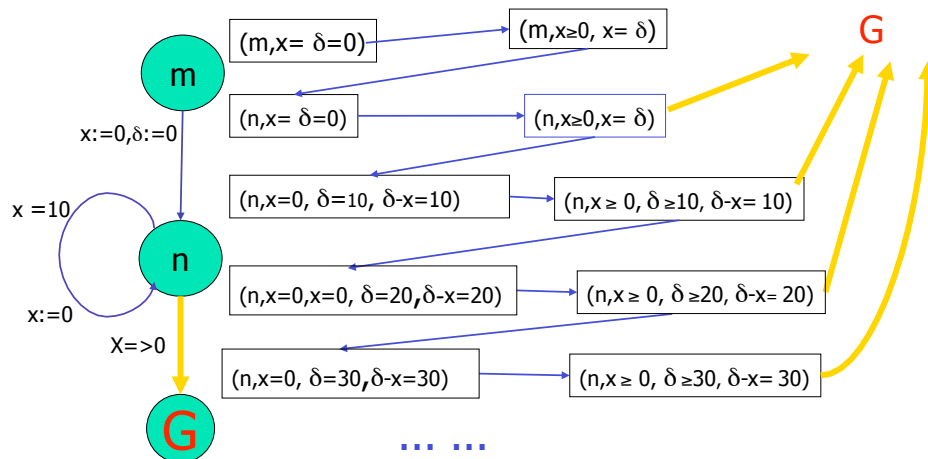
- State-Space Exploration + Use of global variable **Cost** and global clock δ
- Update **Cost** whenever goal state with $\min(\mathbf{C}) < \mathbf{Cost}$ is found:



- Terminates when entire state-space is explored.
- Problem:** The search may never terminate!

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Example (min delay to reach **G**)



The minimal **delay** = 0 but the search may never terminate!
Problem: How to **symbolically** represent the zone **C**.

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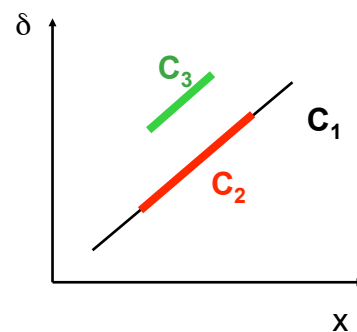
Priced-Zone

- Cost = minimal total time
- \mathbf{C} can be represented as the zone Z^δ , where:
 - Z^δ original (ordinary) DBM plus...
 - δ clock keeping track of the cost/time.
- Delay, Reset, Conjunction etc. on Z are the standard DBM-operations
- Delay-Cost is incremented by Delay-operation on Z^δ .

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Priced-Zone

- Cost = min total time
- \mathbf{C} can be represented as the zone Z^δ , where:
 - Z^δ is the original zone Z extended with the global clock δ keeping track of the cost/time.
 - Delay, Reset, Conjunction etc. on C are the standard DBM-operations
- But inclusion-checking will be different



Then: $C_3 \subseteq C_2 \subseteq C_1$
 But: $C_3 \not\subseteq C_2 \subseteq C_1$

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Solution: $()^+$ -widening operation

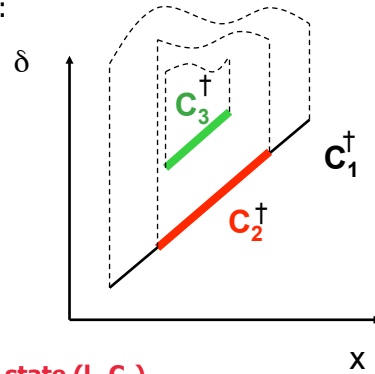
- $()^+$ removes upper bound on the δ -clock:

$$\begin{aligned} \mathbf{C}_3 &\subseteq \mathbf{C}_2 \subseteq \mathbf{C}_1 \\ \mathbf{C}_3^+ &\subseteq \mathbf{C}_2^+ \subseteq \mathbf{C}_1^+ \end{aligned}$$

- In the Algorithm:

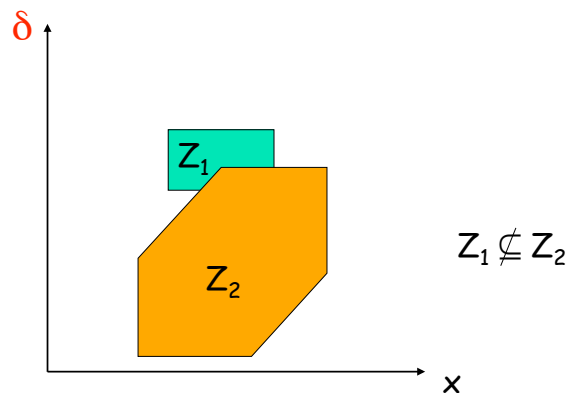
- $\text{Delay}(C^+) = (\text{Delay}(C^+))^+$
- $\text{Reset}(x, C^+) = (\text{Reset}(x, C^+))^+$
- $C_1^+ \wedge g = (C_1^+ \wedge g)^+$

- It suffices to apply $()^+$ to the initial state (I_0, C_0) .



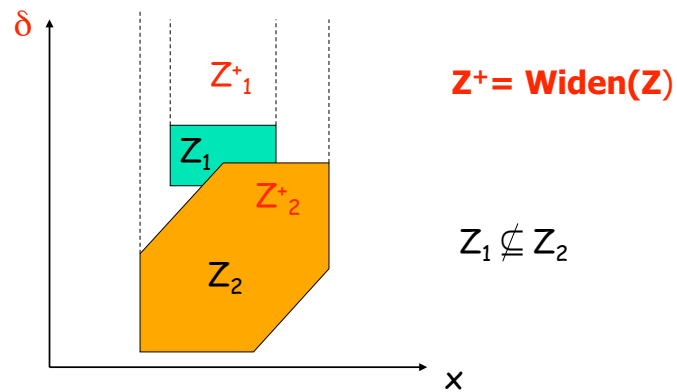
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Example (widening for Min)



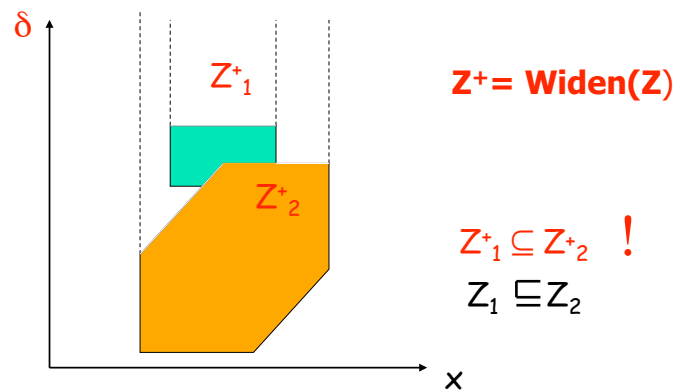
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Example (widening for Min)



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Example (widening for Min)



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An Algorithm (Min)

```

Cost:=∞, Pass := {}, Wait := {(l0,C0)}
while Wait ≠ {} do
  select (l,C) from Wait
  if (l,C) ⊨ P and Min(C)<Cost then Cost:= Min(C)
  if (l,C) ⊆ (l,C') for some (l,C') in Pass then skip
  otherwise add (l,C) to Pass
  and forall (m,C') such that (l,C) → (m,C') :
    add (m,C') to Wait
Return Cost

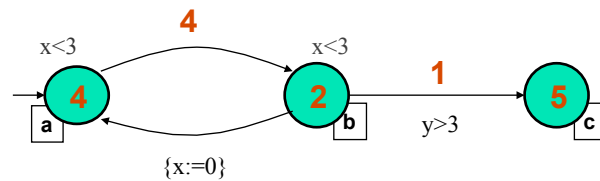
```

One-step reachability relation

Output: Cost = the min cost of a found trace satisfying P .

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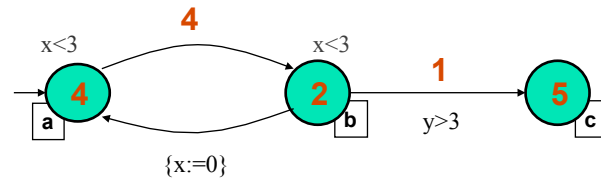
Further reading: **Priced** Timed Automata [Larsen et al]



- Timed Automata + Costs on transitions and locations.
- Uniformly Priced = Same cost in all locations (edges may have different costs).
- Cost of performing transition: Transition cost.
- Cost of performing delay d : ($d \times$ location cost).

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Priced Timed Automata



Trace:

$(a, x=y=0) \xrightarrow{4} (b, x=y=0) \xrightarrow[2.5 \times 2]{\varepsilon(2.5)} (b, x=y=2.5) \xrightarrow{0} (a, x=0, y=2.5)$

Cost of Execution Trace:

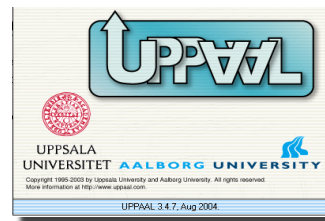
Sum of costs: $4 + 5 + 0 = 9$

Problem: Finding the minimum cost of reaching **c** !

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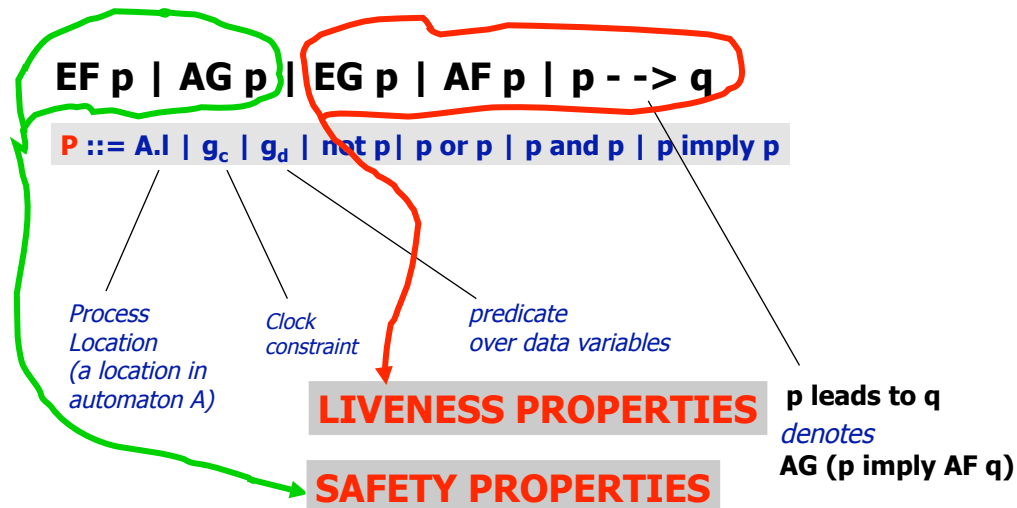
Inside the UPPAAL tool

- Data Structures
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Timed CTL in UPPAAL



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LIVENESS Properties

in UPPAAL

$F ::= EG\ p \mid AF\ p \mid p \rightarrow q$

Possibly always P
is equivalent to $(: AF : P)$

Eventually P
is equivalent to $(: EG : P)$

P leads to Q
is equivalent to $AG\ (P \text{ imply } AF\ Q)$

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Algorithm for checking **AF P** **Eventually P**

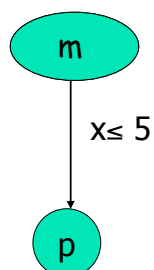
Bouajjani, Tripakis, Yovine'97
On-the-fly symbolic model checking of TCTL

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Question

AF P

"P will be *true for sure* in future"



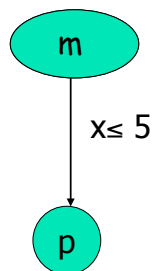
?? Does this automaton satisfy **AF P**

54

Note that

AF P

"P will be true for sure in future"



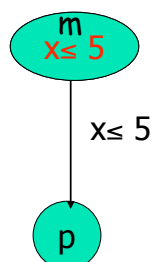
NO !!!!! there is a path:
 (m, x=0) → (m, x=1) → (m, 2) ... (m, x=k) ...
 Idling forever in location m

55

Note that

AF P

"P will be true for sure in future"



This automaton satisfies AF P

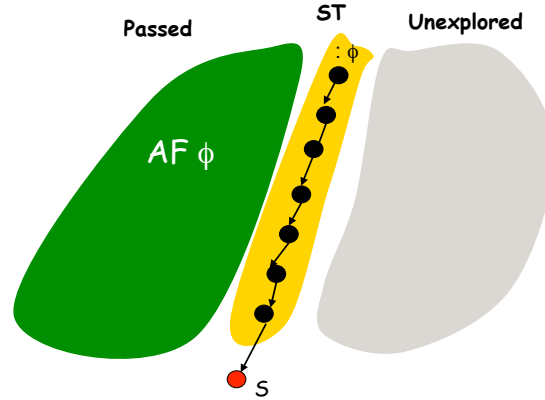
56

Liveness Algorithm

Bouajjani, Tripakis, Yovine, 97

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end
proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $\bar{S} := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```

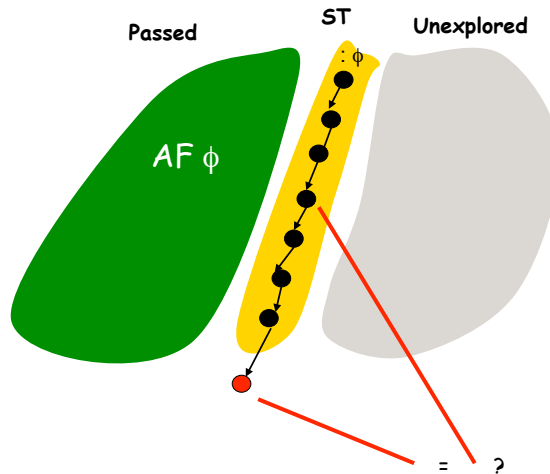


57

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
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  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
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    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```



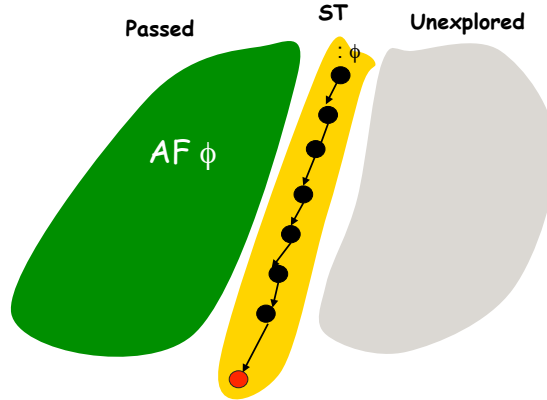
58

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end

proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $\bar{S} := S \wedge \neg\varphi$ 
  • push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```



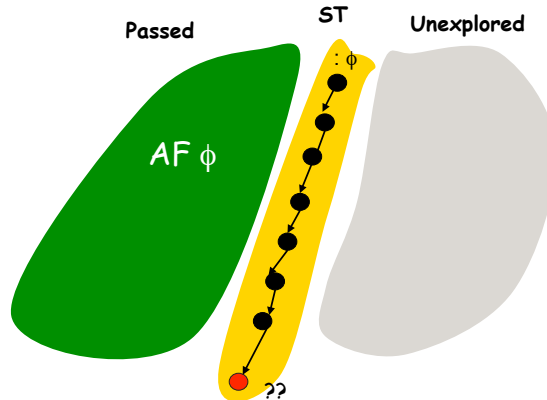
59

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end

proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $\bar{S} := S \wedge \neg\varphi$ 
  • push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
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  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```

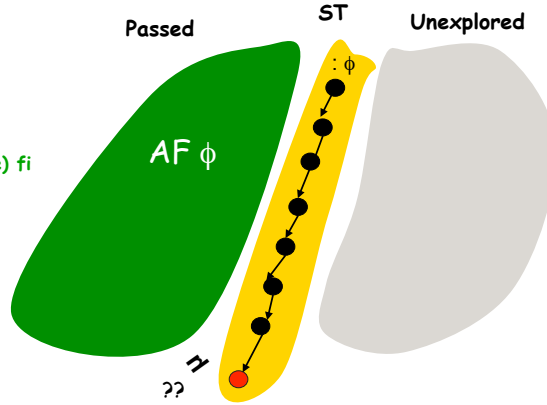


60

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end
proc Search( $S$ )  $\equiv$  if empty( $S$ ) then exit(true) fi
  if loop( $S, ST$ ) then exit(false) fi
   $\bar{S} := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  • if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```

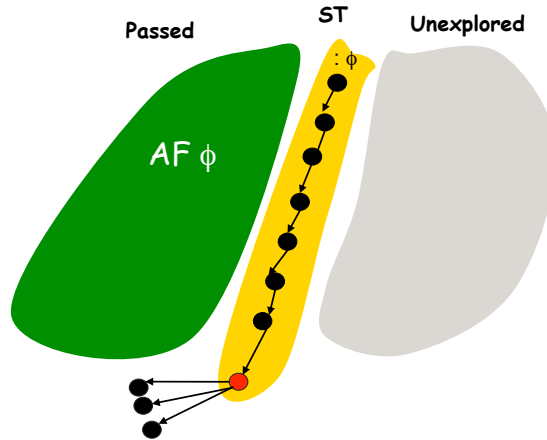


61

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end
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   $\bar{S} := S \wedge \neg\varphi$ 
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    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end
  
```



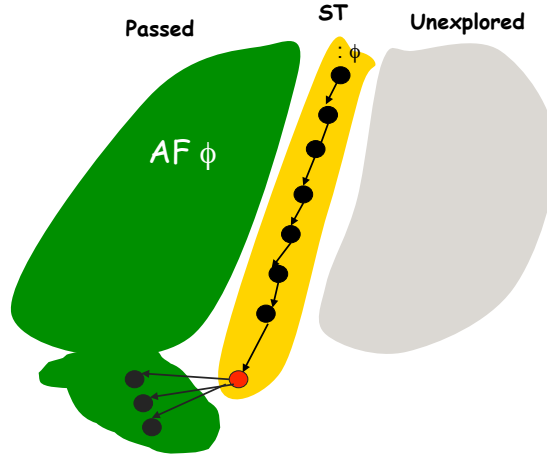
62

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end
proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $\bar{S} := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end

```



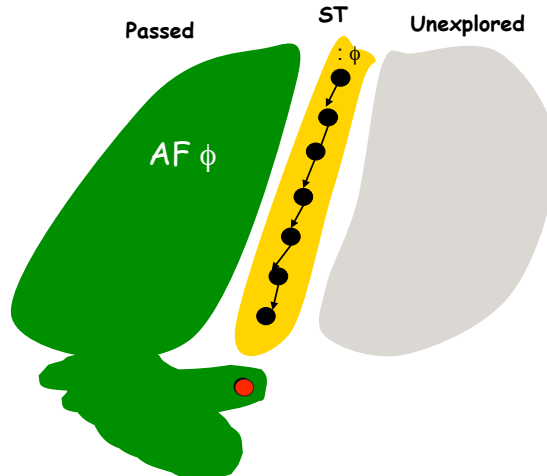
63

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
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  Search(delay( $S_0, \neg\varphi$ ))
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  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
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  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
   $Passed := Passed \cup \{pop(ST)\}$ 
end

```



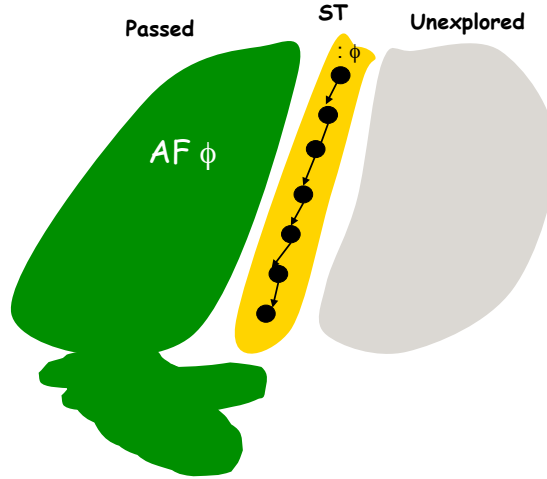
64

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
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   $\bar{S} := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
  ●  $Passed := Passed \cup \{pop(ST)\}$ 
end

```



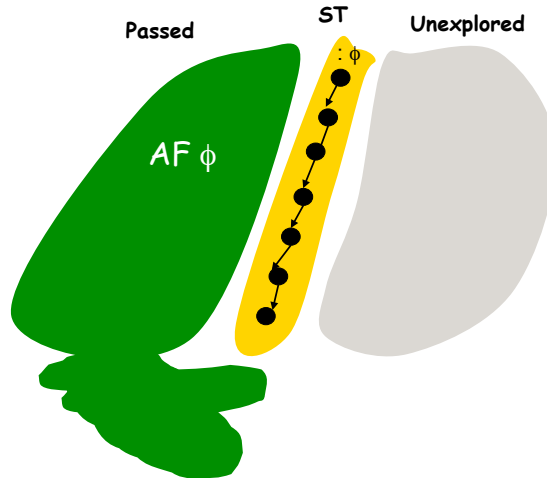
65

Liveness Algorithm

```

proc Eventually( $S_0, \varphi$ )  $\equiv$ 
   $ST := \emptyset$ 
   $Passed := \emptyset$ 
  Search(delay( $S_0, \neg\varphi$ ))
  exit(true)
end
proc Search( $S$ )  $\equiv$ 
  if loop( $S, ST$ ) then exit(false) fi
   $\bar{S} := S \wedge \neg\varphi$ 
  push( $ST, S$ )
  if unbounded( $S$ )  $\vee$  deadlocked( $S$ ) then
    exit(false) fi
  if  $\forall S' \in Passed : S \not\sqsubseteq S'$ 
    then foreach  $S' : S \xrightarrow{a} S'$  do
      Search(delay( $S', \neg\varphi$ ))
    od
  fi
  ●  $Passed := Passed \cup \{pop(ST)\}$ 
end

```



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Question: Time bound synthesis

$AF P$ "P will be true eventually"
But no time bound is given.

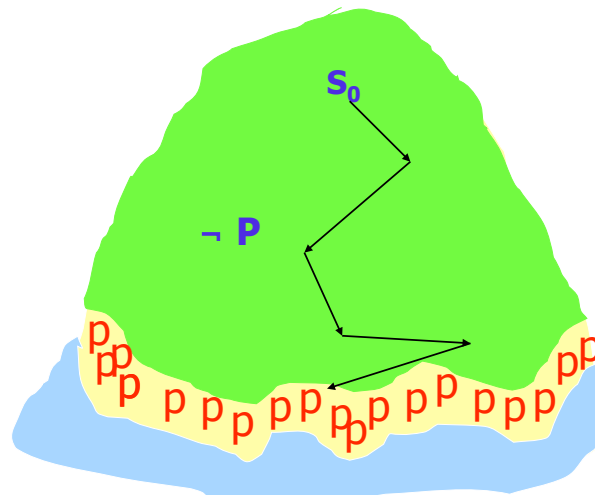
Assume $AF P$ is satisfied by an automaton A.
Can we calculate the **Max** time bound?

OBS: we know how to calculate the **Min** !

67

Assume $AF P$ is satisfied

Find the trace leading to P with the **max** delay



Almost the same algorithm as for synthesizing **Min**

We need to explore the **Green** part

68

An Algorithm (Max)

```

Cost:=0, Pass := {}, Wait := {(l0,C0)}
while Wait ≠ {} do
  select (l,C) from Wait
  if (l,C) ⊨ P and Max(C)>Cost then Cost:= Max(C)
  else if forall (l,C') in Pass: C ⊈ C' then
    add (l,C) to Pass
    forall (m,C') such that (l,C)  $\xrightarrow{\quad}$  (m,C') :
      add (m,C') to Wait
Return Cost
  
```

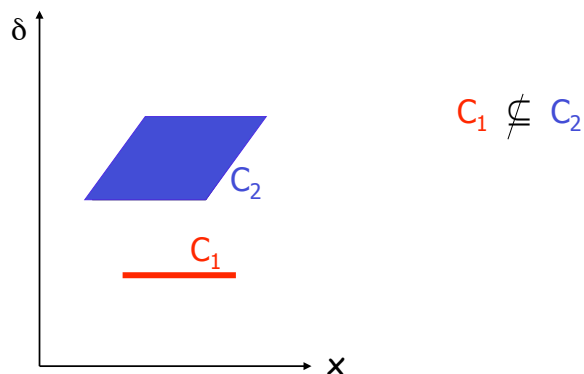
One-step reachability relation

Output: Cost = the min cost of a found trace satisfying P .

BUT: \sqsubseteq is defined on zones where the lower bound of "cost" is removed

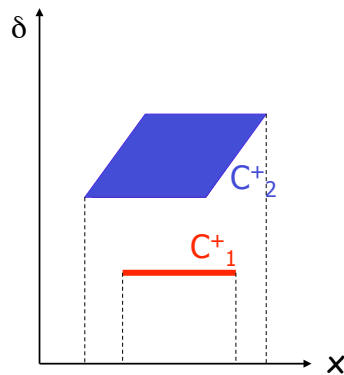
69

Zone-Widening operation for Max



70

Zone-Widening operation for Max



$$C_1 \not\subseteq C_2$$

$$C_1^+ \subseteq C_2^+$$

$$C_1 \sqsubseteq C_2 !$$

71

Inside the UPPAAL tool

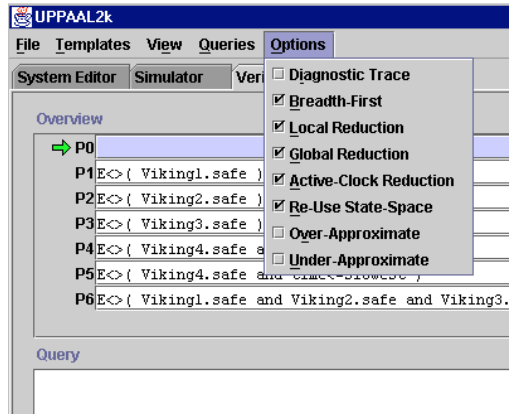
- Data Structures
 - DBM's (Difference Bounds Matrices)
 - Canonical and Minimal Constraints
- Algorithms
 - Reachability analysis
 - Liveness checking
 - Termination

➡ Verification Options



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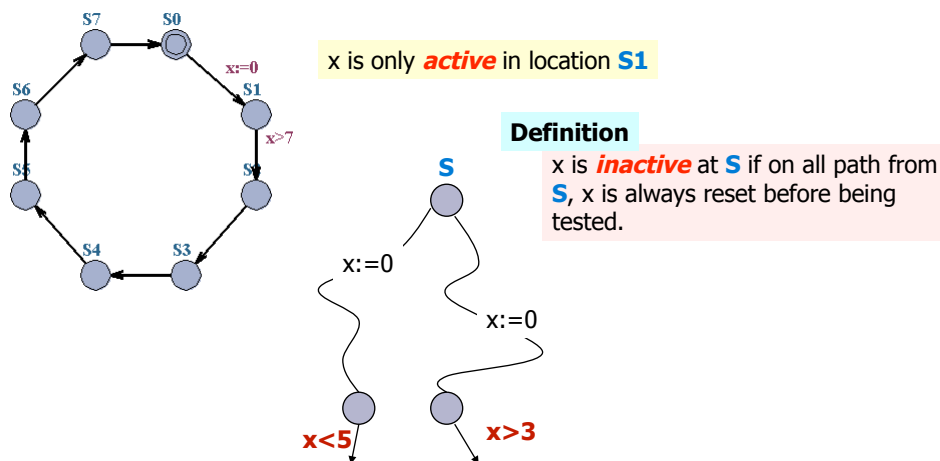
Verification Options



- Diagnostic Trace
- Breadth-First
- Depth-First
- Local Reduction
- Active-Clock Reduction
- Global Reduction
- Re-Use State-Space
- Over-Approximation
- Under-Approximation

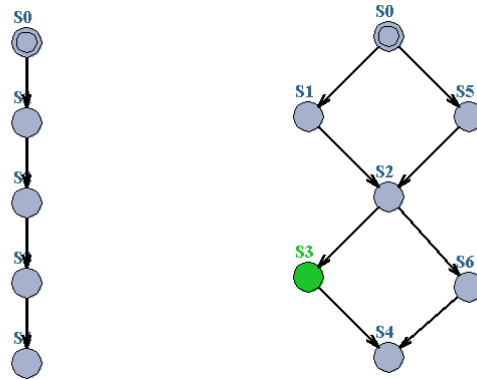
73

Inactive (passive) Clock Reduction



74

Global Reduction (When to store symbolic state)

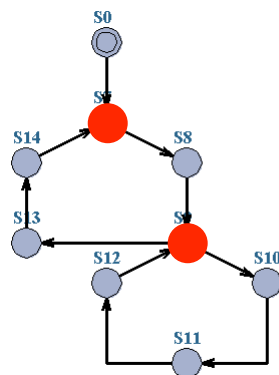


However,
Passed list useful for
efficiency

No Cycles: **Passed** list not needed for *termination*

75

Global Reduction [RTSS97] (When to store symbolic state)



Cycles:
Only symbolic states
involving loop-entry points
need to be saved on **Passed** list

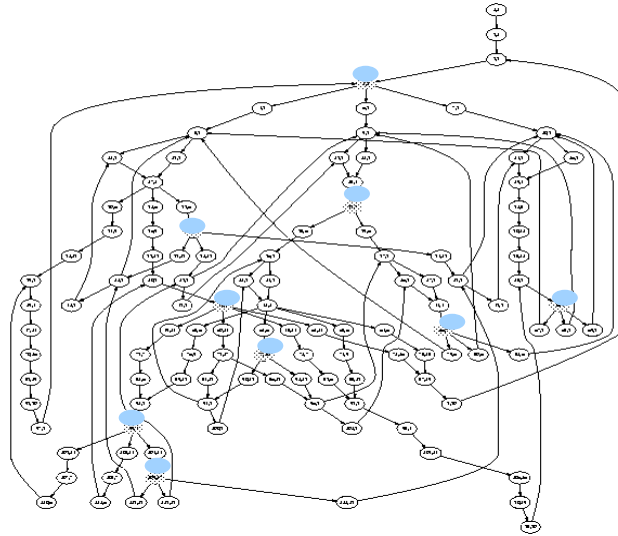
76

To Store Or Not To Store?

117 states_{total}
 ↓
 81 states_{entrypoint}
 ↓
 9 states

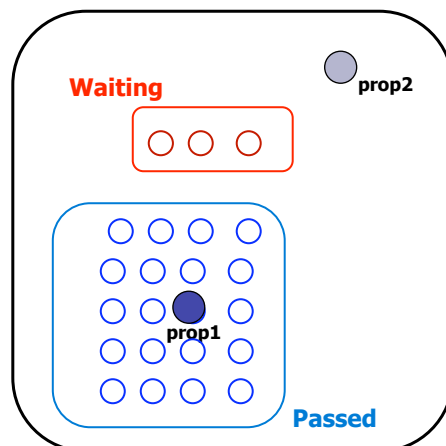
Time OH
 less than 10%

(need to
 re-explore
 some states)



7

Reuse of State Space



A[] prop1

A[] prop2

A[] prop3

A[] prop4

A[] prop5

.

.

.

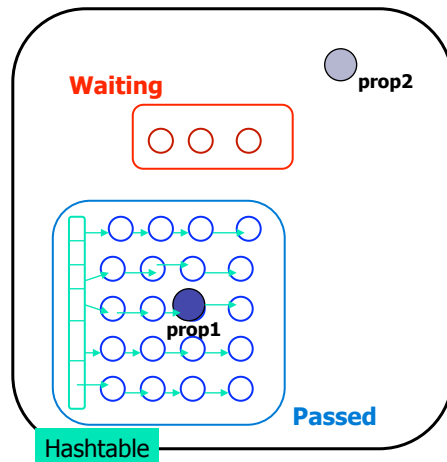
A[] propn

Search
 in existing
Passed
 list before
 continuing
 search

Which order
 to search?

78

Reuse of State Space



A[] prop1

A[] prop2

A[] prop3

A[] prop4

A[] prop5

.

.

.

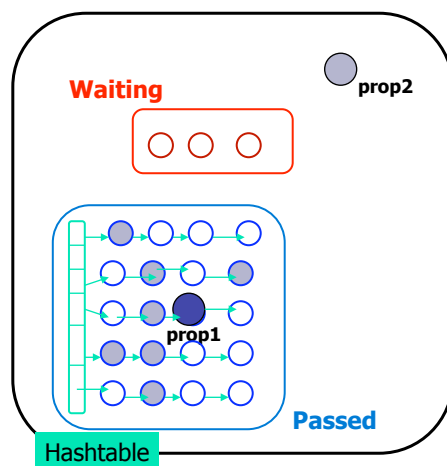
A[] propn

Search
in existing
Passed
list before
continuing
search

Which order
to search?

79

Reuse of State Space



A[] prop1

A[] prop2

A[] prop3

A[] prop4

A[] prop5

.

.

.

A[] propn

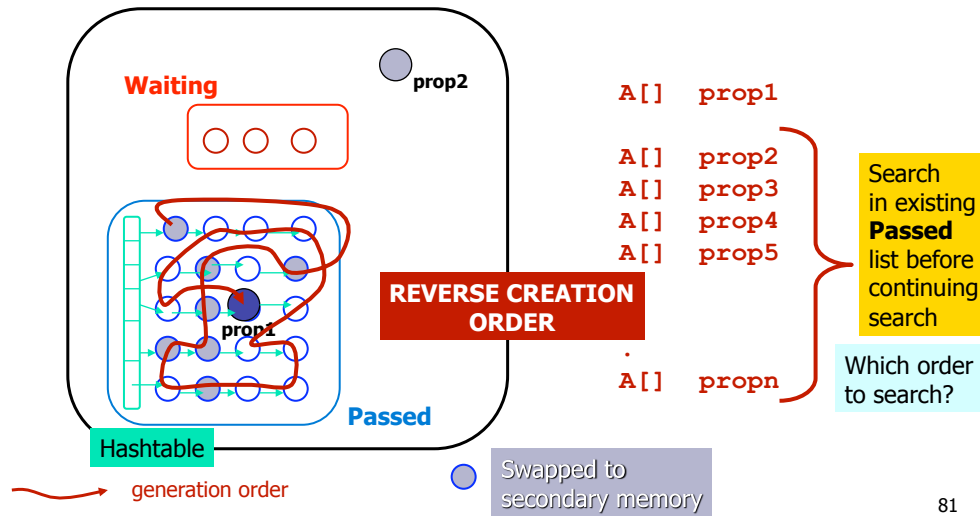
Search
in existing
Passed
list before
continuing
search

Which order
to search?

Swapped to
secondary memory

80

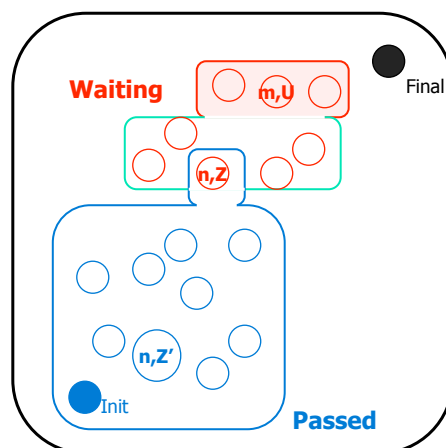
Reuse of State Space



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Under-approximation

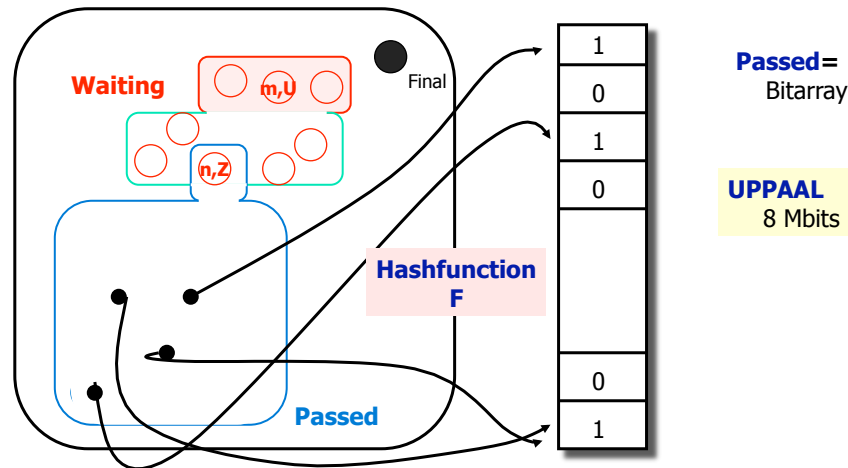
Bitstate Hashing (Holzman, SPIN)



82

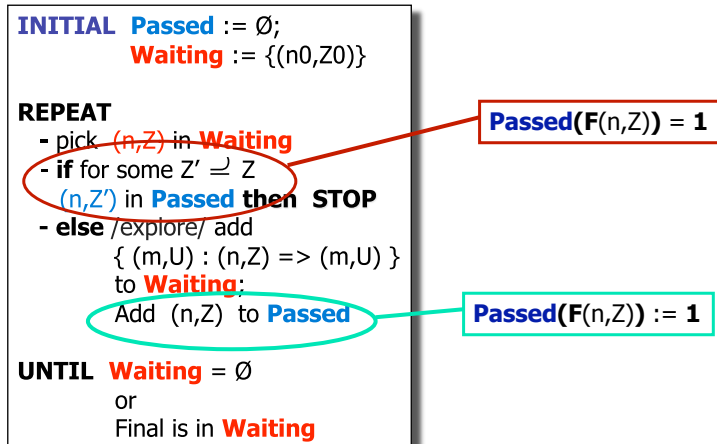
Under-approximation

Bitstate Hashing



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Bit-state Hashing



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Under Approximation

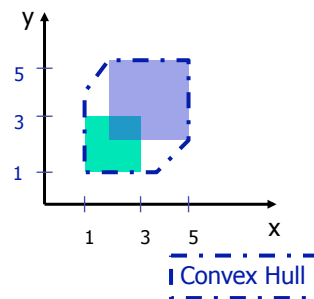
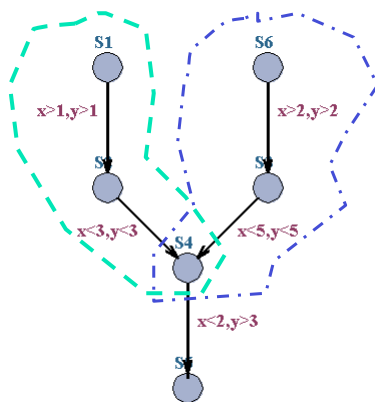
(good for finding Bugs quickly, debugging)

- Positive answer is safe (you can trust)
 - You can trust your tool if it tells:
a state is reachable (it means Reachable!)
- Negative answer is Inconclusive
 - You should not trust your tool if it tells:
a state is non-reachable
 - Some of the branch may be terminated by
conflict (the same hashing value of two states)

85

Over-approximation

Convex Hull



86

Over-Approximation

(good for safety property-checking)

- Positive answer is Inconclusive
 - a state is reachable means Nothing
(you should not trust your tool when it says so)
 - Some of the transitions may be enabled by
Enlarged zones
- Negative answer is safe
 - a state is not reachable means Non-reachable
(you can trust your tool when it says so)

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