Untangling Cycles for Contour Grouping

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Finding Salient Contours by Grouping Edges

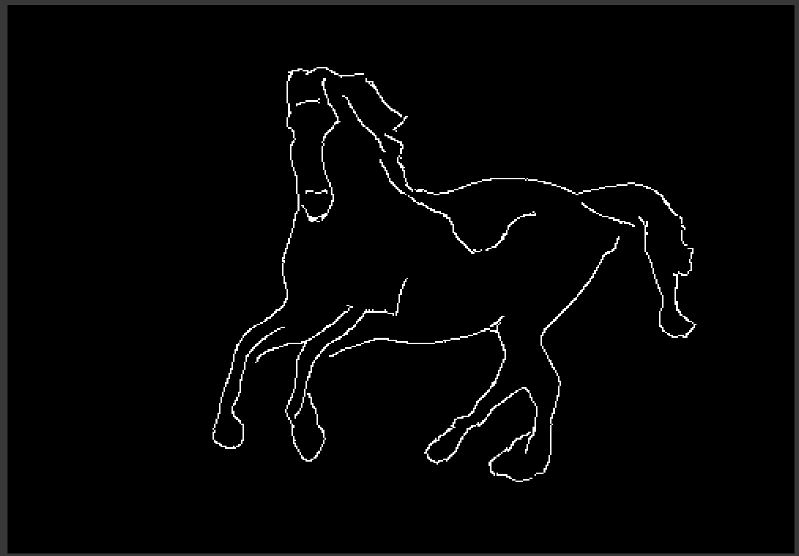
Edge
Detection

Input image

Edgels

Finding Salient Contours by Grouping Edges

Contour
Grouping



edgels

contours

"Long contours are nice to look at",
K. Koffka. *Principles of Gestalt Psychology*.

S. Ullman and A. Shashua. Structural saliency: The detection of globally salient structures using a locally connected network. In *MIT AI Memo*, 1988.

S. Mahamud, L.Williams, K. Thornber, and K. Xu. Segmentation of multiple salient closed contours from real images. *PAMI*, 2003.

T. D. Alter and R. Basri. Extracting salient curves from images:An analysis of the saliency network. *CVPR*, 1996.

A. Amir and M. Lindenbaum. Grouping-based nonadditive verification. *PAMI*, 20(2):186–192, 1998.

B. Fischer and J. M. Buhmann. Path-based clustering for grouping of smooth curves and texture segmentation. *PAMI*, 25(4):513–518, 2003.

D. W. Jacobs. Robust and efficient detection of salient convex groups. *PAMI*, 18(1):23–37, 1996.

G. G. Medioni and G. Guy. Inferring global perceptual contours from local features. In *IUW*, 1993.

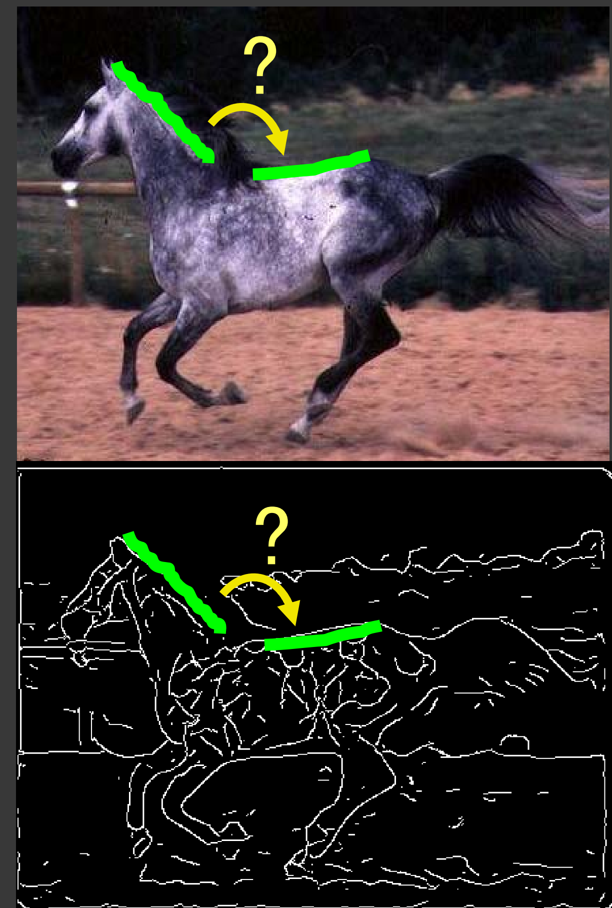
S. Sarkar and P. Soundararajan. Supervised learning of large perceptual organization: Graph spectral partitioning and learning automata. *PAMI*, 22(5):504–525, 2000.

J. H. Elder and S. W. Zucker. Computing contour closure. *Lecture Notes in Computer Science*, 1064, 1996.

Challenges in Real Images

Edge linking fails in clutter:

2D clutter



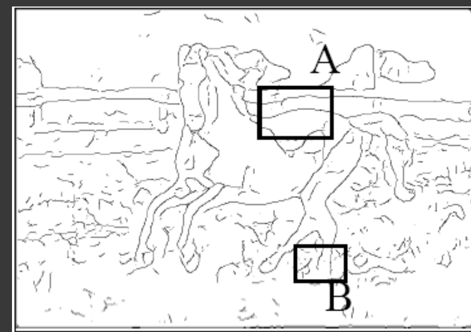
Gap

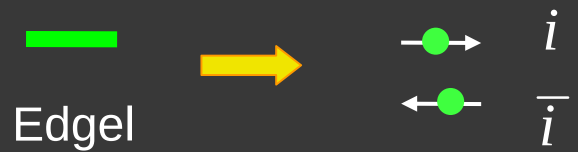
Our Goal

Group salient 1D structures robust to 2D clutter

Image | Edges and detected contours | Image | Edges and detected contours

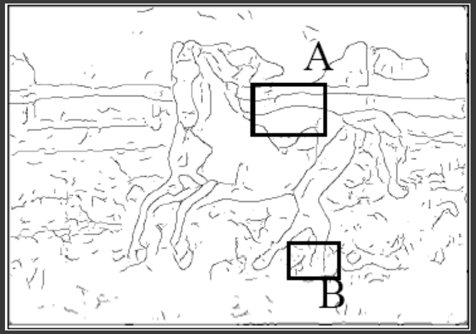
Directed Graph for Grouping *G=(V,E,W)*

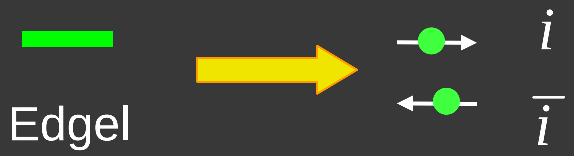




- *V* Duplicate each edgel to $(i,\bar{i})$
- *W* Collinearity
  - Elastic energy

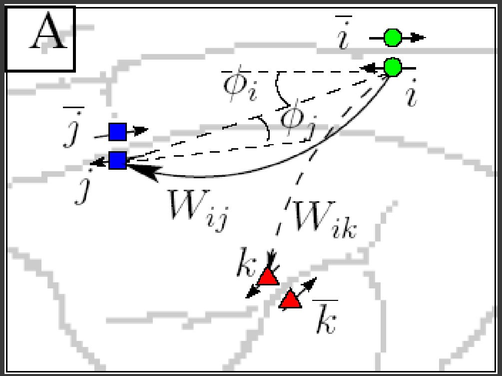
Directed Graph for Grouping *G=(V,E,W)*



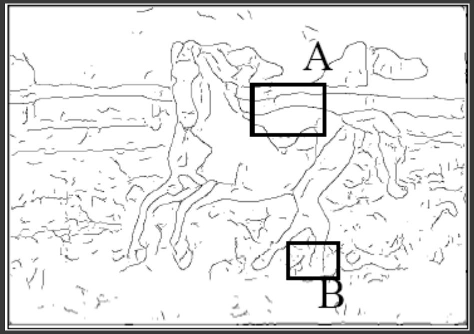


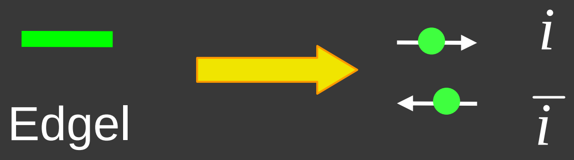
- *V* Duplicate each edgel to $(i, \bar{i})$
- *W* Collinearity
  - Elastic energy

$$W_{ij} = e^{-(1-\cos(|\phi_i|+|\phi_j|))/\sigma^2}$$



Directed Graph for Grouping $G=(V,E,W)$



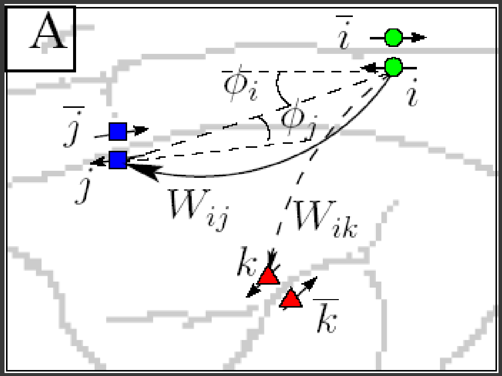


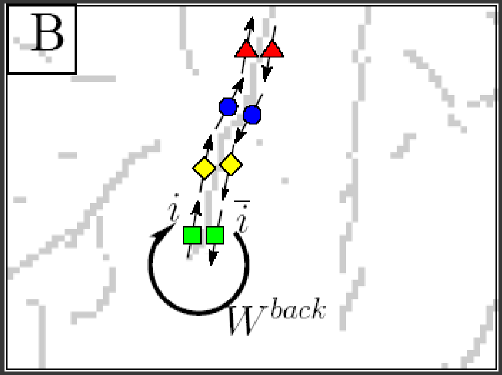
- V Duplicate each edgel to $(i, \bar{i})$
- W Collinearity
  - Elastic energy

$$W_{ij} = e^{-(1-\cos(|\phi_i|+|\phi_j|))/\sigma^2}$$

  - Backward connection W_{ij}^{back}

    open contour: chain → graph cycle

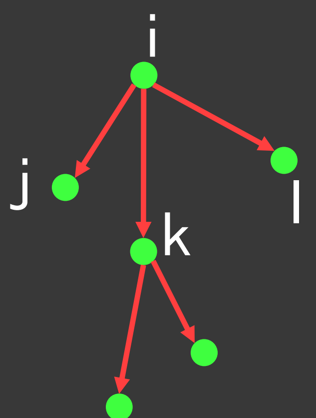


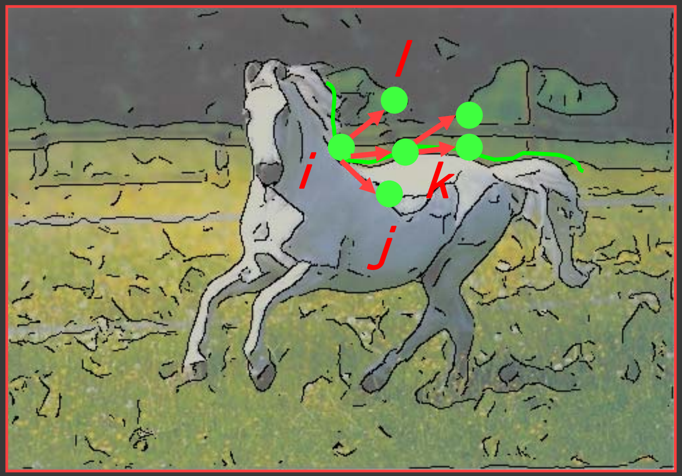


Directed Random Walk

$$P = D^{-1}W$$

$$D = Diag(W \cdot \mathbf{1})$$





$$P_1(j\,|\,i) + P_1(k\,|\,i) + P_1(l\,|\,i) = 1$$

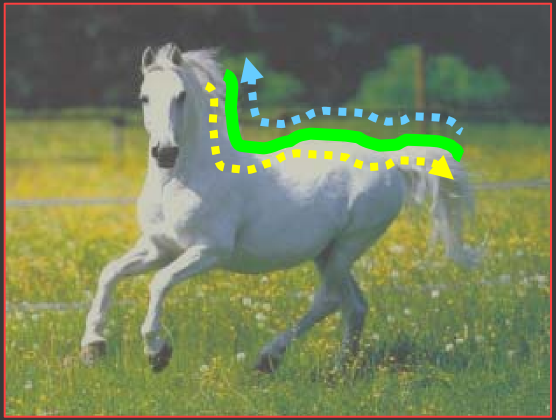
$P_1(j\,|\,i)$ probability of jumping from i to j in one step

Directed Random Walk

$$P = D^{-1}W$$

$$D = Diag(W \cdot \mathbf{1})$$

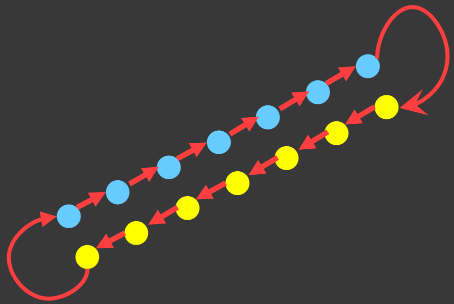
Image contour

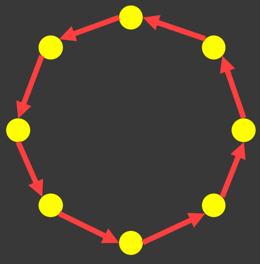


Graph cycle

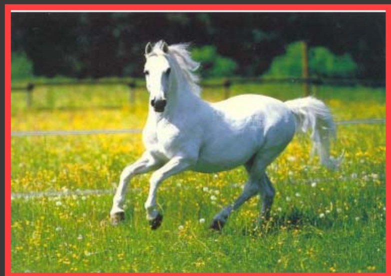
Open contour

Closed contour





Untangling Cycle Algorithm

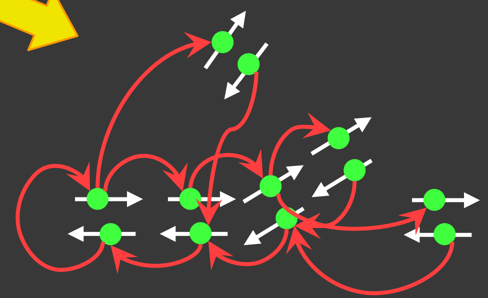


Input image

① Edge detection

Edgels

② Construct *G*



③ Find salient graph cycles

Contour Saliency

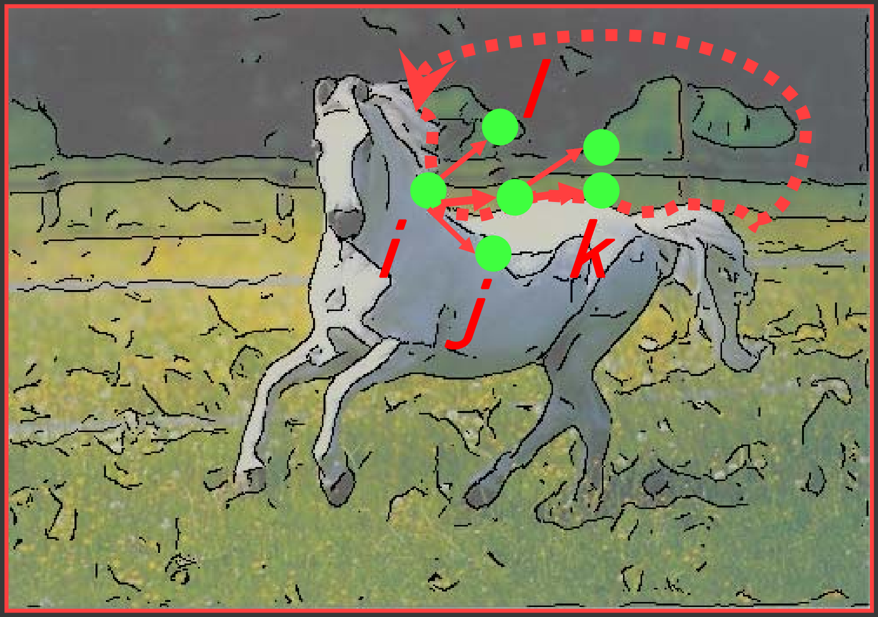
Q: What is the appropriate saliency measure for good cycles (1D contour) and bad cycles (2D clutter)?

Shortest cycle? Longest cycle? Shortest average cycle? …

Persistency of a Random Walk Cycle

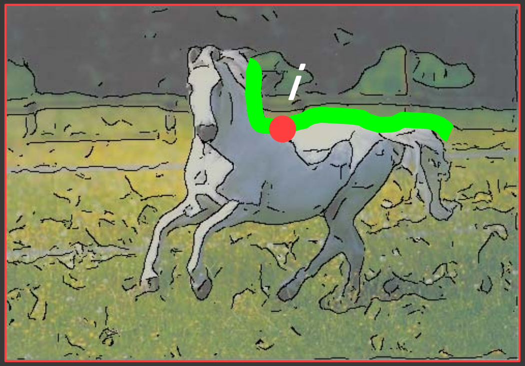
$P_t(i\,|\,i)$ probability of cycling $i \rightarrow i$ in t steps

Check how likely a **random walk cycle** back to starting points after some time t

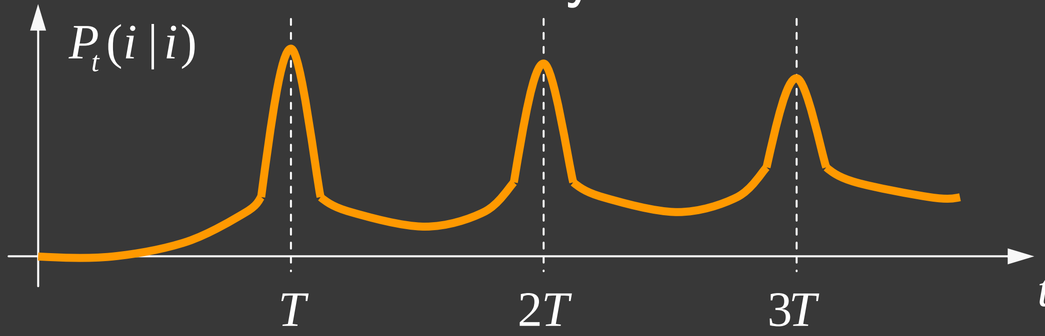


Observation: Persistent Cycles

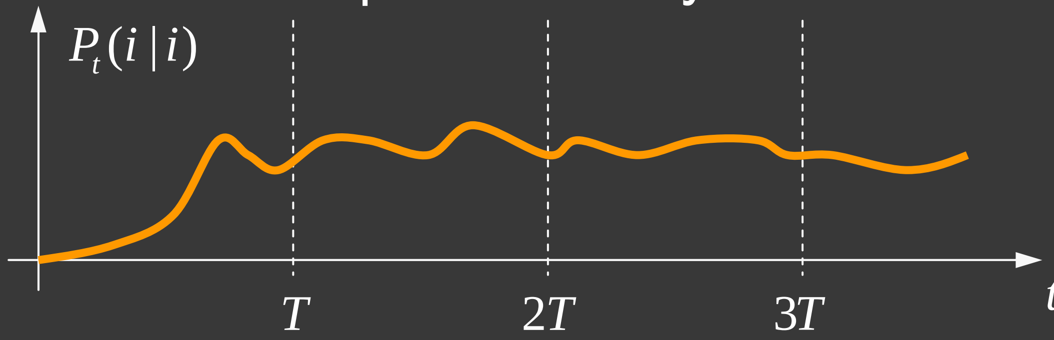
Image contour = Persistent cycles



Persistent cycle



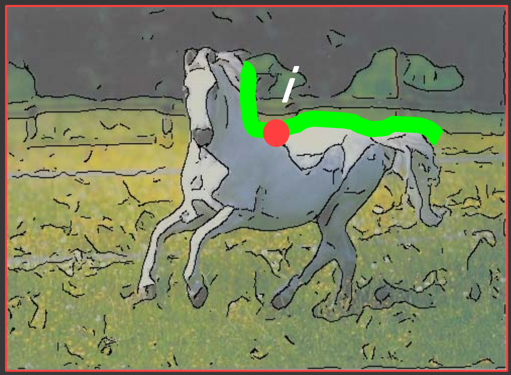
Non-persistent cycle

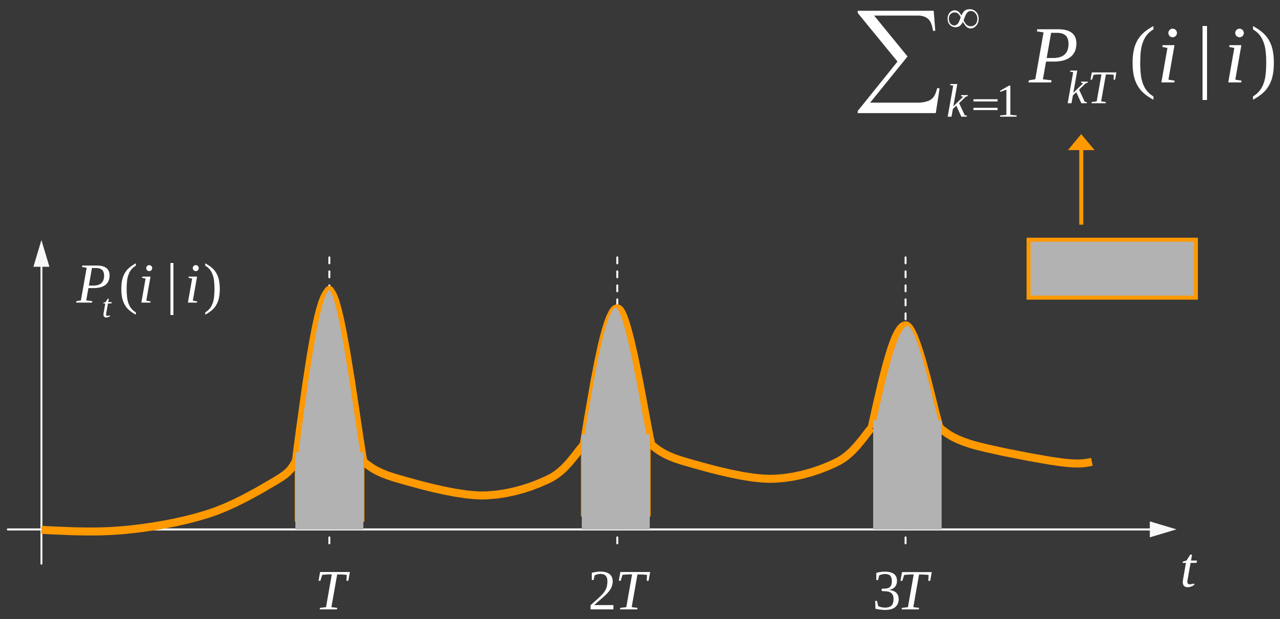


Persistent Cycle Measure

- 'Peakiness' of returning probability: $P_t(i\,|\,i)$

$$R(i,T)=\frac{\sum_{k=1}^{\infty}P_{kT}(i\,|\,i)}{\sum_{k=0}^{\infty}P_{k}(i\,|\,i)}$$





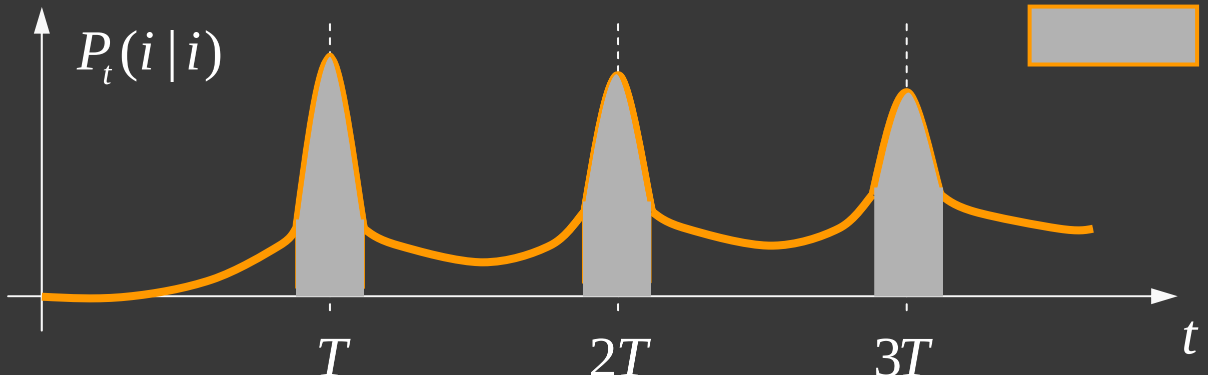
Theorem 'Peakiness': $R(i,T)$ can be computed:

$$R(i,T) = \frac{\sum_j \mathrm{Re}\left(\frac{\lambda_j^T}{1-\lambda_j^T} \cdot U_{ij} V_{ij}\right)}{\sum_j \mathrm{Re}\left(\frac{\lambda_j}{1-\lambda_j} \cdot U_{ij} V_{ij}\right)}$$

$U_{:j}$ $V_{:j}$: left & right eigenvectors of P

$R(i,T)$: dominated by complex eigenvalues λ_j

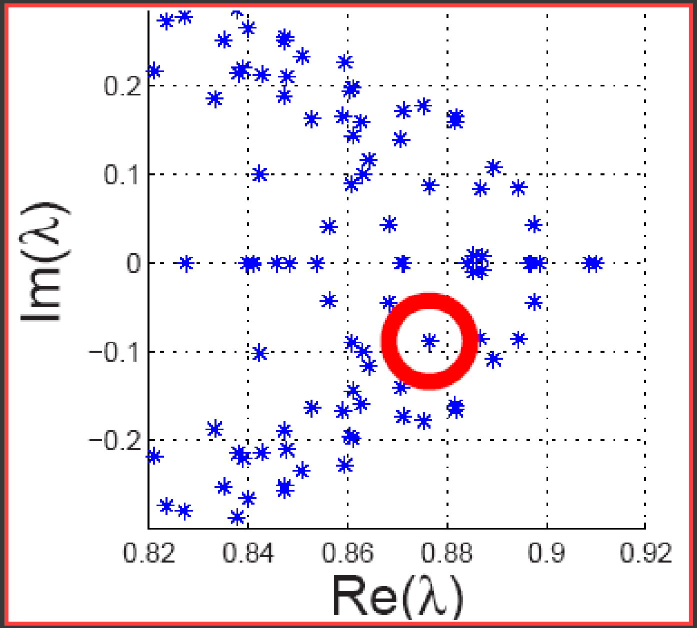
$(\frac{\lambda_j^T}{1-\lambda_j^T})/(\frac{\lambda_j}{1-\lambda_j})$ large for complex λ_j

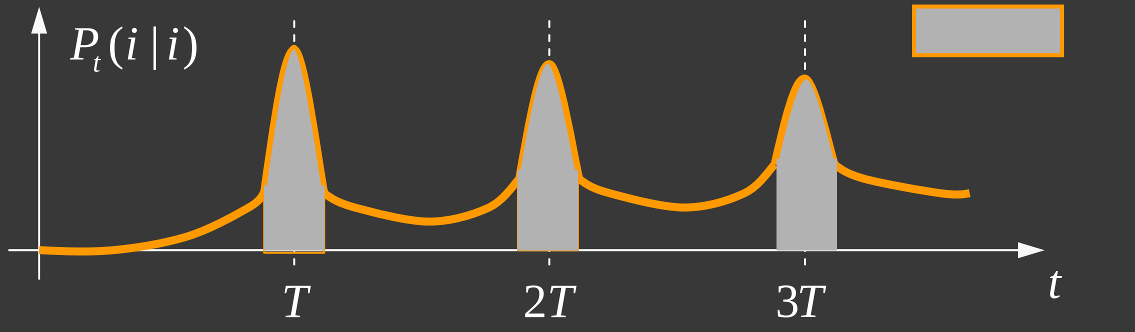


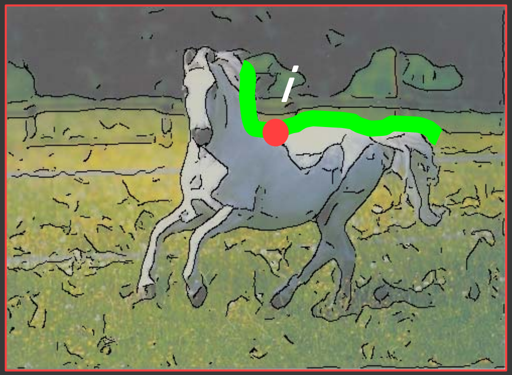
Complex Eigenvalues of Random Walk

One step random walk ← $P^T \cdot x \leftrightarrow \lambda \cdot x$

$R(i,T)$



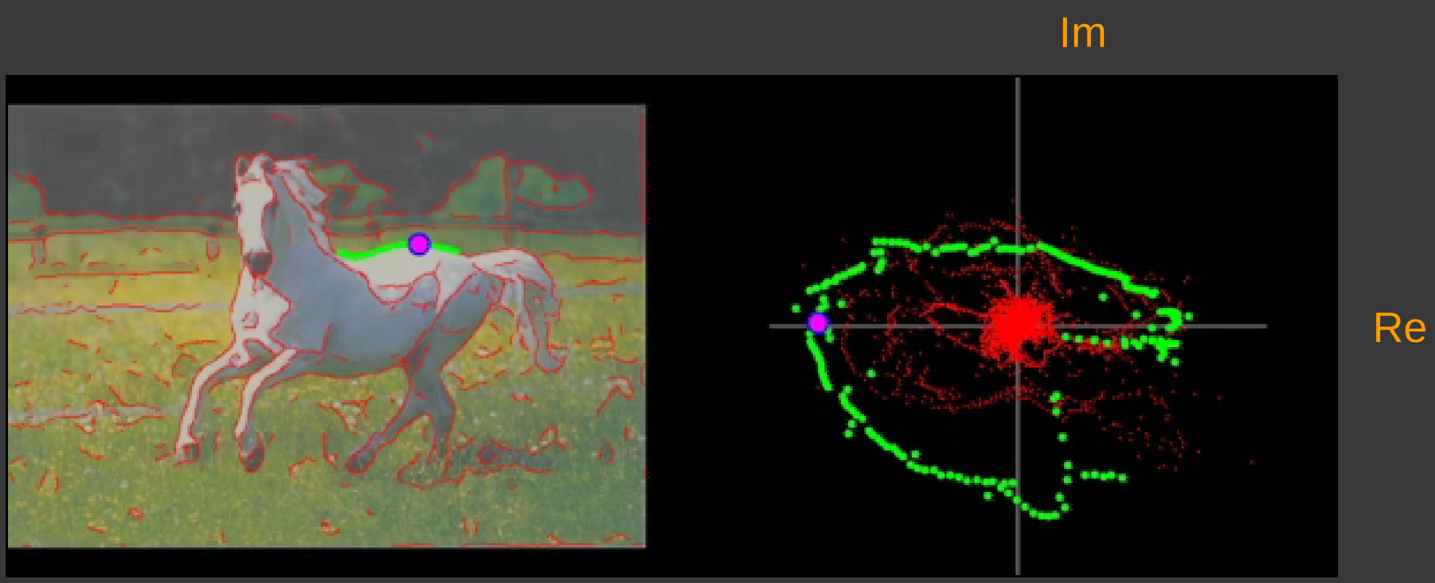




Complex Eigenvector of Random Walk

One step random walk ← $P^T \cdot x \leftrightarrow \lambda \cdot \boxed{x}$ → Rotation in complex vector:

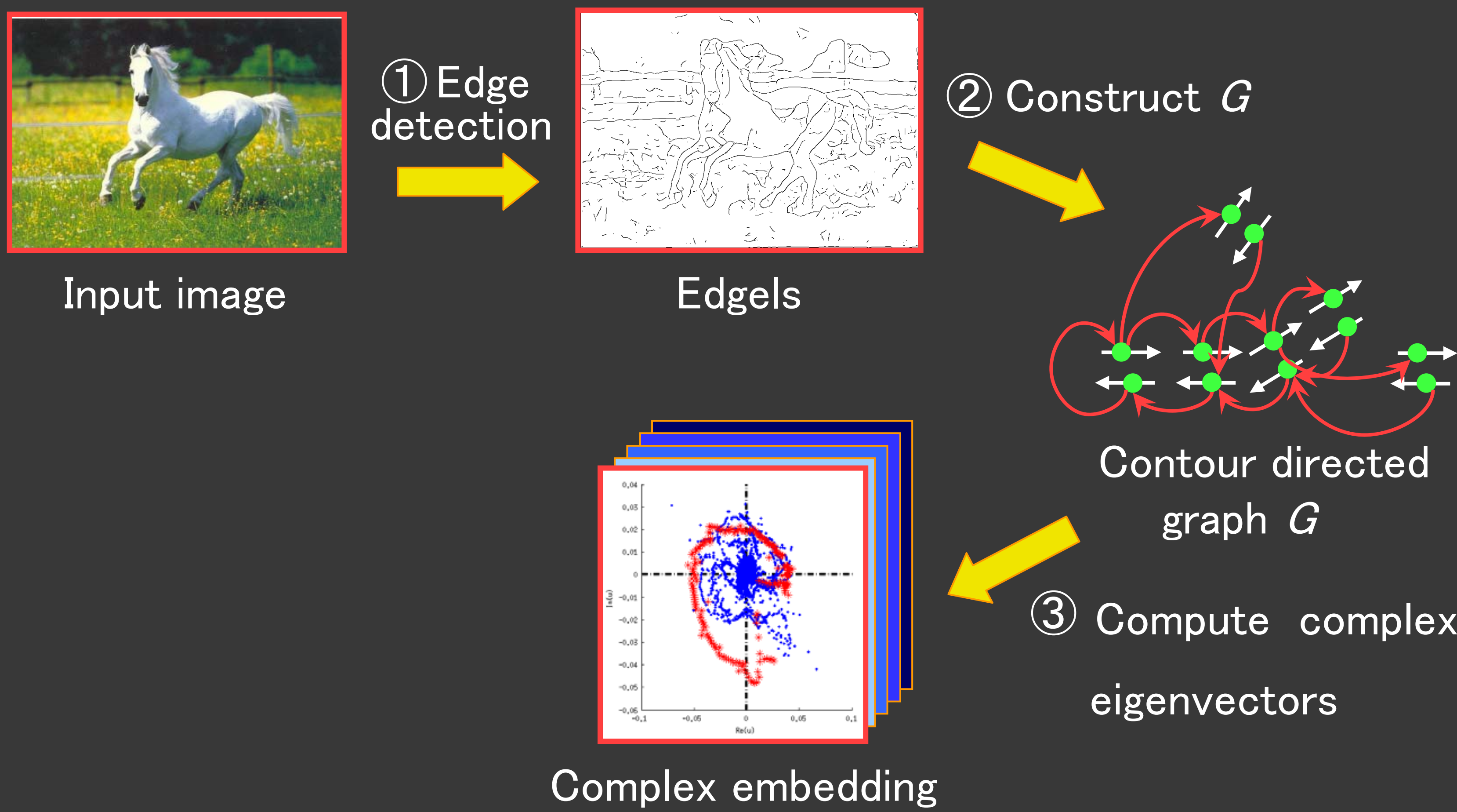
Complex eigenvectors encode both cyclic ordering and likelihood on cycles



Image

Complex eigenvector x

Untangling Cycle Algorithm



Ideal Cost for Circular Embedding

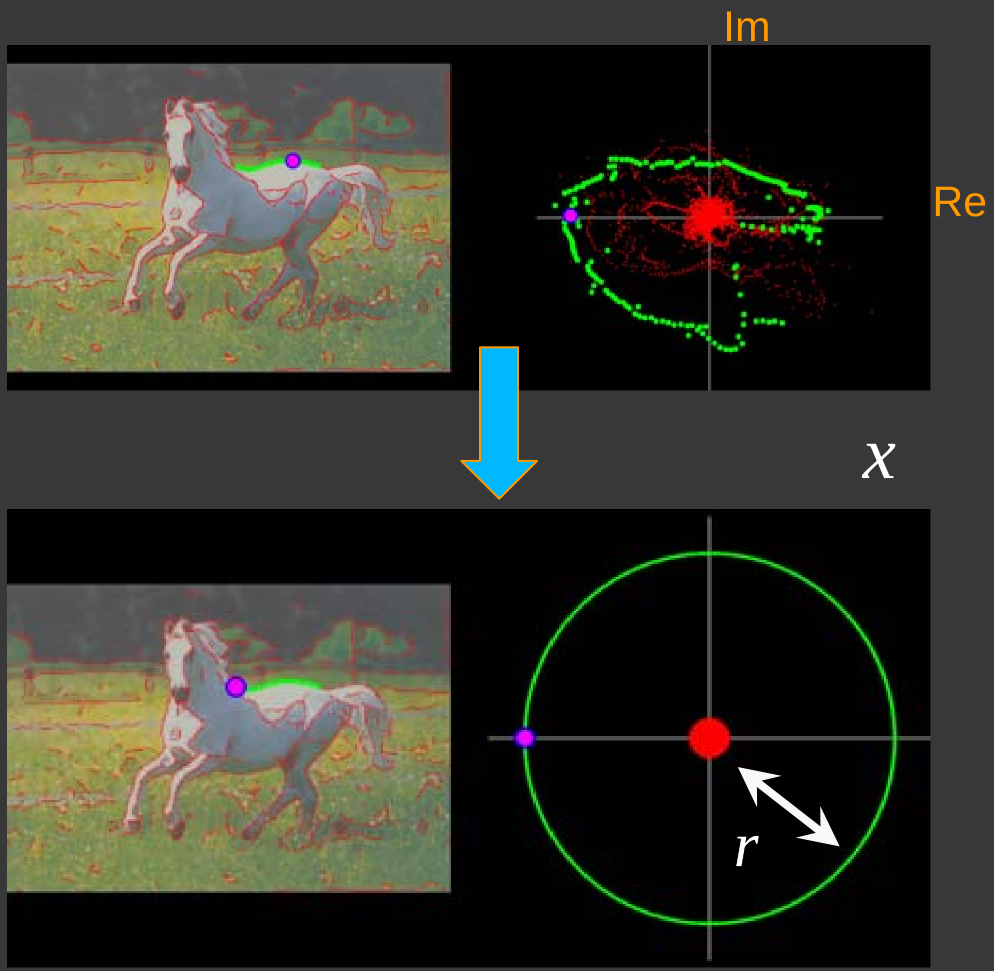


Image　　Ideal circular embedding

Each complex eigenvector gives a circular embedding of the original graph

For a point x in complex plane

$$x(r,\theta) = r \cdot e^{i\theta}$$

Ideal Cost for Circular Embedding

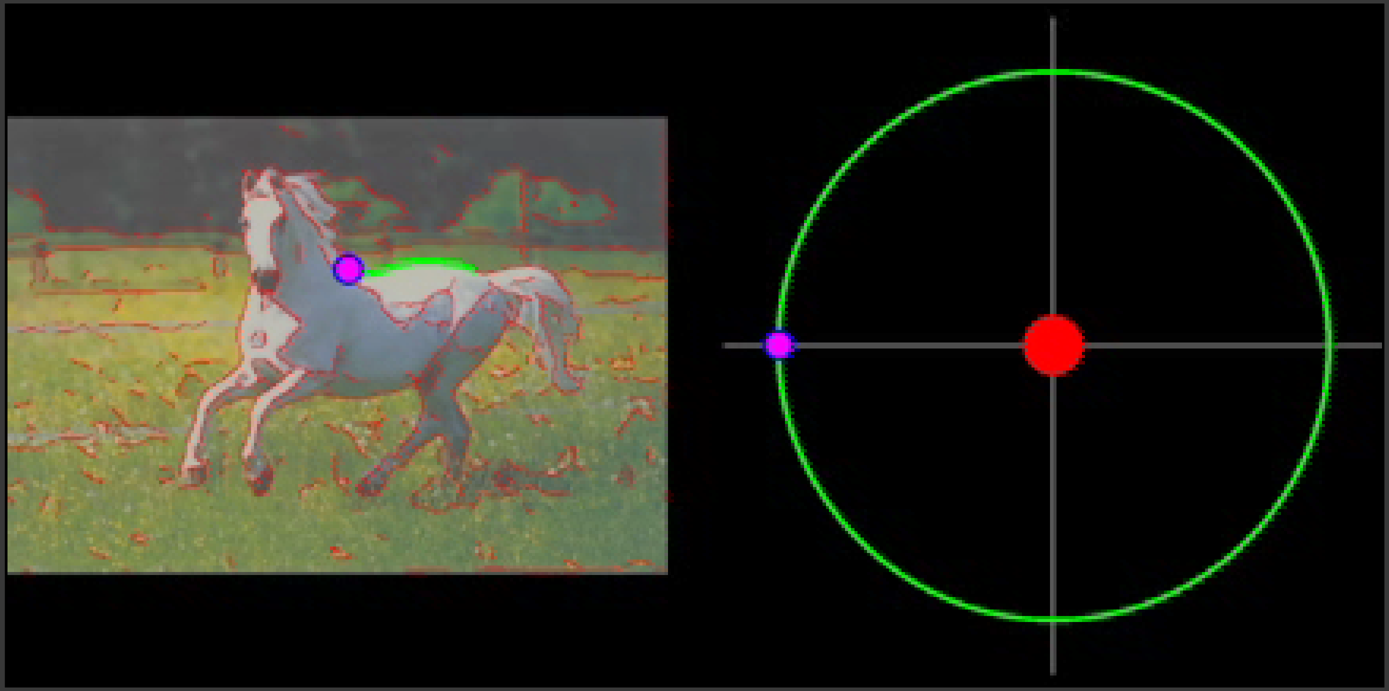
One step random walk ← $P^T \cdot x \leftrightarrow \lambda \cdot x$ → Rotation in circular embedding:

in circular embedding:

$x \rightarrow P^T \cdot x$

$x(r,\theta) = r \cdot e^{i\theta}$

$x \rightarrow \lambda \cdot x$



Image

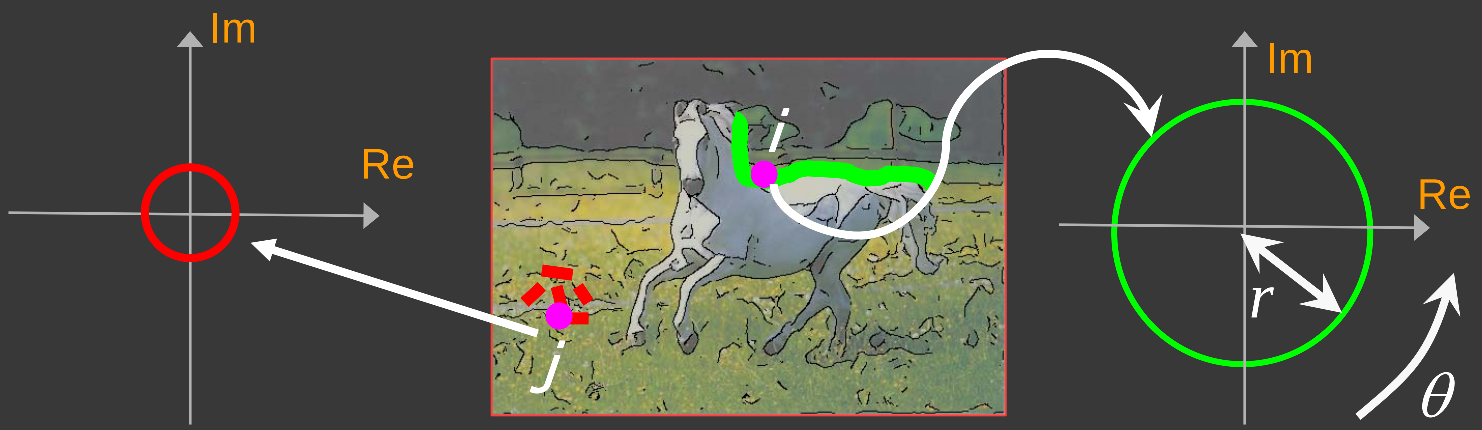
Ideal circular embedding

Ideal Cost of Circular Embedding

we want:

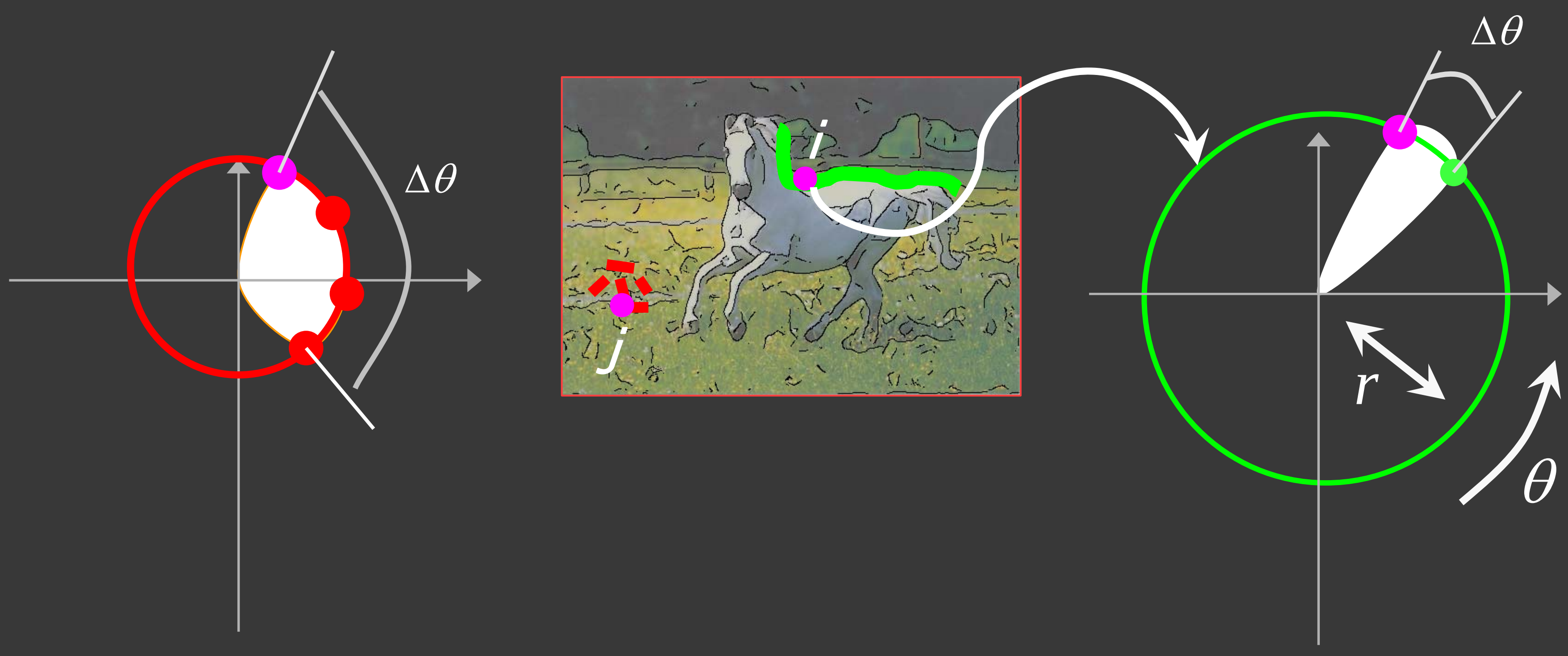
- Good Contour → large circle according to cyclic ordering
- Bad Clutter → core around the origin

$$x(r,\theta) = r \cdot e^{i\theta}$$



Ideal Cost of Circular Embedding

$$r \ * \ \Delta\theta = \text{constant}$$



In clutter, P(j,:) many immediate neighbors for each random walk step

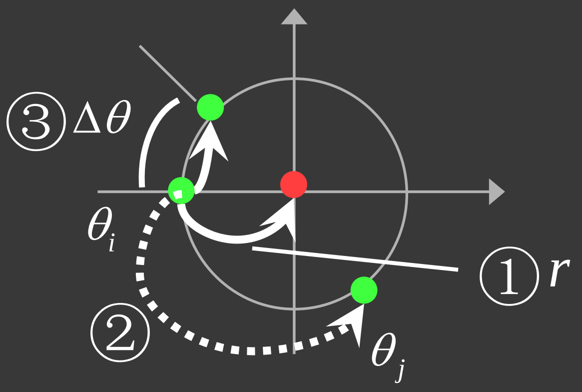
In contour, P(i,:) few immediate neighbors for each random walk step

Circular Embedding Score

We conjecture the ideal circular embedding Max.

$$C_e(r,\theta,\Delta\theta) = \sum_{\substack{\theta_i<\theta_j\le\theta_i+2\Delta\theta \\ r_i>0,r_j>0}} P_{ij} / |S| \cdot \frac{1}{\Delta\theta}$$

$$S = \{(r,\theta) \mid r = r_0\}$$



- r Circle indicator with $r_i \in \{r_0, 0\}$
- θ Phase angles on cycles specifying an ordering
- $\Delta\theta$ Average jumping angle $\Delta\theta = \overline{\theta_j - \theta_i}$

Solution: Complex Eigenvector

$$C_e(r,\theta,\Delta\theta) = \sum_{\substack{\theta_i < \theta_j \le \theta_i + 2\Delta\theta \\ r_i > 0, r_j > 0}} P_{ij} / |S| \cdot \frac{1}{\Delta\theta}$$

Continuous relaxation

$$\max_{u,v \in \mathbb{C}^n} \ \mathrm{Re}(u^H P v) \quad s.t. \ u^H v = c$$

Solution: Complex Eigenvector

$$C_e(r,\theta,\Delta\theta) = \sum_{\substack{\theta_i<\theta_j\le\theta_i+2\Delta\theta \\ r_i>0, r_j>0}} P_{ij} / |S| \cdot \frac{1}{\Delta\theta}$$

Continuous relaxation

$$\max_{u,v\in\mathbb{C}^n} \quad \mathrm{Re}(u^H P v)$$
$$s.t. \quad u^H v = c$$

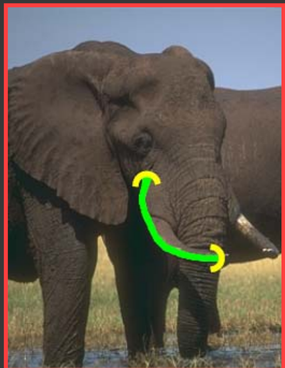
Theorem: All critical points (local maxima) $(u_{\max}, v_{\max})$ of the above are left and right eigenvectors of P

$$P v_{\max} = \lambda v_{\max} \qquad P^T u^*_{\max} = \lambda u^*_{\max}$$

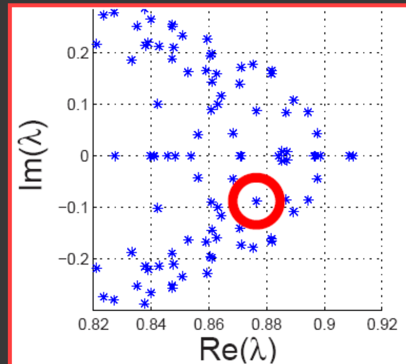
Maximum values are $\max_\lambda(\mathrm{Re}(\lambda \cdot c))$

Discretization

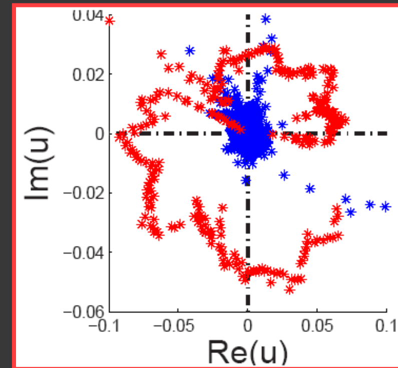
Image



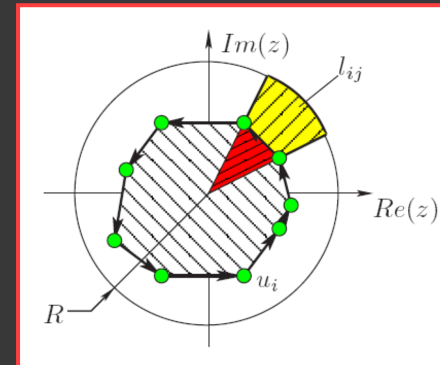
Eigenvalues



One eigenvector



Maximal circular cover



Find embedding cycles with large radius

- Maximal cover area

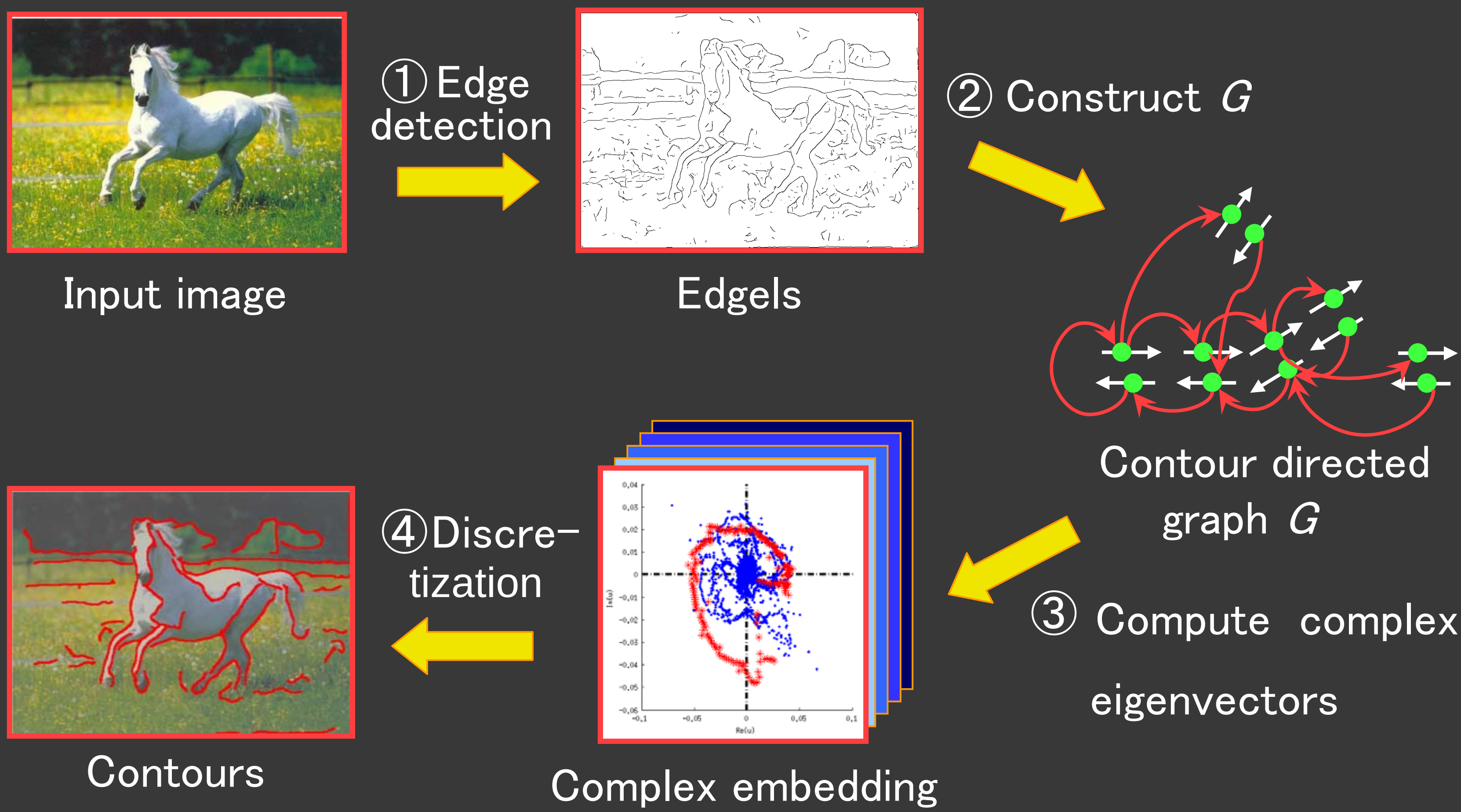
$$\max_{s_1,\ldots,s_k} \sum_{j=1}^{k} A(u_{s_j}, u_{s_{j+1}})$$

$$A(u_{s_j}, u_{s_{j+1}}) = \frac{1}{2}\mathrm{Im}(u_{s_j}^{*} \cdot u_{s_{j+1}})$$

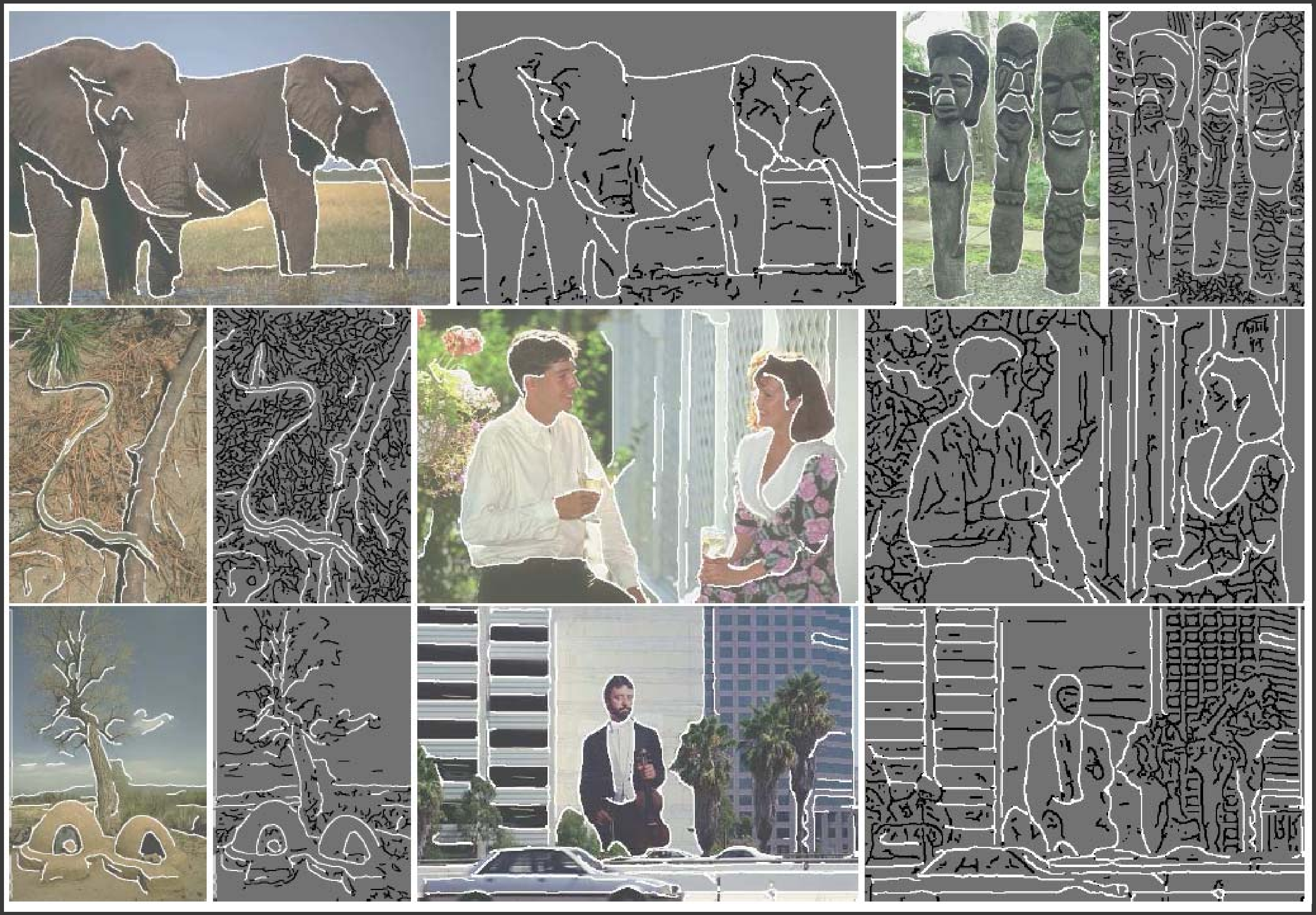
Section area spanned by $u_{s_j}, u_{s_{j+1}}$

- Compute shortest paths in the embedding space

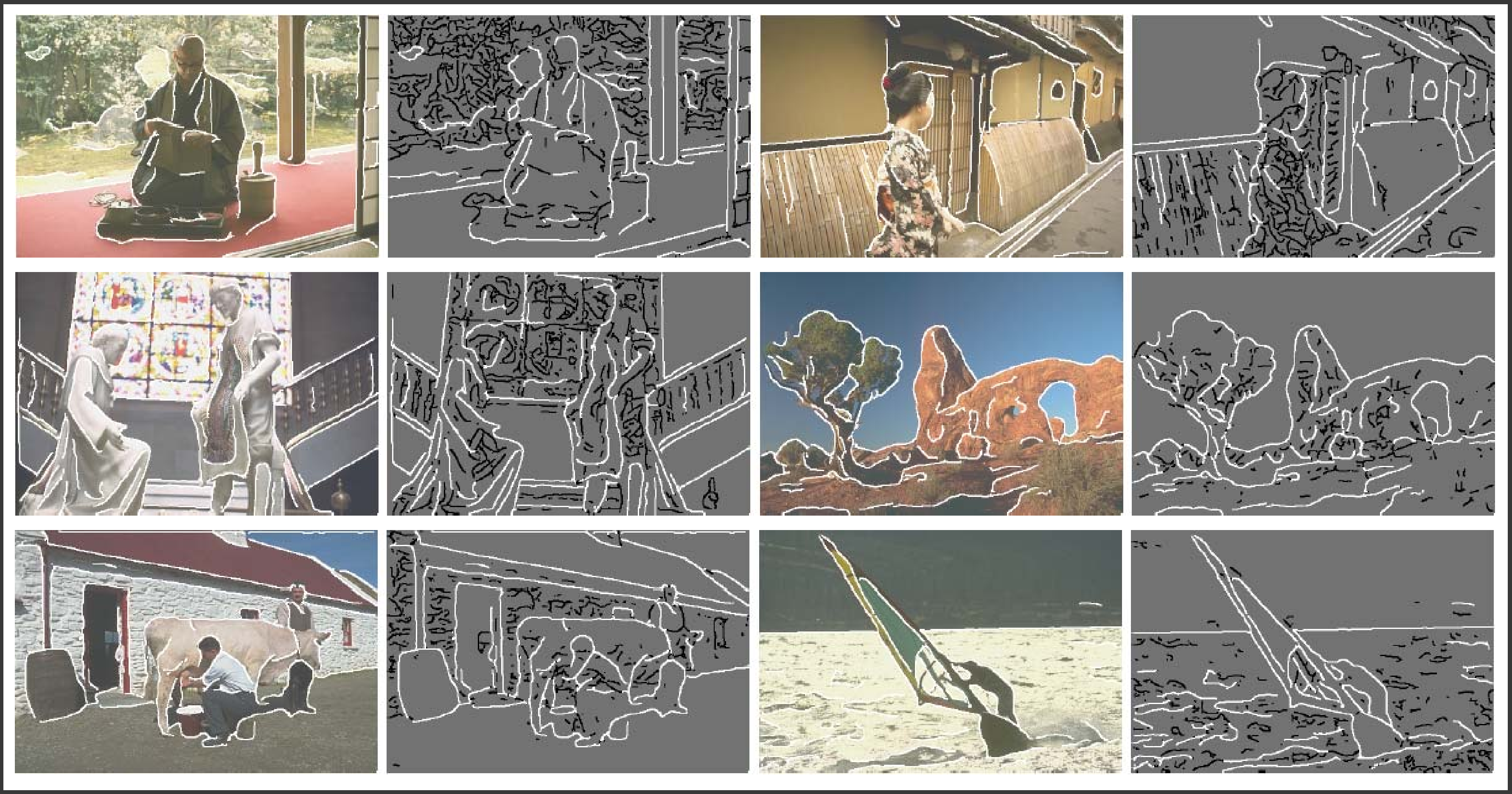
Untangling Cycle Algorithm



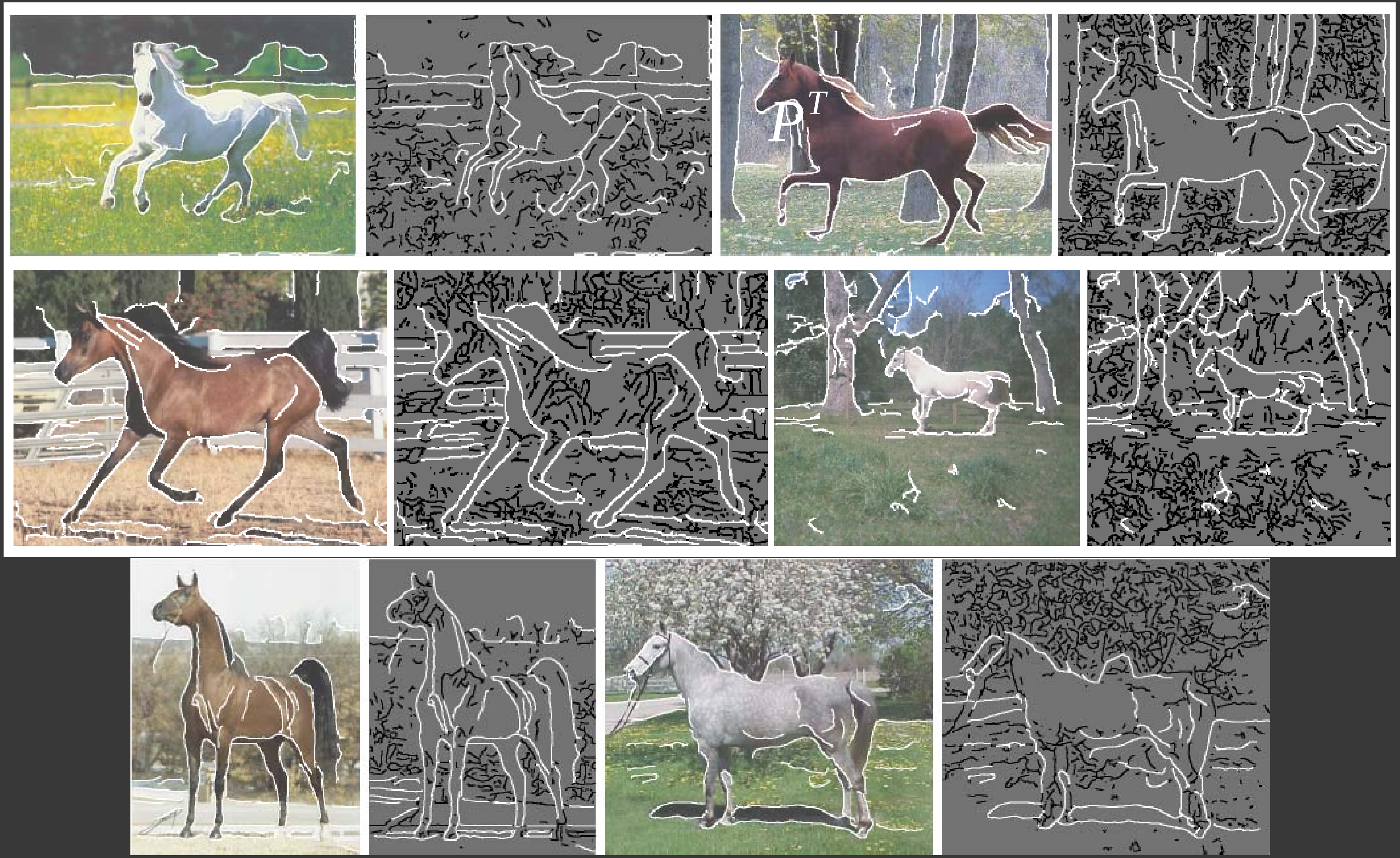
Experiments: BSDS



Experiments: BSDS

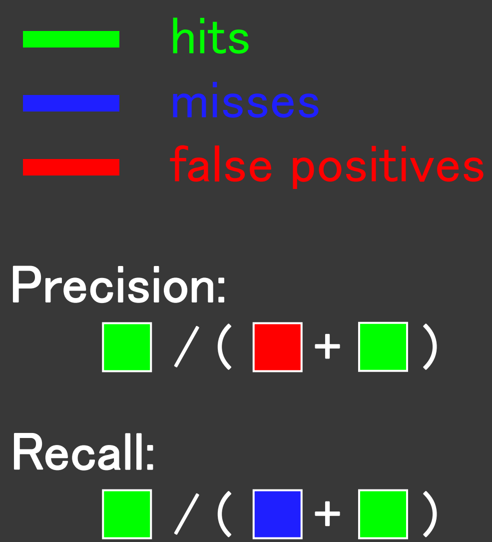


Experiments: Horses



Berkeley Segmentation Benchmark

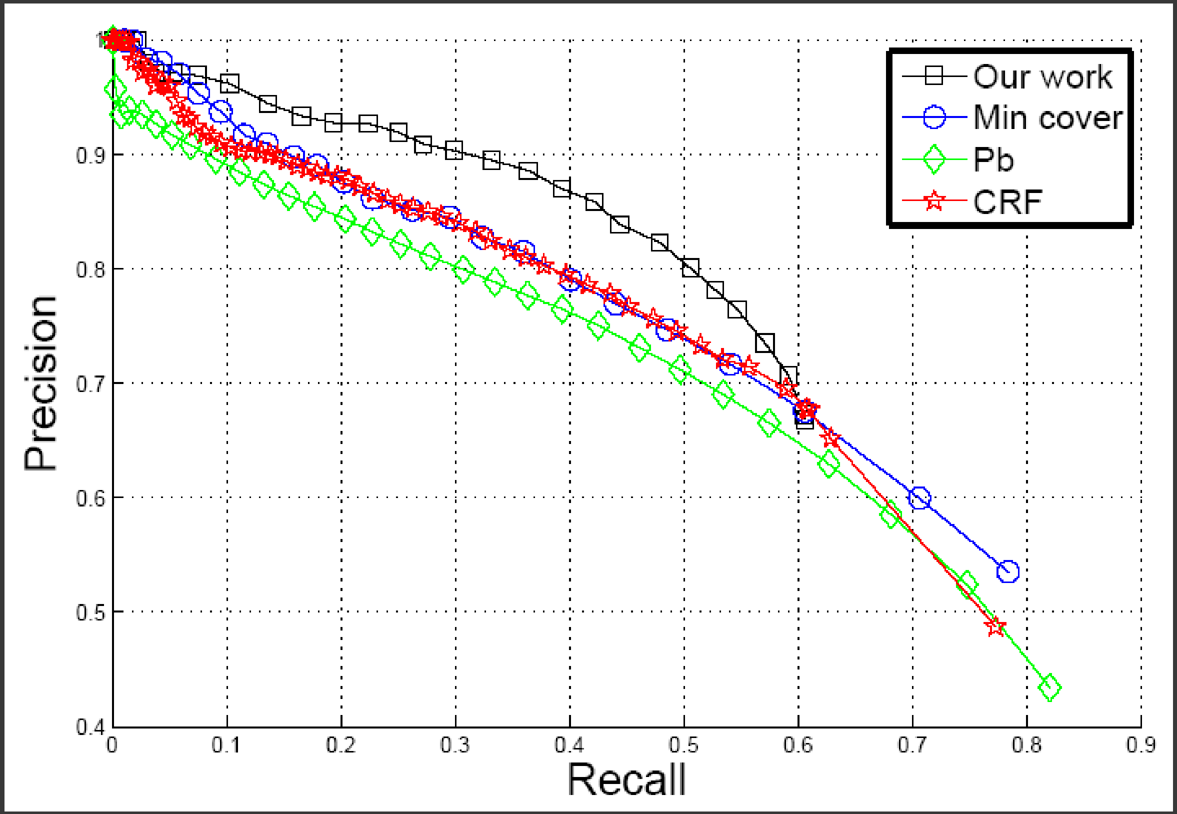
Image | Human labels | Contour



Compare our method to

- **Pb** D. Martin *et al*, PAMI 2004
- **CRF** X. Ren *et al*, ICCV 2005
- **Min cover** P. Felzenszwalb *et al*, WPOCV 2006

Berkeley Segmentation Comparison



P. F. Felzenszwalb and D. McAllester. A min-cover approach for finding salient curves. In *WPOCV*, page 185, 2006.

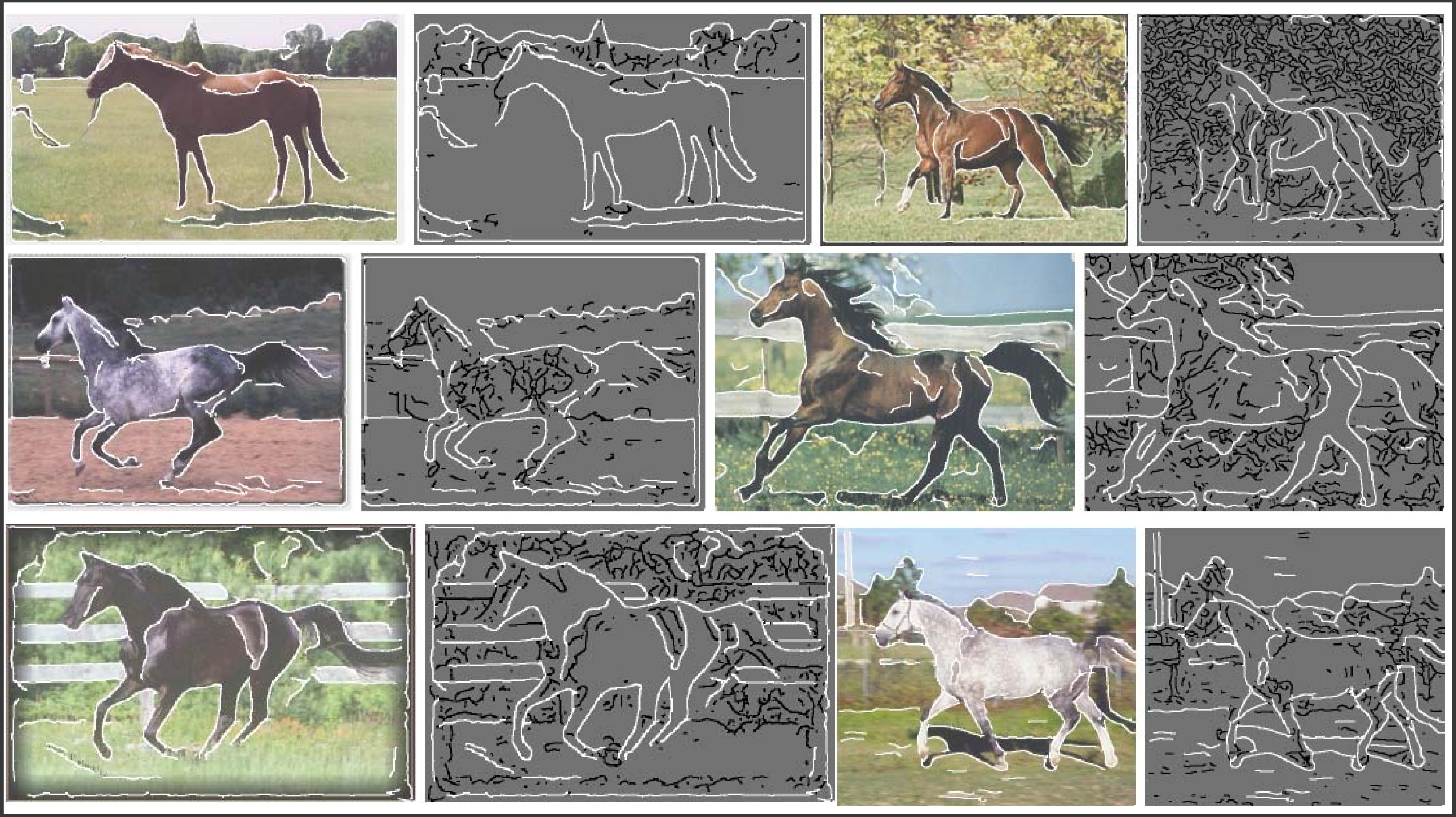
X. Ren, C. Fowlkes, and J. Malik. Scale-invariant contour completion using conditional random fields. In *ICCV*, pages 1214–1221, 2005.

Pb D. Martin *et al*, PAMI 2004

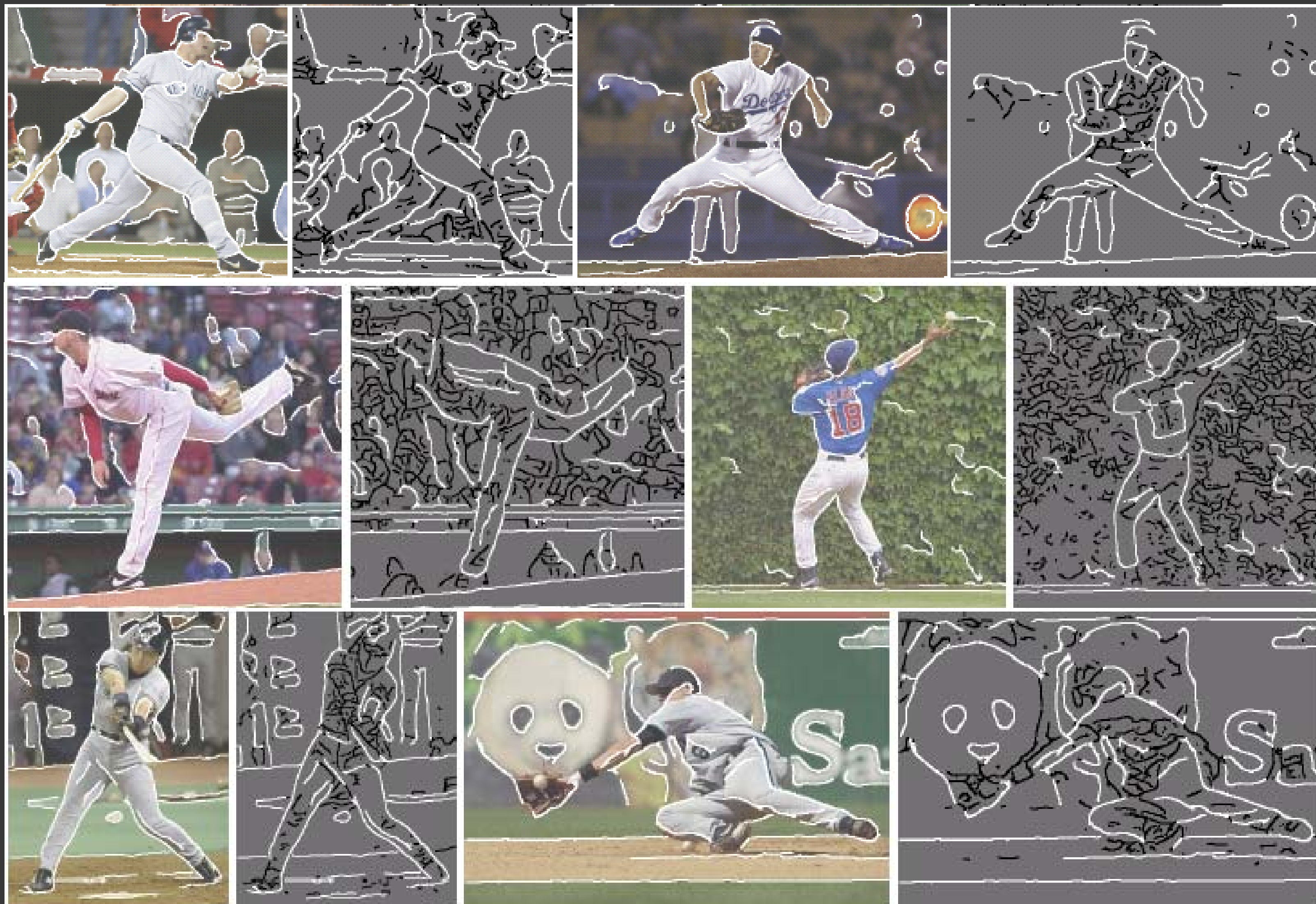
Conclusion

- Utilize topology information for contour grouping
- Persistent cycles: circular/complex embedding
- Untangling cycle cut score: grouping 1D structures

Experiments: Horses

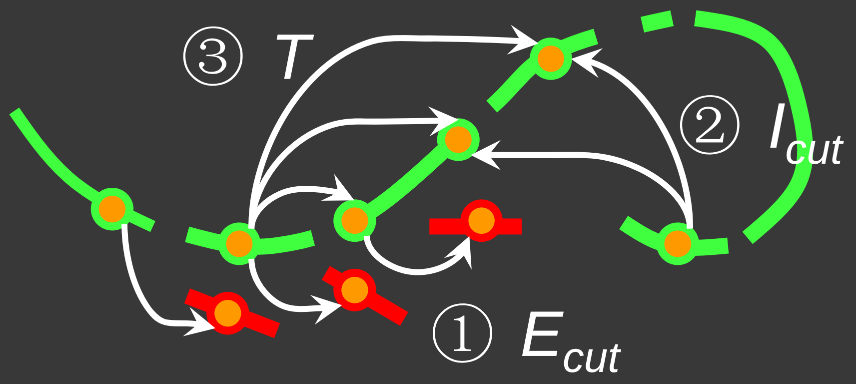


Experiments: Baseball Players



Experiments: Baseball Players

Untangling Cycle Cut Score



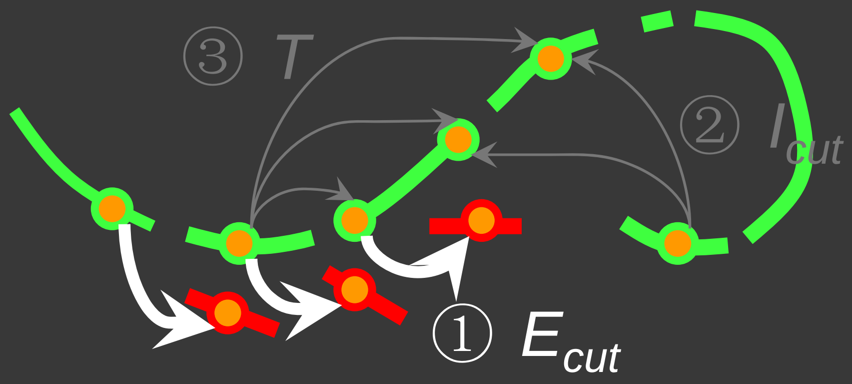
① **External cut (E_{cut})**

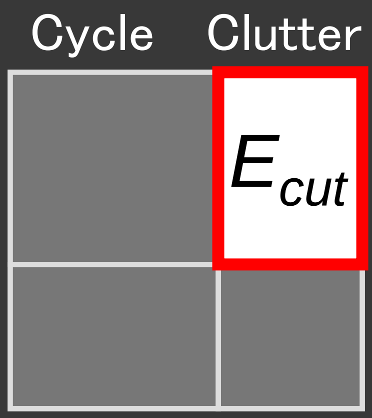
② **Internal cut (I_{cut})**

③ **Tube size (T)**

A discrete graph cut score useful for segmenting persistent cycles from continuous embedding space

External Cut

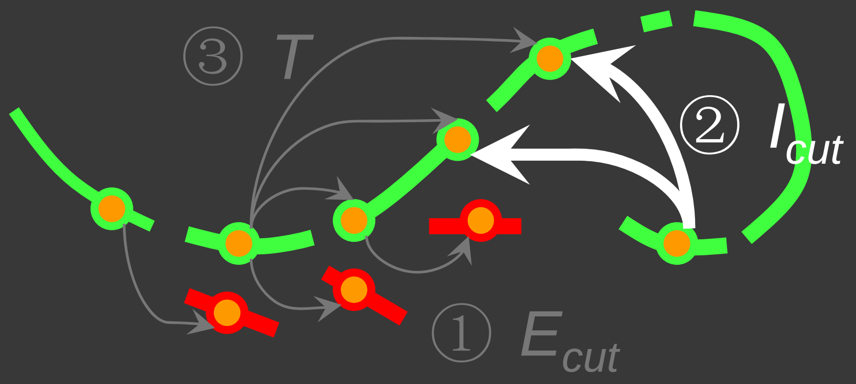


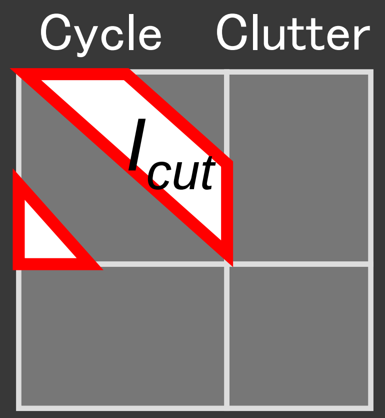


$$E_{cut}(S) = \frac{1}{|S|} \sum_{i \in S,\, j \in (V-S)} P_{ij}$$

- Cut cycle (*S*) from clutter (*V-S*)
- Similar to NCut (2D grouping)

Internal Cut





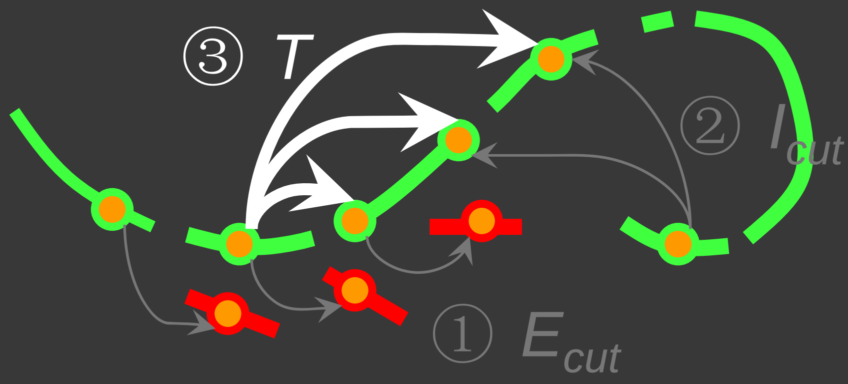
$$I_{cut}(S,O,k) = \frac{1}{|S|} \sum_{(O(i)\geq O(j)) \vee (O(j) \geq O(i)+k)} P_{ij}$$

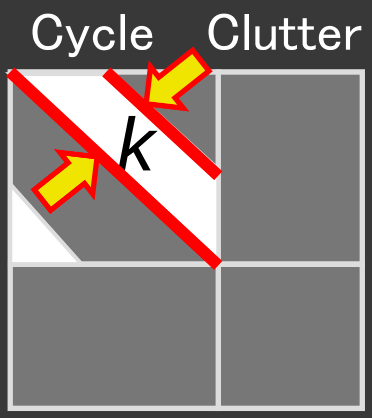
Ordering

$O : S \mapsto S = \{1,2,\ldots,|S|\}$

$$\begin{cases} \text{Forward} & 0 < O(j) - O(i) \leq k \\ \text{Backward} & -|S|/2 \leq O(j) - O(i) \leq 0 \\ \text{Fast-forward} & \text{otherwise} \end{cases}$$

Tube Size





$$T(k) = k / |S|$$

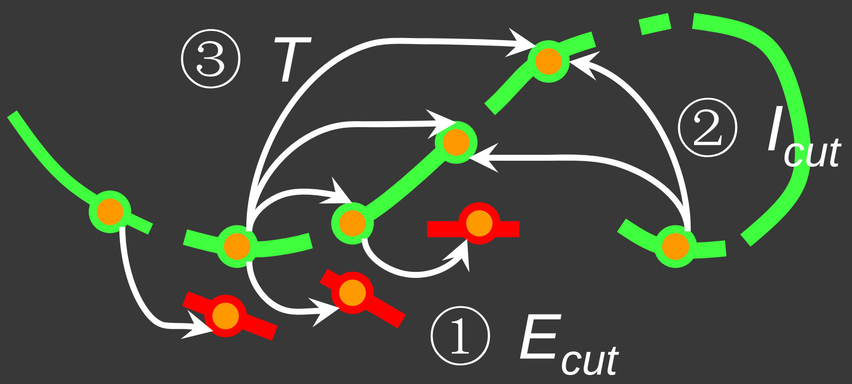
- Thickness: how fat is the cycle?
- Special cases
  - *k=1* ideal case of a cycle
  - *k=|S|* 2D structures

Combining Scores

Maximize Untangling Cycle Cut Score

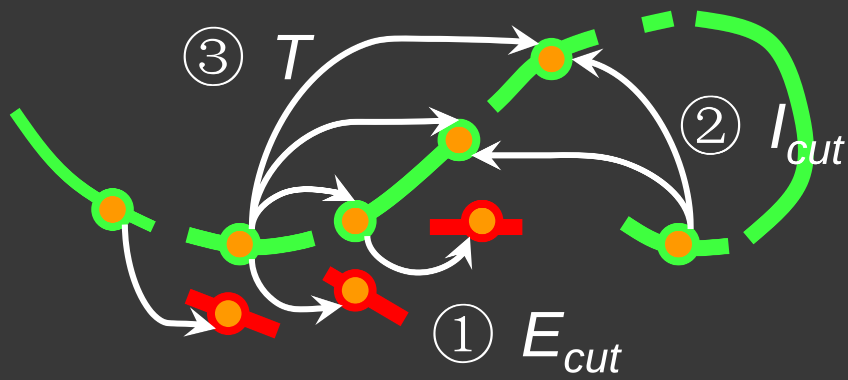
$$C_u(S,O,k) = \frac{1 - E_{cut}(S) - I_{cut}(S,O,k)}{T(k)}$$

- S Subset of graph nodes V
- O Cycle ordering on S
- k Cycle thickness

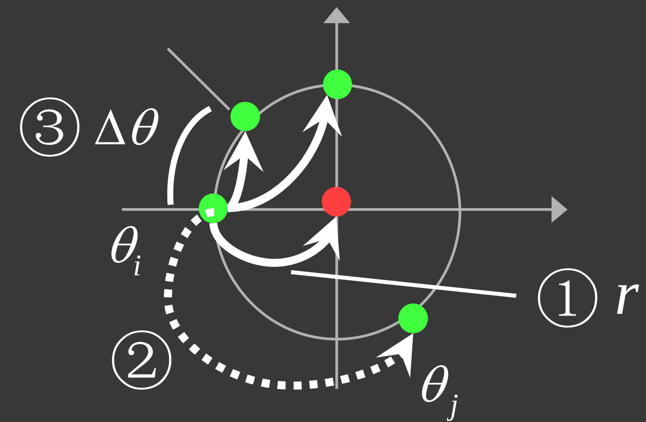


Cut Score Interpretation

Three untangling cycle criteria



Circular embedding



① External cut: $r \Leftrightarrow E_{cut}$

② Internal cut: $\theta \Leftrightarrow I_{cut}$

③ Tube size: $\Delta\theta \Leftrightarrow T$