

## Homework 2

*Release Date: September 13, 2023**Due Date: September 20, 2023*

## 1 Written Questions

- **6.3** from the textbook.
- **6.12abd** from the textbook.
- **MGF/Chernoff.** Let  $X_1, \dots, X_N$  be i.i.d. random variables, with moment generating function  $m_X(t) = \mathbb{E}[e^{tX}]$  and mean  $\mu = \mathbb{E}[X]$ . Consider the average of all  $N$  random variables:  $Y = \frac{1}{N} \sum_{i=1}^N X_i$ . Derive the form of  $m_Y(t)$ , the moment generating function for  $Y$ , and use the Chernoff method to derive a tail bound.
- **Bonus:** Use the Chernoff method to derive a tail bound  $P(X - 1 \geq u)$  for the  $\chi^2$  random variable and for  $u \leq 1$ . A  $\chi^2$  random variable is  $X = Z^2$  where  $Z \sim N(0, 1)$  is a standard normal, and note that  $\chi^2$  has mean 1.

*Hint: consider the MGF when  $t < 1/2$ , and use the fact that  $\frac{e^{-t}}{\sqrt{1-2t}} \leq e^{2t^2}$  for  $|t| \leq 1/4$ .*