Introduction to Logistic Regression and Support Vector Machine

guest lecturer: Ming-Wei Chang
CS 446

Fall, 2009

Before we start

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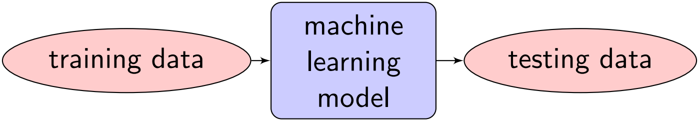
- Feel free to ask questions anytime
- The slides are newly made. Please tell me if you find any mistake.

Today: supervised learning algorithms



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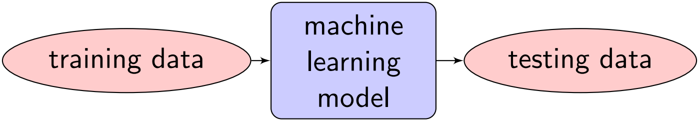

Supervised learning algorithms we have mentioned

- Decision Tree
- Online Learning: Perceptron, Winnow, ...
- Generative Model: Naive Bayes

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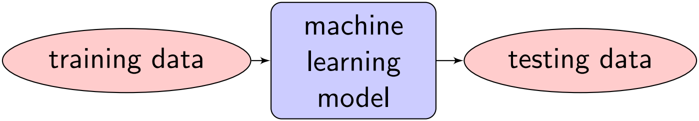

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- Online Learning: Perceptron, Winnow, . . .
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What are we going to talk about today?

- "Modern" supervised learning algorithms
- Specifically, logistic regression and support vector machine

Motivation

- Logistic regression and support vector machine are both very popular!

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  - Using optimization algorithms as training algorithms
  - An important technique we need to be familiar with.
  - Learn not to be afraid of these algorithms

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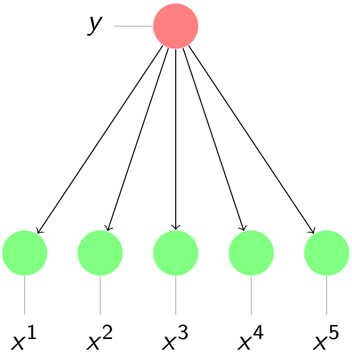
Understand the relationships
between these algorithms and the algorithms we have learned

Review: Naive Bayes

Notations

- **Input**: x, **Output** $y \in \{+1, -1\}$
- Assume each x has m features.
  - We use x^j to represent the j-th features of x



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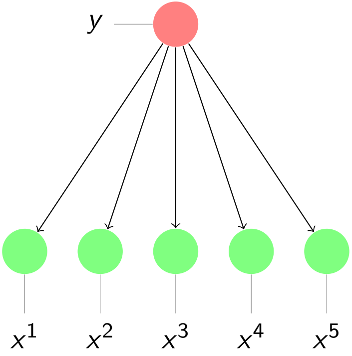
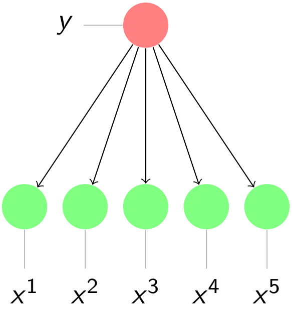


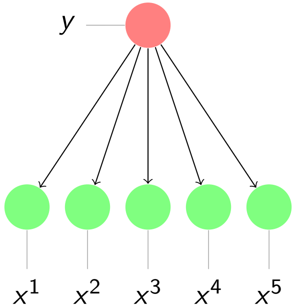
Conditional Independence

Review: Naive Bayes



$$P(y, x) = P(y) \prod_{j=1}^{m} P(x^j | y)$$

Review: Naive Bayes



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Training

- Maximize the likelihood of $P(D) = P(Y, X) = \prod_i^l p(y_i, x_i)$
- Algorithm
  - Estimate $P(y = -1)$ and $P(y = 1)$ by counting
  - Estimate $P(x^j | y)$ by counting

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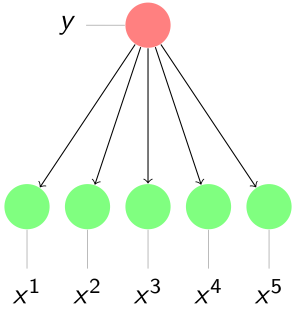

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Testing

- $\frac{P(y=+1|x)}{P(y=-1|x)} = \frac{P(y=+1,x)}{P(y=-1,x)} \geq 1?$

Review: Naive Bayes

The prediction function of a Naive Bayes model is a linear function

- In previous lectures, we have shown that
  $$log \frac{P(y=+1|x)}{P(y=-1|x)} \geq 0 \Rightarrow w^T x + b \geq 0$$
- The counting results can be re-expressed as a linear function

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- Key observation: Naive Bayes cannot express all possible linear functions
  - Intuition: conditional independence assumption
- We will propose a model (logistic regression) that can express all possible linear functions in the next few slides.

Modeling conditional probability using a linear function

Starting point: the predicting function of Naive Bayes

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Using the bias trick, $P(y|x) = \frac{1}{1+e^{-y(w^T x)}}$

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  - For *all possible* w, find the one that maximizes the conditional likelihood
    - drop the conditional independence assumption!

Logistic regression: the final objective function

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Finding w as an optimization problem

$$\begin{aligned} w &= \underset{w}{\text{argmax}} \log P(Y|X, w) = \underset{w}{\text{argmin}} - \log P(Y|X, w) \\ &= \underset{w}{\text{argmin}} - \sum_{i=1}^{l} \log \frac{1}{1 + e^{-y_i(w^T x_i)}} \\ &= \underset{w}{\text{argmin}} \sum_{i=1}^{l} \log(1 + e^{-y_i(w^T x_i)}) \end{aligned}$$

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Properties of this optimization problem

- A convex optimization problem

Adding regularization

Explanation

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  - In order to minimize the empirical loss, w will tend to be large
- Therefore, to prevent over-fitting, we add a regularization term

- Regularization Term

$$\min_w \frac{1}{2} w^T w + C \sum_{i=1}^{l} \log(1 + e^{-y_i(w^T x_i)})$$

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- Empirical Loss
- balance parameter

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Low cost per iteration	–	High cost per iteration
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Currently: Limited memory BFGS is very popular in NLP community

Logistic regression versus Naive Bayes

	Logistic regression	Naive Bayes
Training	maximize $P(Y\|X)$	maximize $P(Y,X)$
Training Algorithm	optimization algorithms	counting
Testing	$P(y\|x) \geq 0.5$?	$P(y\|x) \geq 0.5$?

Table: Comparison between Naive Bayes and logistic regression

Logistic regression versus Naive Bayes

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Table: Comparison between Naive Bayes and logistic regression

- LR and NB are both linear functions in the testing phase
- However, their training agendas are very different

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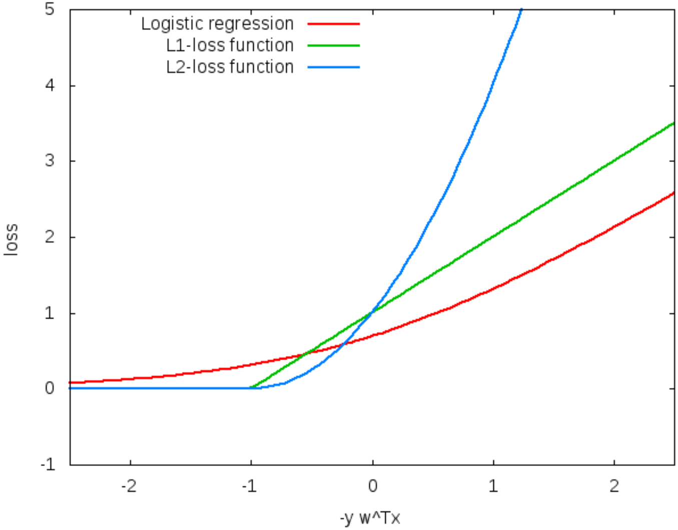
- L2-loss SVM

$$\min_w \frac{1}{2} w^T w + C \sum_{i=1}^{l} max(0, 1 - y_i w^T x_i)^2$$

Compare these loss functions



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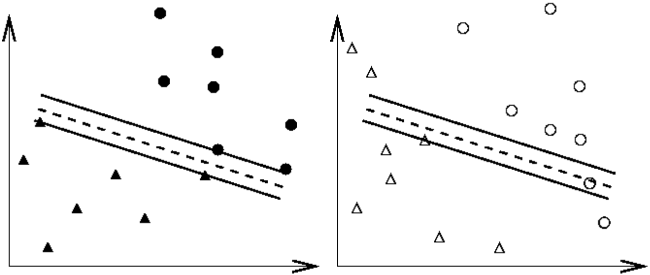
Learning theory: Link to SVM theory notes

Balance between regularization and empirical loss



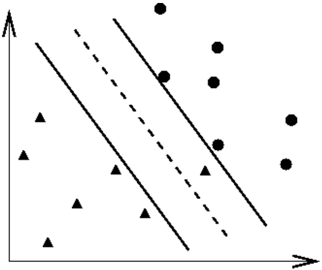
(a) Training data and an over-fitting classifier

(b) Testing data and an over-fitting classifier

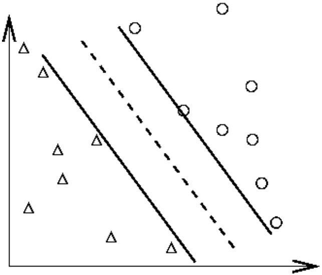
The maximal margin line with 0 training error

Best?

Balance between regularization and empirical loss



(c) Training data and a better classifier



(d) Testing data and a better classifier

If we allow some training error, we can find a better line

We need to balance the regularization term and the empirically loss term

Problem of model selection. Select balance parameter with cross validation

Primal and Dual Formulations

Explaining the primal-dual relationship

- Link to the lecture notes: 07-LecSvm-opt.pdf

Why primal-dual relationship is useful

- Link to a talk by Professor Chih-Jen Lin in 2005.
  - Optimization, Support Vector Machines, and Machine Learning. Talk in DIS, University of Rome and IASI, CNR, Italy. September 1-2, 2005.
- We will only use the slides from page 11-20.
- Link to notes

Nonlinear SVM

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    - This can be a non linear function for x

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$$s.t. \quad 1 - y_i w^T \phi(x)_i \leq \xi_i$$

$$\xi_i \geq 0, \forall i = 1 \dots l$$

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$$s.t. \quad 1 - y_i w^T \phi(x)_i \leq \xi_i$$

$$\xi_i \geq 0, \forall i = 1 \ldots l$$

Dual

$$\min_{\alpha} \quad \frac{1}{2}\alpha^T Q\alpha - eT\alpha$$

$$s.t. \quad \forall i, 0 \leq \alpha_i \leq C$$

where Q is a l-by-l matrix with $Q_{ij} = y_i y_j K(x_i, x_j)$

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    - This can be a non linear function for x

Primal

$$\begin{aligned} \min_{w,\xi_i} \quad & \frac{1}{2} w^T w + C \sum_{i=1}^{l} \xi_i \\ s.t. \quad & 1 - y_i w^T \phi(x)_i \leq \xi_i \\ & \xi_i \geq 0, \forall i = 1 \dots l \end{aligned}$$

Dual

$$\begin{aligned} \min_{\alpha} \quad & \frac{1}{2} \alpha^T Q \alpha - eT\alpha \\ s.t. \quad & \forall i, 0 \leq \alpha_i \leq C \end{aligned}$$

where Q is a l-by-l matrix
with $Q_{ij} = y_i y_j K(x_i, x_j)$

Same for Kernel perceptron: find a linear function on $\phi(x)$

Demo

Solving SVM

- Both primal and dual problems have constraints
  - We can not use the gradient descent algorithm
- For linear dual SVM, there is a simple optimization algorithm
  - Coordinate descent method!

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$$s.t. \quad \forall i, 0 \leq \alpha_i \leq C$$

  - # of α_i = # of training example
  - The idea: pick one example i. Optimize α_i only

Coordinate Descent Algorithm

Algorithm

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$$\alpha_i' = \alpha_i + s$$

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where d is a vector of $l - 1$ zeros. The i-th component of d is 1.

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    where d is a vector of $l - 1$ zeros. The i-th component of d is 1.
  - It is a single variable problem. We know how to solve this.

Coordinate Descent Algorithm

- Assume that the optimal s is s^*. We can update α_i using:

$$\alpha_i' = \alpha_i + s^*$$

- Given that $w = \sum_i^l \alpha_i y_i x_i$, this is equivalent to is equivalent to

$$w \leftarrow w + (\alpha_i' - \alpha_i) y_i x_i$$

- Isn't this familiar?

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Relationships between linear classifiers

- NB, LR, Perceptron and SVM are all linear classifiers
- NB and LR have the same interpretation for conditional probability

$$P(y|x, w) = \frac{1}{1 + e^{-y(w^T x)}} \quad (2)$$

- The difference between LR and SVM are their loss functions
  - But they are quite similar!
- Perceptron algorithm and the coordinate descent algorithm for SVM are very similar

Summary

Logistic regression

- Maximizes $P(Y|X)$ while Naive Bayes maximizes the joint probability $P(Y, X)$
- Model the conditional probability using a linear line. Drop the conditional independence assumption
- Many available methods of optimizing the objective function

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Support Vector Machine

- Similar to Logistic Regression; Different Loss function
- Maximizes Margin; Has many nice theoretical properties
- Interesting Primal-Dual relationship
  - Allows us to choose the easier one to solve
- Many available methods of optimizing the objective function
  - The linear dual coordinate descent method turns out to be similar to Perceptron