Where are we?

- Algorithms
  - DTs
  - Perceptron + Winnow
  - Gradient Descent
  - NN
- Theory
  - Mistake Bound
  - PAC Learning
- We have a formal notion of "learnability"
  - We understand Generalization
    - How will your algorithm do on the next example?
  - How it depends on the hypothesis class (VC dim)
    - and other complexity parameters
- Algorithmic Implications of the theory?

Boosting

- Boosting is (today) a general learning paradigm for putting together a Strong Learner, given a collection (possibly infinite) of Weak Learners.
- The original Boosting Algorithm was proposed as an answer to a theoretical question in PAC learning. [The Strength of Weak Learnability; Schapire, 89]
- Consequently, Boosting has interesting theoretical implications, e.g., on the relations between PAC learnability and compression.
  - If a concept class is efficiently PAC learnable then it is efficiently PAC learnable by an algorithm whose required memory is bounded by a polynomial in n, size c and log(1/ε).
  - There is no concept class for which efficient PAC learnability requires that the entire sample be contained in memory at one time – there is always another algorithm that "forgets" most of the sample.

Boosting Notes

- However, the key contribution of Boosting has been practical, as a way to compose a good learner from many weak learners.
- It is a member of a family of Ensemble Algorithms, but has stronger guarantees than others.

- A Boosting demo is available at http://cseweb.ucsd.edu/~yfreund/adaboost/

- Example
- Theory of Boosting
  - Simple & insightful

Boosting Motivation

Example: "How May I Help You?"

[Gorin et al.]

- goal: automatically categorize type of call requested by phone customer (Collect, CallingCard, PersonToPerson, etc.)
  - yes I'd like to place a collect call long distance please (Collect)
  - operator I need to make a call but I need to bill it to my office (ThirdNumber)
  - yes I'd like to place a call on my master card please (CallingCard)
  - I just called a number in sioux city and I musta rang the wrong number because I got the wrong party and I would like to have that taken off of my bill (BillingCredit)
- observation:
  - easy to find "rules of thumb" that are "often" correct
    - e.g.: "IF 'card' occurs in utterance
      THEN predict 'CallingCard' "
  - hard to find single highly accurate prediction rule

The Boosting Approach

- Algorithm
  - Select a small subset of examples
  - Derive a rough rule of thumb
  - Examine 2nd set of examples
  - Derive 2nd rule of thumb
  - Repeat T times
  - Combine the learned rules into a single hypothesis
- Questions:
  - How to choose subsets of examples to examine on each round?
  - How to combine all the rules of thumb into single prediction rule?
- Boosting
  - General method of converting rough rules of thumb into highly accurate prediction rule

Theoretical Motivation

- "Strong" PAC algorithm:
  - for any distribution
  - $\forall \epsilon, \delta > 0$
  - Given polynomially many random examples
  - Finds hypothesis with error $\leq \epsilon$ with probability $\geq$ (1-δ)

- "Weak" PAC algorithm
  - Same, but only for some $\epsilon \leq$ ½ - γ

- [Kearns & Valiant '88]:
  - Does weak learnability imply strong learnability?
  - Anecdote: the importance of the distribution free assumption
    - It does not hold if PAC is restricted to only the uniform distribution, say

History

- [Schapire ’89]:
  - First provable boosting algorithm
  - Call weak learner three times on three modified distributions
  - Get slight boost in accuracy
  - apply recursively
- [Freund ’90]:
  - “Optimal” algorithm that “boosts by majority”
- [Drucker, Schapire & Simard ’92]:
  - First experiments using boosting
  - Limited by practical drawbacks
- [Freund & Schapire ’95]:
  - Introduced “AdaBoost” algorithm
  - Strong practical advantages over previous boosting algorithms
- AdaBoost was followed by a huge number of papers and practical applications

Some lessons for Ph.D. students

A Formal View of Boosting

- Given training set $(x_1, y_1), \ldots (x_m, y_m)$
- $y_i \in \{-1, +1\}$ is the correct label of instance $x_i \in X$
- For $t = 1, \ldots, T$
  - Construct a distribution D_t on $\{1,\ldots m\}$
  - Find weak hypothesis ("rule of thumb")

    $$h_t : X \to \{-1, +1\}$$

    with small error ϵ_t on D_t:

    $$\epsilon_t = Pr_D [h_t (x_i) \neg= y_i]$$
- Output: final hypothesis H_{final}

Adaboost

- Constructing D_t on {1,…m}:
  - $D_1(i) = 1/m$
  - Given D_t and h_t :
  - $D_{t+1} = D_t(i)/z_t \times e^{-\alpha_t}$ if $y_i = h_t(x_i)$

    $D_t(i)/z_t \times e^{+\alpha_t}$ if $y_i \neg= h_t(x_i)$

    $= D_t(i)/z_t \times \exp(-\alpha_t\, y_i\, h_t(x_i))$

    where z_t = normalization constant

    and

    $\alpha_t = ½ \ln\{ (1 - \epsilon_t)/\epsilon_t \}$

Think about unwrapping it all the way to 1/m

$Z_t = \sum_i D_t(i) \exp(-\alpha_t\, y_i\, h_t(x_i))$

< 1; smaller weight

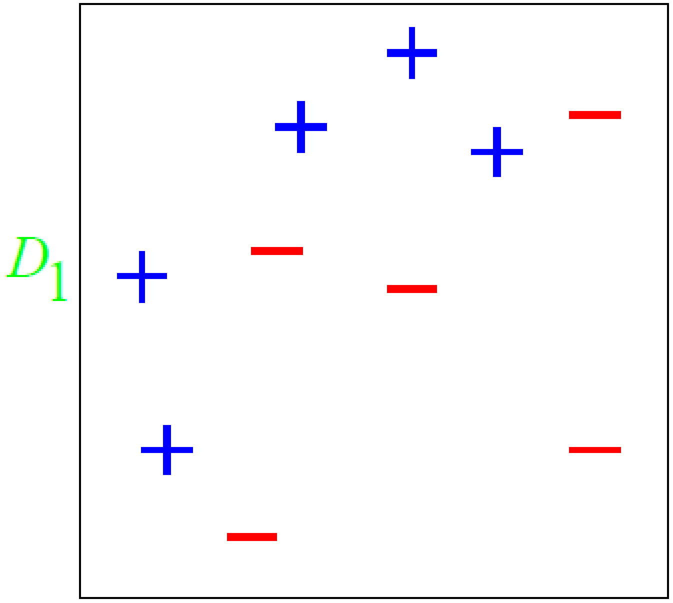
> 1; larger weight

Notes about α_t: $e^{+\alpha_t} = \mathrm{sqrt}\{(1 - \epsilon_t)/\epsilon_t\} > 1$

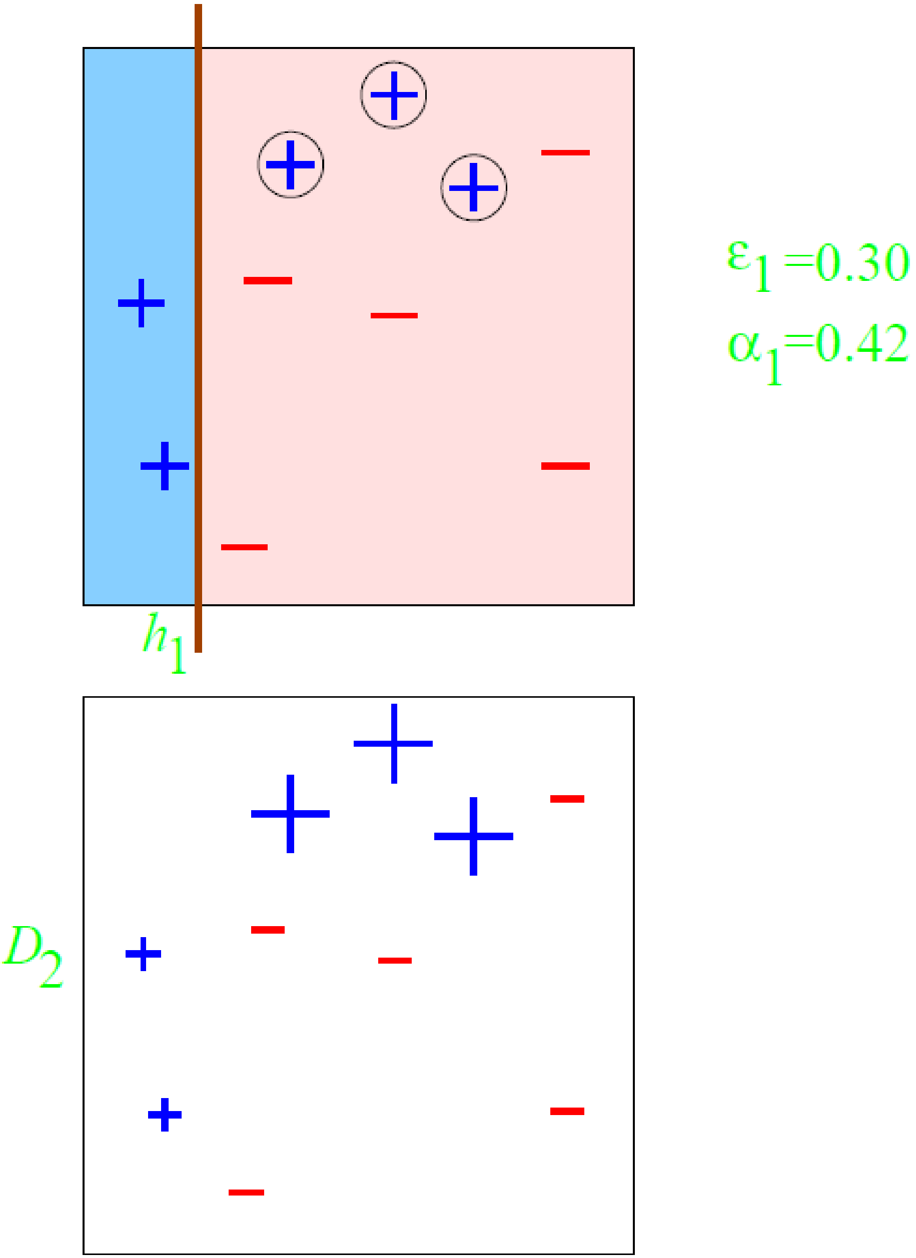
- Positive due to the weak learning assumption
- Examples that we predicted correctly are demoted, others promoted
- Sensible weighting scheme: better hypothesis (smaller error) → larger weight

- Final hypothesis: $H_{final}(x) = \mathrm{sign}(\sum_t \alpha_t h_t(x))$

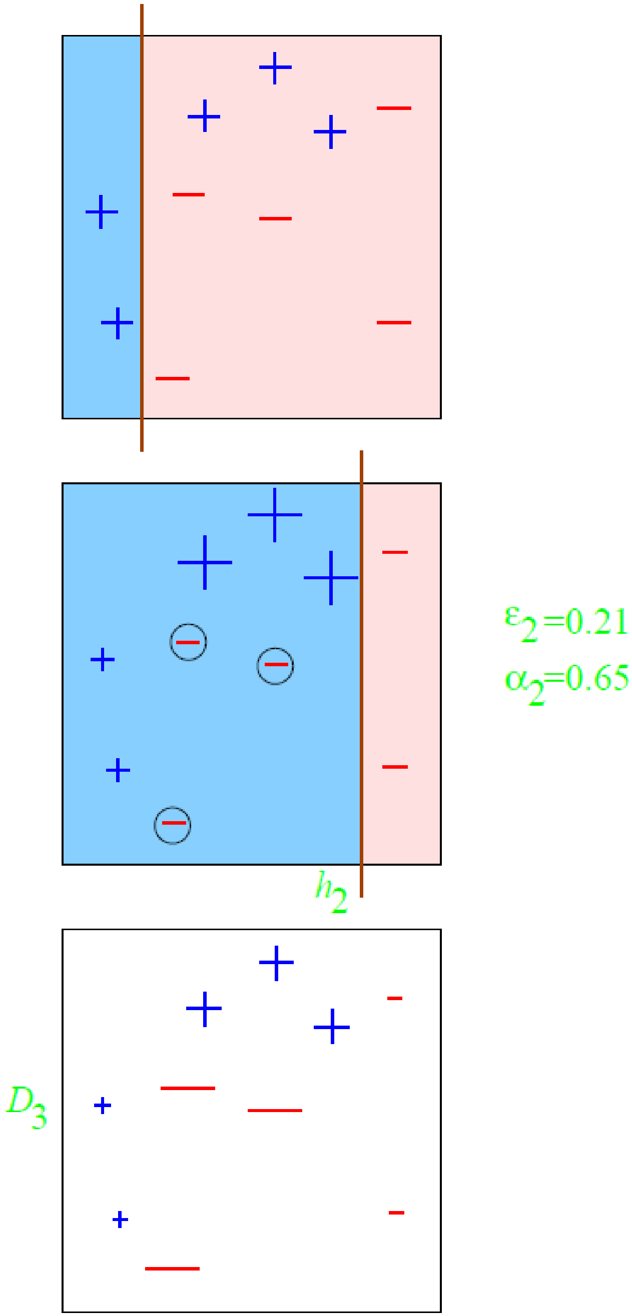
A Toy Example



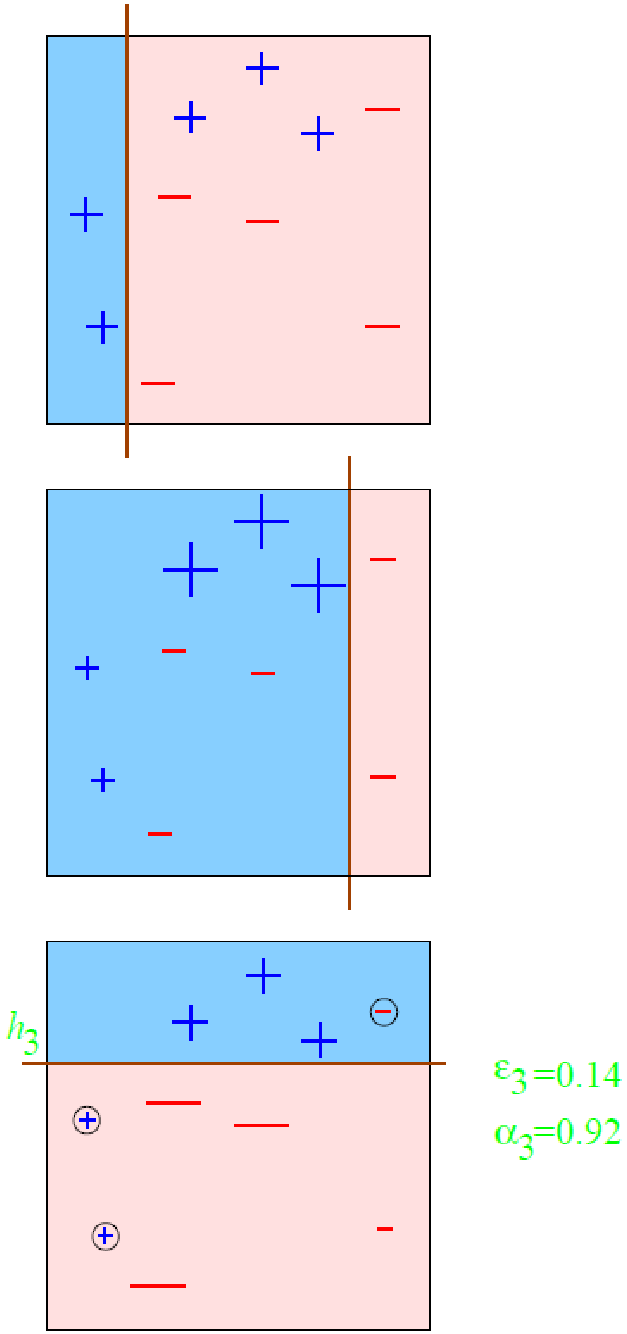
Round 1



Round 2

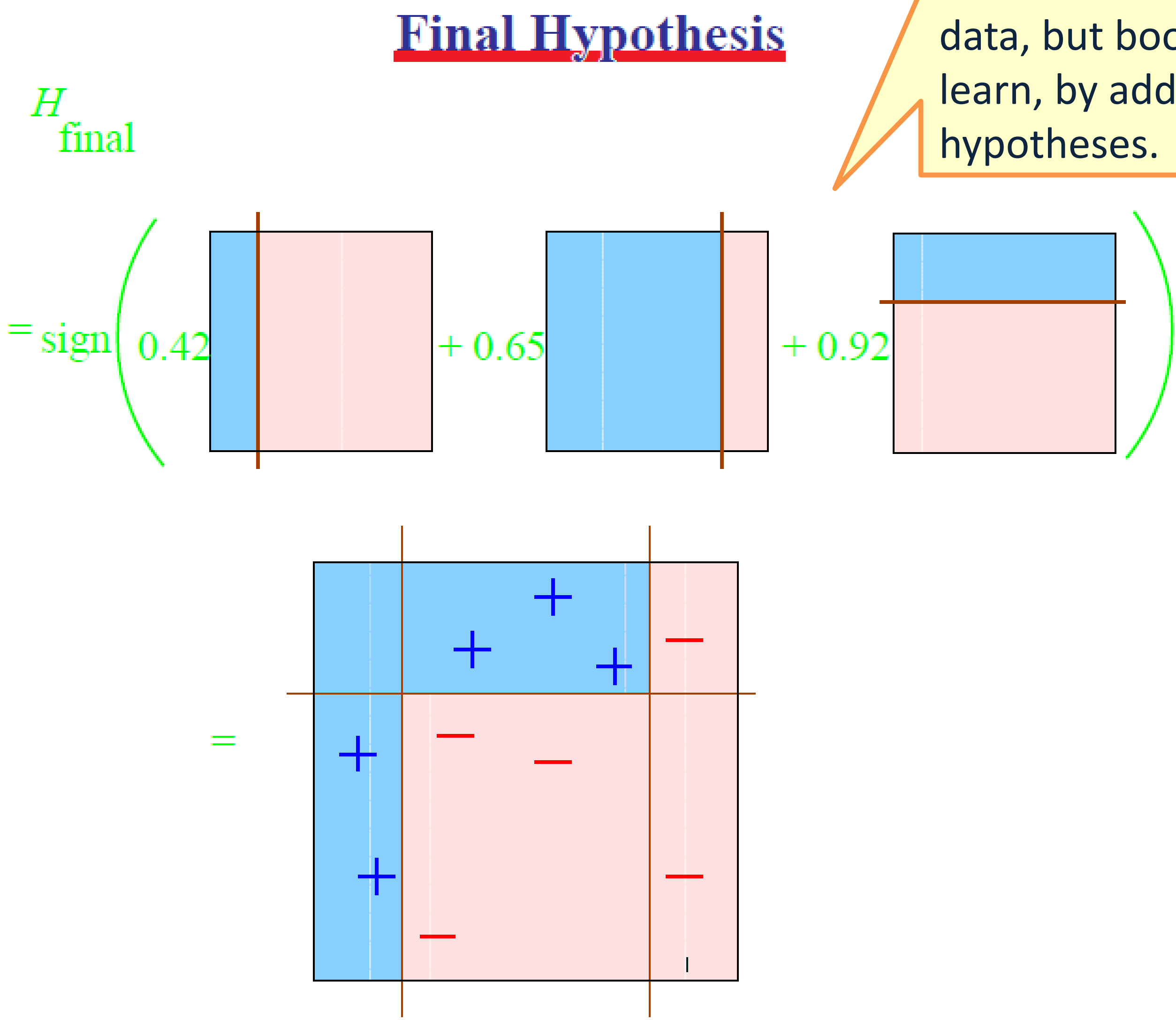


Round 3



A Toy Example

A cool and important note about the final hypothesis: it is possible that the combined hypothesis makes no mistakes on the training data, but boosting can still learn, by adding more weak hypotheses.



Analyzing Adaboost

- Theorem:
  - run AdaBoost
  - let $\epsilon_t = 1/2 - \gamma_t$
  - then

1. Why is the theorem stated in terms of minimizing training error? Is that what we want?
2. What does the bound mean?

$$\text{training error}(H_{\text{final}}) \le \prod_t \left[2\sqrt{\epsilon_t(1-\epsilon_t)}\right]$$
$$= \prod_t \sqrt{1-4\gamma_t^2}$$
$$\le \exp\left(-2\sum_t \gamma_t^2\right)$$

Need to prove only the first inequality, the rest is algebra.

$\epsilon_t (1- \epsilon_t) = (1/2-\gamma_t)(1/2+\gamma_t)) = 1/4 - \gamma_t^2$

$1-(2\gamma_t)^2 \le \exp(-(2\gamma_t)^2)$

- so: if $\forall t: \gamma_t \ge \gamma > 0$
  then $\text{training error}(H_{\text{final}}) \le e^{-2\gamma^2 T}$
- adaptive:
  - does not need to know γ or T a priori
  - can exploit $\gamma_t \gg \gamma$

AdaBoost Proof (1)

Need to prove only the first inequality, the rest is algebra.

- let $f(x) = \sum_t \alpha_t h_t(x) \Rightarrow H_{\text{final}}(x) = \text{sign}(f(x))$
- *Step 1*: unwrapping recursion:

The final "weight" of the i-th example

$$D_{\text{final}}(i) = \frac{1}{m} \cdot \frac{\exp\left(-y_i \sum_t \alpha_t h_t(x_i)\right)}{\prod_t Z_t}$$

$$= \frac{1}{m} \cdot \frac{e^{-y_i f(x_i)}}{\prod_t Z_t}$$

AdaBoost Proof (2)

- *Step 2*: training error$(H_{\text{final}}) \leq \prod_t Z_t$
- Proof:
  - $H_{\text{final}}(x) \neq y \Rightarrow yf(x) \leq 0 \Rightarrow e^{-yf(x)} \geq 1$
  - so:

$$\text{training error}(H_{\text{final}}) = \frac{1}{m}\sum_i \begin{cases} 1 & \text{if } y_i \neq H_{\text{final}}(x_i) \\ 0 & \text{else} \end{cases}$$

$$\leq \frac{1}{m}\sum_i e^{-y_i f(x_i)}$$

$$= \sum_i D_{\text{final}}(i) \prod_t Z_t$$

$$= \prod_t Z_t$$

The definition of training error

Always holds for mistakes (see above)

Using Step 1

D is a distribution over the m examples

Why does it work?
The Weak Learning Hypothesis

AdaBoost Proof(3)

A strong assumption due to the "for all distributions". But – works well in practice

- *Step 3*: $Z_t = 2\sqrt{\epsilon_t(1-\epsilon_t)}$
- Proof:

By definition of Z_t; it's a normalization term

$$Z_t = \sum_i D_t(i) \exp(-\alpha_t \, y_i \, h_t(x_i))$$

Splitting the sum to "mistakes" and no-mistakes"

$$= \sum_{i:y_i \neq h_t(x_i)} D_t(i) e^{\alpha_t} + \sum_{i:y_i = h_t(x_i)} D_t(i) e^{-\alpha_t}$$

The definition of ϵ_t

$$= \epsilon_t \, e^{\alpha_t} + (1-\epsilon_t) \, e^{-\alpha_t}$$

The definition of α_t

$e^{+\alpha_t}$ = sqrt{(1 - ϵ_t)/ϵ_t }>1

$$= 2\sqrt{\epsilon_t(1-\epsilon_t)}$$

Steps 2 and 3 together prove the Theorem.
→ The error of the final hypothesis can be as low as you want.

Boosting The Confidence

- Unlike Boosting the accuracy (ε), Boosting the confidence (δ) is easy.

- Let's fix the accuracy parameter to ε.
- Suppose that we have a learning algorithm L such that for any target concept $c \in C$ and any distribution D, L outputs h s.t. $error(h) < \varepsilon$ with confidence at least $1 - \delta_0$, where $\delta_0 = 1/q(n, size(c))$, for some polynomial q.

- Then, if we are willing to tolerate a slightly higher hypothesis error, $\varepsilon + \gamma$ ($\gamma > 0$, arbitrarily small) then we can achieve arbitrary high confidence $1 - \delta$.

Boosting The Confidence(2)

- Idea: Given the algorithm L, we construct a new algorithm L' that simulates algorithm L k times (k will be determined later) on independent samples from the same distribution

- Let $h_1, \ldots h_k$ be the hypotheses produced. Then, since the simulations are independent, the probability that all of $h_1,. h_k$ have error $>\varepsilon$ is as most $(1-\delta_0)^k$. Otherwise, at least one h_j is good.
- Solving $(1-\delta_0)^k < \delta/2$ yields that value of k we need,

$$k > (1/\delta_0) \ln(2/\delta)$$

- There is still a need to show how L' works. It would work by using the h_i that makes the fewest mistakes on the sample S; we need to compute how large S should be to guarantee that it does not make too many mistakes. [Kearns and Vazirani's book]

Summary of Ensemble Methods

- Boosting
- Bagging
- Random Forests

Boosting

- Initialization:
  - Weigh all training samples equally
- Iteration Step:
  - Train model on (weighted) train set
  - Compute error of model on train set
  - Increase weights on training cases model gets wrong!!!
- Typically requires 100's to 1000's of iterations
- Return final model:
  - Carefully weighted prediction of each model

Boosting: Different Perspectives

- Boosting is a maximum-margin method
  (Schapire et al. 1998, Rosset et al. 2004)
  - Trades lower margin on easy cases for higher margin on harder cases

- Boosting is an additive logistic regression model (Friedman, Hastie and Tibshirani 2000)
  - Tries to fit the logit of the true conditional probabilities

- Boosting is an *equalizer*
  (Breiman 1998) (Friedman, Hastie, Tibshirani 2000)
  - Weighted proportion of times example is misclassified by base learners tends to be the same for all training cases

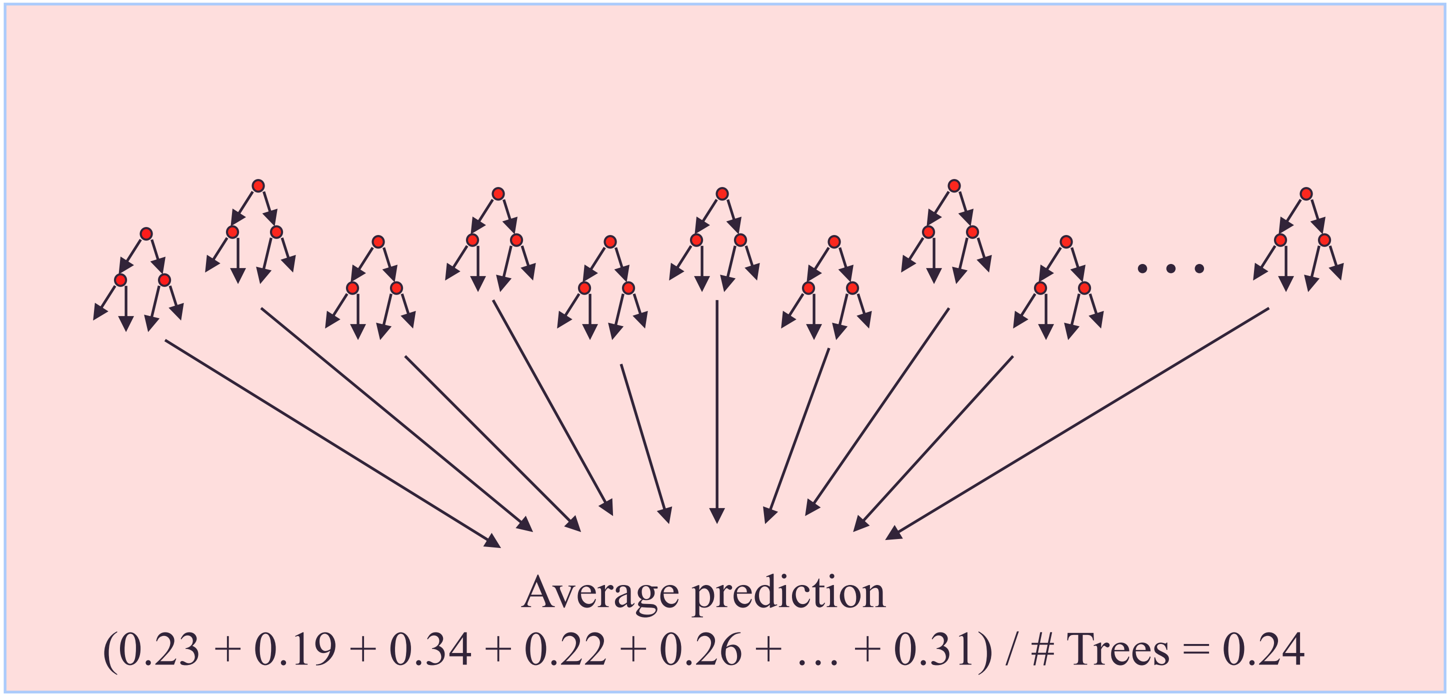
- Boosting is a linear classifier, but does not give well calibrated probability estimate.

Bagging

- Bagging predictors is a method for generating multiple versions of a predictor and using these to get an aggregated predictor.
- The aggregation averages over the versions when predicting a numerical outcome and does a plurality vote when predicting a class.
- The multiple versions are formed by making bootstrap replicates of the learning set and using these as new learning sets.
  - That is, use samples of the data, with repetition
- Tests on real and simulated data sets using classification and regression trees and subset selection in linear regression show that bagging can give substantial gains in accuracy.
- The vital element is the instability of the prediction method. If perturbing the learning set can cause significant changes in the predictor constructed then bagging can improve accuracy.

Example: Bagged Decision Trees

- Draw 100 bootstrap samples of data
- Train trees on each sample → 100 trees
- Average prediction of trees on out-of-bag samples



Random Forests (Bagged Trees++)

- Draw ***1000+*** bootstrap samples of data
- ***Draw sample of available attributes at each split***
- Train trees on each sample/attribute set → ***1000+*** trees
- Average prediction of trees on out-of-bag samples