CIS 1600 Recitation 8
Expectation, Rooted Trees

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Expectation

- The weighted average (proportional to the probabilities) of the possible values of $X$.

- $\mathbb{E}[X]$ is the value we would expect to obtain if we repeated a random experiment many times and took the average of the outcomes of $X$.

$$\mathbb{E}[X] = \sum_i i \cdot Pr[X = i]$$
Linearity of Expectation (LOE)

- The expectation of the sum of random variables equals the sum of their expectations.

- For random variables $X_1, X_2, \ldots, X_n$ on the same probability space $\Omega$ and $c_1, c_2, \ldots, c_n \in \mathbb{R}$

$$E\left[\sum_{i=1}^{n} c_i X_i\right] = \sum_{i=1}^{n} c_i E[X_i]$$

- Random variables do not have to be independent.
Rooted Trees

- A tree in which one vertex is distinguished from the others and is called the root.
- The level of a vertex $v$ is the number of edges along the unique path between $v$ and the root.
- The height of a rooted tree is the maximum level of any vertex in the tree.
The children of $v$ are neighbors of $v$ that are one level away from the root than $v$. If $u$ is a child of $v$, then $v$ is the parent of $u$. If two vertices share the same parent, they are siblings.

Given vertices $v$ and $w$, if $v$ lies on the unique path between $w$ and the root, then $v$ is an ancestor of $w$ and $w$ is a descendant of $v$. 