Problem 1:
We have three wooden buckets, $A, B, C$ and we throw $n \geq 3$ metal keys in them. The key throws are mutually independent and each key is equally likely to land in each of the three buckets.

(a) Let $A$ be the event that after all keys are thrown, bucket $A$ has at least one key in it and similarly associate an event $B$ with $B$. Are $A$ and $B$ independent? Justify your answer.

(b) Compute the probability that after all keys are thrown, each of the three buckets has at least one key in it. Justify your answer.
Problem 2: Let $T$ be a tree where the maximum degree is $\Delta$. Prove that $T$ has at least $\Delta$ leaves by contradiction.