CIS 1600
Recitation Guide - Week 5

Topics Covered: Strong Induction, Intro to Graphs, Intro to Probability

Problem 1: Suppose we have the following sequence:

\[ a_1 = 1 \quad a_2 = 3 \quad a_i = a_{i-2} + 2a_{i-1}, \quad i \in \mathbb{Z}, i \geq 3 \]

Use induction to prove that for all integers \( n \geq 1 \), \( a_n \) is odd.
Problem 2: Our favorite head TAs Andrew and Ishaan are playing a game in which there are two non-empty bags with an equal number of marbles in them. In this game, the two players take turns removing marbles from one of the bags. In each turn, the player can remove any positive number of marbles as long as they are all from the same bag. The winner of the game is the player that removes the last marble. In Andrew and Ishaan’s current configuration, both bags initially start with the same number of marbles. Prove that one of them can guarantee to always win.
Problem 3: Compute the probability of the event “when we roll $n$ (distinguishable) fair dice, any $k$ of the dice show a 4 while the other $n-k$ do not show a 4”. Assume $n \geq 3$ and $\frac{n}{2} < k < n$. 