Problem 1:

Prove using induction that for any positive integer $n$ and for any integers $d_0, d_1, \ldots, d_{n-1} \in [0..9]$ we have:

$$\sum_{j=0}^{n-1} d_j \cdot 10^j < 10^n$$
Problem 2:

Give a combinatorial proof for the following:

\[ 2^n - 1 = \sum_{k=0}^{n-1} 2^k \]
Problem 3: Cindy is proctoring a makeup exam and she needs to distribute scratch paper and pens to students. She starts with 10 sheets of scratch paper and 10 pens. She begins handing out the paper and pens to the students, but after the 6th student, Cindy discovers that she has run out of supplies. Most importantly, she does not remember when her supplies ran out (meaning she could have given all of her supplies to the first student). She cannot tell the difference between any two sheets of scratch paper and between any two pens. However, she can easily tell the difference between a sheet of scratch paper and a pen.

How many ways could Cindy have distributed the scratch paper and pens to the different students?