Topics Covered: Relations, Probabilistic Method

Problem 1:
Define an equivalence relation \( R \) on the set \( \{1, 2, 3, \ldots, 100\} \) with the restriction that there are exactly 2 equivalence classes. Find an \( R \) such that it maximizes the size of the relation, and then show that the size is maximized.
Problem 2:

Consider a set $A$ with $n \geq 1$ elements. We color independently each of the elements of $A$ red with probability $\frac{1}{3}$ and blue with probability $\frac{2}{3}$. Let $R$ be the “is the same color as” relation on $A$, i.e. if $a$ is the same color as $b$, then $(a, b) \in R$.

a) Is $R$ an equivalence relation? If so, what are its equivalence classes?

b) Calculate the expected value of $|R|$. 

Problem 3:
Let $G$ be a bipartite graph with $|V| = n$. Suppose you give each vertex its own list more than $\log_2 n$ possible colors. Show that it is possible to provide a valid coloring of $G$ choosing each vertex’s color from the list.